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Electric field of rotating uniformly charged ball

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The scalar potential and electric field strength of uniformly charged ball rotating at constant angular velocity are calculated. The achieved calculation accuracy is determined by small terms containing the fourth power of the speed of light in denominator and can be further improved. The calculation method is based on use of spherical harmonics, which ensures rapid convergence of results. The rotation of the ball causes the electric field strength vector to deflect from the radial directions toward the poles of the ball's axis of rotation.

Keywords: scalar potential; field strength; electromagnetic field; rotating charged ball.

1. Introduction

Rotation of a uniformly charged ball significantly complicates expressions for electric field in comparison with the case when the ball is fixed and does not rotate. This is due to the fact that rotating charges have acceleration, so that the methods using Lorentz transformations or special theory of relativity as the first approximation usually give a solution in the form of infinite series of terms. For example, in [1] the method of retarded potentials was used, however, integrals appear in the solutions that are not expressed in terms of ordinary functions. This makes it difficult to obtain simple formulas that could give the required accuracy of calculations. The approximation method used in [2-3] first gives a solution for potential of the first order of smallness, and therefore requires repeated solution of equations to increase the accuracy at each step.

One of the standard methods for calculating electric potential is associated with the use of superposition principle, when the initial system of charges is divided into parts. For each part, its own potential is calculated, and the sum of parts' potentials, which in some cases is

represented as an integral, is considered as potential of the system of charges. For moving charges, it turns out to be more convenient to use additional conditions present in each problem. For example, in the presence of charged rotating bodies, it is reasonable to choose a coordinate system corresponding to the shape of a given body, and also to take into account symmetry of the system's parts. In addition, instead of direct integration of differential equations of electric field simultaneously with respect to several variables, it is easier to use the method of separation of variables, followed by integration with respect to one variable.

According to [4], the use of vector spherical polynomials in harmonic method is one of the best ways to obtain solutions for components of vector potential and magnetic field of moving charge configurations, since it provides the highest rate of convergence of solutions. The same turns out to be true in the case of calculating quantities characterizing electric field.

In this paper, our purpose is to determine scalar potential and electric field strength of a uniformly charged ball rotating at a constant angular velocity. The use of harmonic method makes it possible to find the required quantities in an explicit form, in principle, with any given accuracy. As will be shown below, expressions for potential and field strength are obtained in a simple form, are expressed in terms of elementary functions and are convenient for calculations.

2. Basic relations

In case of rotation at constant angular velocity ω, the Lorentz factor γ of an arbitrarily chosen charged particle located inside rotating uniformly charged ball, within the framework of special theory of relativity, is determined by the formula:

$$\gamma = \frac{1}{\sqrt{1-\dfrac{\omega^2 r^2 \sin^2\theta}{c^2}}}\,, \tag{1}$$

where r is the current radius, c is the speed of light, the angle θ of spherical coordinate system is measured from the axis OZ, which is the axis of rotation.

In the case under consideration, all physical quantities in the system, including scalar potential and electric field strength, do not depend on time. This simplifies equation for determining the scalar potential φ [5]:

$$\Delta\varphi = -\frac{\gamma\rho_{0q}}{\varepsilon_0}. \tag{2}$$

In (2) ρ_{0q} is invariant charge density of typical particles uniformly filling the spherical volume of system; ε_0 is vacuum permittivity.

In spherical coordinates, the Laplacian of a scalar function $f = f(r,\theta,\phi)$ has the following form:

$$\Delta f = \frac{1}{r}\frac{\partial^2}{\partial r^2}(rf) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\phi^2}. \tag{3}$$

It should be noted that the ball's uniform rotation leads to the fact that all physical quantities can depend only on the radial coordinate r and on the angle θ, but do not depend on the angle ϕ. We will substitute ϕ from (1) into (2) and will use Laplacian expression (3) for $\Delta\varphi$. Taking into account that $\frac{\partial^2\varphi}{\partial\phi^2}$, we have:

$$\frac{\partial^2(r\varphi)}{\partial r^2} + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\varphi}{\partial\theta}\right) = -\frac{\rho_{0q}r}{\varepsilon_0\sqrt{1-\frac{\omega^2 r^2\sin^2\theta}{c^2}}}. \tag{4}$$

