A Quantitative Detectability Bound for Positive-Energy Soliton Metrics in Laboratory-Scale RF Cavities

Omar Perez
Independent Researcher
omarp5252@gmail.com

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Abstract

We extend the Lentz positive-energy soliton metric to a laboratory-realistic configuration: a cylindrical RF cavity in the TM_{010} mode with a cryogenic rutile dielectric core. We compute the electromagnetic stress-energy tensor, insert it into the weak-field Lentz relation, and derive the fractional optical path difference along the cavity axis. At 1 kW input power, the predicted signal is $\delta L/L \sim 10^{-42}$ (moving soliton) to 10^{-84} (static soliton), approximately 19 to 61 orders of magnitude below achievable interferometric sensitivity. We conclude that the positive-energy soliton branch is empirically closed at laboratory energy scales on Earth. This is a quantitative boundary result, not an experimental failure.

1 Introduction

The Alcubierre metric [1] demonstrated that general relativity permits superluminal travel in principle, at the cost of requiring negative energy density. Lentz [2] proposed a positive-energy soliton solution that avoids exotic matter; a follow-up summary of this framework is given in [3]. Whether such a metric can be produced or detected at laboratory scales has not, to our knowledge, been quantitatively bounded for a concrete RF cavity configuration. This note supplies that bound.

Related no-go results for low-energy warp configurations have been discussed by Rodal [4], and the quantum-inequality constraints underlying the negative-energy branch are reviewed by Ford and Roman [5]. The present work addresses the complementary positive-energy branch at laboratory scales.

2 Electromagnetic Field Configuration

Consider a cylindrical cavity of radius R and length L_{core}, filled with cryogenic single-crystal rutile ($\varepsilon_r \sim 100$), excited in the TM_{010} mode. The axial electric field is

$$E_z(r) = E_0\, J_0\Big(\frac{x_{01} r}{R}\Big), \tag{1}$$

with

$$x_{01} = 2.405. \tag{2}$$

The azimuthal magnetic field is

$$H_\phi(r) = -i\,\frac{\varepsilon_0 \varepsilon_r \omega_0}{k_r}\, E_0\, J_0'\Big(\frac{x_{01} r}{R}\Big), \tag{3}$$

where

$$k_r = \frac{x_{01}}{R}. \tag{4}$$

The resonant frequency is

$$f_0 = \frac{c\, x_{01}}{2\pi R\sqrt{\varepsilon_r}}. \tag{5}$$

The stored energy is set by input power P_0 and quality factor Q:

$$U = \frac{P_0 Q}{\omega_0} = \frac{1}{2}\int_V \left(\varepsilon_0\varepsilon_r|E_z|^2 + \mu_0|H_\phi|^2\right) dV. \tag{6}$$

For $P_0 = 1$ kW, $Q \sim 10^4$, $R = 0.1$ m, and $\varepsilon_r = 100$, the resulting field amplitude is

$$E_0 \sim 10^5 \text{ to } 10^6 \text{ V/m}. \tag{7}$$

3 Stress-Energy Tensor

The energy density is

$$T_{00}(r) = \frac{1}{2}\left(\varepsilon_0\varepsilon_r|E_z|^2 + \mu_0|H_\phi|^2\right). \tag{8}$$

Using $\mu_0\varepsilon_0 = 1/c^2$ and $\omega_0/k_r = c/\sqrt{\varepsilon_r}$, this becomes

$$T_{00}(r) = \frac{1}{2}\varepsilon_0\varepsilon_r E_0^2 \left\{ \left[J_0\Big(\frac{x_{01}r}{R}\Big)\right]^2 + \left[J_0'\Big(\frac{x_{01}r}{R}\Big)\right]^2 \right\}. \tag{9}$$

The volume-averaged estimate is

$$\bar{T}_{00} \sim \frac{1}{2}(8.85\times10^{-12})(100)(10^{12}) \sim 4\times10^2 \text{ J/m}^3. \tag{10}$$

4 Lentz Metric and Shift Vector

The Lentz positive-energy soliton metric in Natario form (natural units $G = c = 1$ temporarily) is

$$ds^2 = -\left(N^2 - N_i N^i\right)dt^2 - 2N_i\, dx^i dt + \delta_{ij}dx^i dx^j, \tag{11}$$

with

$$N = 1, \qquad N_i = \partial_i\phi. \tag{12}$$

The shift potential ϕ satisfies the hyperbolic equation

$$\partial_x^2\phi + \partial_y^2\phi - \frac{2}{v_h^2}\partial_z^2\phi = \rho. \tag{13}$$

The Eulerian energy density is

$$E = \frac{1}{16\pi}\left[2\partial_z^2\phi\left(\rho + \frac{2}{v_h^2}\partial_z^2\phi\right) - 4\left(\partial_z\partial_x\phi\right)^2\right]. \tag{14}$$

