Three Interpretations of the Dirac Spinor: Representation Theory, Geometry, and a Primitive Chiral Doublet

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Abstract

Three apparently independent lines of investigation suggest that the four-component Dirac spinor contains a deeper two-state structure than is usually recognised. Peter Woit [1] observed that a Standard Model lepton generation can be identified with components of the spinor representation associated with a natural $U(2)$ structure. Robert A. Wilson has argued [2] that the Dirac algebra possesses an intrinsic doubled geometry associated with $\mathfrak{so}(4,2)$. Independently, a vacuum-polarisation model of particle structure suggests that the primitive physical states are not the conventional electron chiralities but instead the pair (ν_L, e_R) [3]. We compare these three approaches and show that, although they arise from very different motivations, they all focus attention on a common two-state structure underlying the Dirac equation. This suggests that the familiar decomposition of the Dirac spinor into left- and right-handed components may not be the unique physical interpretation of the underlying mathematics.

1 Introduction

The Dirac equation is traditionally interpreted as describing a single fermion possessing two chiral components,

$$\Psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}. \tag{1}$$

This interpretation is so deeply embedded in modern particle theory that it is rarely questioned. However, neither the Clifford algebra nor the gamma matrices themselves specify the physical meaning of the two-component decomposition. They merely require that the spinor space possess a doubled structure. Three distinct approaches have recently highlighted this point. The first is Peter Woit's observation that the spinor representation associated with a natural $U(2)$ structure contains precisely the quantum numbers of one Standard Model lepton generation. The second is Robert Wilson's investigation of the Dirac algebra as a real form of $\mathfrak{so}(4,2)$, suggesting that the doubled nature of Dirac spinors reflects a deeper geometric decomposition. The third is a vacuum-polarisation interpretation in which the

primitive physical states are identified not as two chiralities of one particle but as the physically distinct states

$$(\nu_L, e_R).$$

The aim of this paper is to compare these interpretations and identify their common mathematical core.

2 Woit's $U(2)$ Interpretation

Woit observed that selecting a complex structure within Euclidean four-space naturally introduces

$$U(2) \subset Spin^c(4).$$

The associated spinor representation can then be written as an exterior algebra

$$\Lambda^*(\mathbf{C}^2) = \Lambda^0 \oplus \Lambda^1 \oplus \Lambda^2.$$

The components can be identified schematically as

$$\Lambda^0 \leftrightarrow \nu_R, \tag{2}$$

$$\Lambda^1 \leftrightarrow (\nu_L, e_L), \tag{3}$$

$$\Lambda^2 \leftrightarrow e_R. \tag{4}$$

The remarkable feature of this construction is that an entire lepton generation emerges from spinorial representation theory alone. No dynamical assumptions are required. No masses are derived. No vacuum structure is specified. The result is therefore purely kinematical. Nevertheless it demonstrates that Standard Model particle content appears naturally inside spinor geometry.

3 Wilson's Geometric Doubling

Wilson approaches the problem from a completely different direction. Starting from the observation that the Dirac algebra may be regarded as the complex Clifford algebra associated with $\mathfrak{so}(4,2)$, he emphasises the existence of an intrinsic doubling within the algebraic structure itself. In one interpretation,

$$(4,2) = (2,1) \oplus (2,1),$$

suggesting two coupled geometric sectors. The significance of this proposal is not the specific geometric interpretation but the insistence that the doubling present in the Dirac equation reflects something physically meaningful rather than being merely an algebraic convenience. Wilson therefore addresses a different question from Woit. Woit asks:

> Why do Standard Model fermions fit naturally into spinor representations?

Wilson instead asks:

> Why does the spinor possess a doubled structure at all?

4 The Primitive Doublet Proposal

The present author approaches the problem from a third direction. Earlier work on vacuum polarisation, mass generation and confinement suggested that observed fractional quark charges are effective long-distance quantities rather than fundamental charges. In that framework the physically primitive states are argued to be

$$\nu_L, \qquad e_R. \tag{5}$$

All other observed fermions emerge from subsequent vacuum dressing. The crucial observation is that the conventional Dirac spinor

$$(\psi_L, \psi_R)$$

can therefore be reinterpreted as

$$(\nu_L, e_R). \tag{6}$$

The mathematics remains unchanged. Only the physical interpretation changes. This transforms the Dirac equation from

one particle with two chiral components

into

two primitive chiral states.