Let us denote the external electric potential by φ_o and the potential inside rotating ball by φ_i. The equation for φ_o follows from (4), where the right-hand side must be equal to zero due to absence of charges outside the ball:

$$\frac{\partial^2(r\varphi_o)}{\partial r^2} + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\varphi_o}{\partial\theta}\right) = 0. \tag{5}$$

Equation (5) is solved in a standard way, when the potential is represented as a product of two functions, each of which depends only on one variable: $\varphi_o = R(r)Q(\theta)$. According to [6], taking into account symmetry of the problem and the potential gauge at infinity, for external potential we obtain the following:

$$\varphi_o \approx \frac{C_{20}C_{40}}{r} + \frac{C_{22}C_{52}}{r^3}\left(3\cos^2\theta - 1\right) + \frac{C_{24}C_{54}}{r^5}\left(35\cos^4\theta - 30\cos^2\theta + 3\right) + \\ + \frac{C_{26}C_{56}}{r^7}\left(231\cos^6\theta - 315\cos^4\theta + 105\cos^2\theta - 5\right). \tag{6}$$

In (6) the quantities C_{kn} are constant coefficients to be determined.

The φ_o (6) appears in the problem of a potential with azimuthal symmetry associated with shift in the angle ϕ. While the potential symmetry with respect to the angle θ between northern and southern hemispheres leads to the fact that in (6) only even Legendre polynomials are present in parentheses, with an accuracy up to constant factors.

3. The internal electric potential

The electric potential inside a rotating uniformly charged ball is equal to the sum

$$\varphi_i = \varphi_{i1} + \varphi_{i2}, \tag{7}$$

where φ_{i1} is the general solution of homogeneous equation (4), taken with zero right-hand side; φ_{i2} is the particular solution of non-homogeneous equation (4).

For φ_{i1} we can write similarly to [6]:

$$\varphi_{i1} \approx D_{40}\left(D_{10} + \frac{D_{20}}{r}\right) + D_{52}\left(D_{12}r^2 + \frac{D_{22}}{r^3}\right)\left(3\cos^2\theta - 1\right) + \\ + D_{54}\left(D_{14}r^4 + \frac{D_{24}}{r^5}\right)\left(35\cos^4\theta - 30\cos^2\theta + 3\right) + \\ + D_{56}\left(D_{16}r^6 + \frac{D_{26}}{r^7}\right)\left(231\cos^6\theta - 315\cos^4\theta + 105\cos^2\theta - 5\right). \tag{8}$$

In (8) the quantities D_{kn} are constant coefficients to be determined. Further, we will assume that $D_{22} = 0$, $D_{24} = 0$, $D_{26} = 0$, in order to avoid infinity at the center of the ball at $r = 0$. This allows us to simplify the form of solution of (8):

$$\varphi_{i1} \approx D_{40}\left(D_{10}+\frac{D_{20}}{r}\right)+D_{52}D_{12}r^2\left(3\cos^2\theta-1\right)+$$
$$+D_{54}D_{14}r^4\left(35\cos^4\theta-30\cos^2\theta+3\right)+ \qquad (9)$$
$$+D_{56}D_{16}r^6\left(231\cos^6\theta-315\cos^4\theta+105\cos^2\theta-5\right).$$

Now we will substitute φ with φ_{i2} in (4) and will seek a particular solution of the following equation:

$$\frac{\partial^2\left(r\varphi_{i2}\right)}{\partial r^2}+\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\varphi_{i2}}{\partial\theta}\right)=-\frac{\rho_{0q}r}{\varepsilon_0\sqrt{1-\dfrac{\omega^2r^2\sin^2\theta}{c^2}}}. \qquad (10)$$