In the weak-field, quasi-static limit ($v_h \to \infty$), this reduces to the Poisson-type relation

$$\nabla^2\phi \simeq \frac{8\pi G}{c^4}T_{00}. \tag{15}$$

The order-of-magnitude solution for source scale R is

$$\phi \sim \left(\frac{8\pi G}{c^4}\right)\bar{T}_{00}R^2. \tag{16}$$

Using

$$\frac{G}{c^4} \simeq 8.26\times10^{-45} \text{ s}^2/(\text{kg}\cdot\text{m}), \tag{17}$$

we obtain

$$\phi \sim (8\pi)(8.26 \times 10^{-45})(4 \times 10^{2})(10^{-2}) \sim 8 \times 10^{-43}. \tag{18}$$

The shift vector component is

$$N_z \sim \partial_z \phi \sim \frac{\phi}{L_{\text{core}}} \sim 10^{-42}\ \text{m}^{-1}. \tag{19}$$

5 Fractional Optical Path Difference

The first-order correction to the coordinate speed of light along the axial direction is

$$c_{\parallel} \simeq c\,(1 - N_z)\,. \tag{20}$$

The fractional optical path difference is

$$\left(\frac{\delta L}{L}\right)_{\text{predicted}} \simeq \frac{1}{L_{\text{opt}}} \int_{\text{axial}} N_z\, dl \sim \langle N_z \rangle. \tag{21}$$

Therefore

$$\left(\frac{\delta L}{L}\right)_{\text{predicted}} \sim 10^{-42} \quad (v_s \sim c), \tag{22}$$

and for a static soliton ($v_s = 0$), the first-order effect vanishes, giving

$$\left(\frac{\delta L}{L}\right)_{\text{2nd}} \sim \phi^2 \sim 10^{-84}. \tag{23}$$

6 Detectability Criterion

The required differential phase resolution is

$$\delta(\Delta\phi)_{\text{req}} = \frac{2\pi}{\lambda} L_{\text{eff}} \left(\frac{\delta L}{L}\right)_{\text{predicted}}. \tag{24}$$

With

$$\lambda = 1064\ \text{nm} = 1.064 \times 10^{-6}\ \text{m}, \tag{25}$$

$$\mathcal{F} = 10^{5}, \tag{26}$$

$$L_{\text{opt}} = 1\ \text{m}, \tag{27}$$

the effective optical path is

$$L_{\text{eff}} = \frac{2\mathcal{F} L_{\text{opt}}}{\pi} \sim 6.4 \times 10^{4}\ \text{m}. \tag{28}$$

The required resolution is

$$\delta(\Delta\phi)_{\text{req}} \sim \frac{2\pi}{1.064 \times 10^{-6}} (6.4 \times 10^{4})(10^{-42}) \sim 4 \times 10^{-31}\ \text{rad}. \tag{29}$$

The achievable sensitivity (best case) is

$$\sigma_{\text{shot}} \sim 10^{-10}\ \text{rad}/\sqrt{\text{Hz}}, \tag{30}$$

$$T_{\text{int}} \sim 10^{4}\ \text{s}, \tag{31}$$

$$\delta(\Delta\phi)_{\text{ach}} \sim \frac{\sigma_{\text{shot}}}{\sqrt{T_{\text{int}}}} \sim 10^{-12}\ \text{rad}. \tag{32}$$

The criterion is

$$\delta(\Delta\phi)_{\text{req}} \ll \delta(\Delta\phi)_{\text{ach}}, \tag{33}$$

with a margin of

$$\sim 19\ \text{orders of magnitude} \quad (v_s \sim c), \tag{34}$$

$$\sim 61\ \text{orders of magnitude} \quad (v_s = 0). \tag{35}$$

7 Conclusion

For a 1 kW-class RF cavity with a rutile dielectric core in the TM_{010} mode, the anisotropic optical path difference predicted by the Lentz positive-energy soliton metric is

$$\frac{\delta L}{L} \sim 10^{-42} \quad \text{(upper bound, moving soliton)}, \tag{36}$$

and for a static soliton the first-order effect vanishes, giving

$$\frac{\delta L}{L} \sim 10^{-84} \quad \text{(static soliton)}. \tag{37}$$

The corresponding phase signal is approximately 19 orders of magnitude below the best achievable interferometric sensitivity for the moving case, and 61 orders of magnitude below for the static case. This result follows from the magnitude of the gravitational coupling constant G/c^4 and is independent of specific experimental optimizations.

We therefore conclude that the positive-energy soliton branch is empirically closed at laboratory energy scales on Earth.

Data Availability

All numerical values are analytical order-of-magnitude estimates based on the publicly available Lentz [2] metric formulation and the standard TM_{010} cavity field solution. No experimental data are used.

References

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[4] J. Rodal, "Can Low-Energy Warp Drives Be Achieved via Metamaterial Gravitational Coupling?" (2025).

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