5 The Tensor Product Structure

The Weyl representation takes the familiar form

$$\gamma^0 = \sigma_1 \otimes I, \tag{7}$$
$$\gamma^i = i\sigma_2 \otimes \sigma_i. \tag{8}$$

Defining

$$\tau_i = \sigma_i \otimes I, \tag{9}$$
$$s_i = I \otimes \sigma_i, \tag{10}$$

yields

$$[\tau_i, \tau_j] = 2i\epsilon_{ijk}\tau_k, \tag{11}$$
$$[s_i, s_j] = 2i\epsilon_{ijk}s_k, \tag{12}$$
$$[\tau_i, s_j] = 0. \tag{13}$$

The Clifford algebra therefore contains two mutually commuting Pauli structures. Conventionally one writes

$$\gamma^\mu = (\psi_L, \psi_R) \otimes (\uparrow, \downarrow). \tag{14}$$

The primitive-doublet interpretation instead proposes

$$\gamma^\mu = (\nu_L, e_R) \otimes (\uparrow, \downarrow). \tag{15}$$

Nothing within the Clifford algebra itself requires the first factor to represent chirality. That assignment is an external physical interpretation. On the left-hand side of Equation (15), γ^μ represents the spacetime vector of operators (gamma matrices), whereas the right-hand side represents the decomposition of the spinor state Ψ (or the underlying spinor vector space) into an internal two-state sector and a spin SU(2) sector. We could change the left-hand side from γ^μ to Ψ or specify the state space decomposition explicitly. For example: $\Psi \in \mathcal{H}_{\text{internal}} \otimes \mathcal{H}_{\text{spin}}$. This emphasises the clear distinction between the operator algebra and the state representation space.

6 Comparison of the Three Approaches

The three constructions may be summarised as follows.

Feature	Woit	Wilson	Primitive Doublet
Focus	Representation theory	Geometry	Vacuum dynamics
Explains doubling	Partially	Yes	Assumes it
Identifies special role for ν_L	Yes	Indirectly	Yes
Identifies special role for e_R	Yes	No	Yes
Changes Dirac interpretation	No	No	Yes

The surprising feature of this comparison is that Woit's spinor analysis appears considerably closer to the physical particle content of the primitive-doublet interpretation than does Wilson's geometric approach. Conversely, Wilson provides a possible explanation for why the doubled structure exists in the first place, something neither Woit nor the present proposal addresses directly.

7 A Possible Synthesis

The three approaches suggest a natural hierarchy. At the deepest level there may exist an underlying doubled geometric structure, as suggested by Wilson. This doubled structure naturally generates a four-component spinor space possessing the tensor-product form of the Weyl representation. Within that spinor space, Woit's construction demonstrates that the representation content naturally resembles a Standard Model lepton generation. Finally, the vacuum-polarisation model proposes a specific physical interpretation of the fundamental two-state sector,

$$(\nu_L, e_R),$$

from which the observed particle spectrum emerges through dressing, mass generation and confinement. The resulting chain of ideas is

$$\mathfrak{so}(4,2) \longrightarrow \text{doubled spinor} \longrightarrow (\nu_L, e_R) \longrightarrow \text{observed fermions}.$$

Whether this sequence is mathematically correct remains unknown. However, it demonstrates that representation theory, Clifford algebra, and vacuum dynamics all appear to focus attention on the same underlying two-state structure.

8 Conclusion

The conventional interpretation of the Dirac equation identifies its two Weyl components as left- and right-handed parts of a single particle. The mathematics itself does not require this interpretation. Three independent approaches suggest alternative possibilities. Woit's $U(2)$ representation-theoretic analysis reveals a natural Standard Model-like lepton structure within spinor geometry. Wilson's work suggests that the doubled structure of the Dirac spinor reflects a deeper geometric decomposition associated with $\mathfrak{so}(4,2)$. The vacuum-polarisation approach argues that the primitive physical states may instead be (ν_L, e_R). Taken together these observations suggest that the doubled structure of the Dirac spinor may possess a more direct physical significance than is usually assumed. The central open question is therefore not whether the Dirac spinor contains a doubled structure. It plainly does. The question is what that doubling physically represents.

9 References

[1] Woit, P., https://arxiv.org/abs/hep-th/0206135

[2] Wilson, R. A., https://robwilson1.wordpress.com/2026/10/15/the-principle-of-unitarity/

[3] Cook, N. B., https://quantumfieldtheory.substack.com/p/the-final-theory-of-the-dirac-equation