In (10), we will make a change of variables in the form $\rho=r\sin\theta$ and will assume that φ_{i2} is a function of the variable ρ, that is $\varphi_{i2}=\varphi_{i2}\left(\rho\right)$. With this in mind, we can write:

$$\frac{\partial\varphi_{i2}}{\partial\theta}=\frac{\partial\varphi_{i2}}{\partial\rho}\frac{\partial\rho}{\partial\theta}=r\cos\theta\frac{\partial\varphi_{i2}}{\partial\rho}.$$

$$\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\varphi_{i2}}{\partial\theta}\right)=\frac{\partial}{\partial\theta}\left(r\sin\theta\cos\theta\frac{\partial\varphi_{i2}}{\partial\rho}\right)=$$
$$=r\cos^2\theta\frac{\partial\varphi_{i2}}{\partial\rho}-r\sin^2\theta\frac{\partial\varphi_{i2}}{\partial\rho}+r\sin\theta\cos\theta\frac{\partial\rho}{\partial\theta}\frac{\partial}{\partial\rho}\left(\frac{\partial\varphi_{i2}}{\partial\rho}\right)=$$
$$=r\cos^2\theta\frac{\partial\varphi_{i2}}{\partial\rho}-r\sin^2\theta\frac{\partial\varphi_{i2}}{\partial\rho}+r^2\sin\theta\cos^2\theta\frac{\partial^2\varphi_{i2}}{\partial\rho^2}.$$

$$\frac{\partial}{\partial r}\left(r\varphi_{i2}\right)=\varphi_{i2}+r\frac{\partial\varphi_{i2}}{\partial r}=\varphi_{i2}+r\frac{\partial\varphi_{i2}}{\partial\rho}\frac{\partial\rho}{\partial r}=\varphi_{i2}+r\sin\theta\frac{\partial\varphi_{i2}}{\partial\rho}$$

$$\frac{\partial^2}{\partial r^2}\left(r\varphi_{i2}\right)=\frac{\partial}{\partial r}\left[\frac{\partial}{\partial r}\left(r\varphi_{i2}\right)\right]=\frac{\partial}{\partial r}\left(\varphi_{i2}+r\sin\theta\frac{\partial\varphi_{i2}}{\partial\rho}\right)=$$
$$=\frac{\partial\varphi_{i2}}{\partial\rho}\frac{\partial\rho}{\partial r}+\sin\theta\frac{\partial\varphi_{i2}}{\partial\rho}+r\sin\theta\frac{\partial\rho}{\partial r}\frac{\partial}{\partial\rho}\left(\frac{\partial\varphi_{i2}}{\partial\rho}\right)=$$
$$=2\sin\theta\frac{\partial\varphi_{i2}}{\partial\rho}+r\sin^2\theta\frac{\partial^2\varphi_{i2}}{\partial\rho^2}. \tag{11}$$

An equation follows from (10-11):

$$\frac{\partial^2\varphi_{i2}}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial\varphi_{i2}}{\partial\rho}=-\frac{\rho_{0q}}{\varepsilon_0\sqrt{1-\frac{\omega^2\rho^2}{c^2}}}. \tag{12}$$

In (12), we can make a change of variables in the form $\frac{\partial\varphi_{i2}}{\partial\rho}=y\left(\rho\right)$, which leads to the first-degree equation:

$$\frac{\partial y\left(\rho\right)}{\partial\rho}+\frac{1}{\rho}y\left(\rho\right)=-\frac{\rho_{0q}}{\varepsilon_0\sqrt{1-\frac{\omega^2\rho^2}{c^2}}}. \tag{13}$$

Since the homogeneous equation

$$\frac{\partial y\left(\rho\right)}{\partial\rho}+\frac{1}{\rho}y\left(\rho\right)=0$$

has the solution $y\left(\rho\right)=\frac{D}{\rho}$, where D is a constant, then the solution of (13) will be sought in the form $y\left(\rho\right)=\frac{D\left(\rho\right)}{\rho}$, where $D\left(\rho\right)$ is a certain function of ρ.

Substitution of $y\left(\rho\right)=\frac{D\left(\rho\right)}{\rho}$ into (13) leads to the equation

$$\frac{\partial D(\rho)}{\partial \rho} = -\frac{\rho_{0q}\rho}{\varepsilon_0\sqrt{1-\frac{\omega^2\rho^2}{c^2}}},$$

and to its solution

$$D(\rho) = \frac{c^2\rho_{0q}\sqrt{1-\frac{\omega^2\rho^2}{c^2}}}{\omega^2\varepsilon_0} + C_1 \ .$$

This allows us to find the required quantities:

$$\frac{\partial \varphi_{i2}}{\partial \rho} = y(\rho) = \frac{D(\rho)}{\rho} = \frac{c^2\rho_{0q}\sqrt{1-\frac{\omega^2\rho^2}{c^2}}}{\omega^2\varepsilon_0\rho} + \frac{C_1}{\rho},$$

$$\varphi_{i2} = \frac{c^2\rho_{0q}}{\omega^2\varepsilon_0}\left[\sqrt{1-\frac{\omega^2\rho^2}{c^2}} - \ln\frac{1+\sqrt{1-\frac{\omega^2\rho^2}{c^2}}}{\frac{\omega\rho}{c}}\right] + C_1\ln\rho + C_2. \qquad (14)$$

Let us substitute $\rho = r\sin\theta$ into (14) and make the change $C_1\ln\rho + C_2 = C_3\ln\frac{\omega r\sin\theta}{c}$, where C_1, C_2 and C_3 are constant coefficients. Next, we will substitute (9) and (14) into (7) and will find potential inside the rotating charged ball:

$$\begin{aligned}
&\varphi_i \approx D_{40}\left(D_{10} + \frac{D_{20}}{r}\right) + D_{52}D_{12}r^2\left(3\cos^2\theta - 1\right) + \\
&+ D_{54}D_{14}r^4\left(35\cos^4\theta - 30\cos^2\theta + 3\right) + \\
&+ D_{56}D_{16}r^6\left(231\cos^6\theta - 315\cos^4\theta + 105\cos^2\theta - 5\right) + \\
&+ \frac{c^2\rho_{0q}}{\omega^2\varepsilon_0}\left[\sqrt{1-\frac{\omega^2 r^2\sin^2\theta}{c^2}} - \ln\left(1+\sqrt{1-\frac{\omega^2 r^2\sin^2\theta}{c^2}}\right)\right] + \left(\frac{c^2\rho_{0q}}{\omega^2\varepsilon_0} + C_3\right)\ln\frac{\omega r\sin\theta}{c}.
\end{aligned}$$

(15)

At the center of the ball at $r=0$, as well as at $\theta=0$, the potential cannot be infinite and must not depend on the angle θ. Consequently, in (15) $D_{20}=0$, $\frac{c^2\rho_{0q}}{\omega^2\varepsilon_0}+C_3=0$, and the expression for the potential is simplified:

$$\begin{aligned}\varphi_i \approx\ & D_{10}D_{40}+D_{52}D_{12}r^2\left(3\cos^2\theta-1\right)+\\ &+D_{54}D_{14}r^4\left(35\cos^4\theta-30\cos^2\theta+3\right)+\\ &+D_{56}D_{16}r^6\left(231\cos^6\theta-315\cos^4\theta+105\cos^2\theta-5\right)+\\ &+\frac{c^2\rho_{0q}}{\omega^2\varepsilon_0}\left[\sqrt{1-\frac{\omega^2r^2\sin^2\theta}{c^2}}-\ln\left(1+\sqrt{1-\frac{\omega^2r^2\sin^2\theta}{c^2}}\right)\right].\end{aligned} \tag{16}$$

4. The electric field strength

In order to calculate the electric field strength, we need to know not only the scalar potential φ, but also the vector potential $\mathbf{A}$:

$$\mathbf{E}=-\nabla\varphi-\frac{\partial\mathbf{A}}{\partial t}. \tag{17}$$

Due to rotation of the charged ball at constant angular velocity, the vector potential $\mathbf{A}$ does not depend on time, so that in (17) $\frac{\partial\mathbf{A}}{\partial t}=0$. Using the expression for gradient of arbitrary scalar function f in spherical coordinates,

$$\nabla f=\frac{\partial f}{\partial r}\mathbf{e}_r+\frac{1}{r}\frac{\partial f}{\partial\theta}\mathbf{e}_\theta+\frac{1}{r\sin\theta}\frac{\partial f}{\partial\phi}\mathbf{e}_\phi\ , \tag{18}$$

where the quantities $\mathbf{e}_r$, $\mathbf{e}_\theta$ and $\mathbf{e}_\phi$ are unit vectors of spherical coordinate system, we will substitute f in (18) with φ in order to use in (17). Next, substituting external scalar potential φ_o (6) instead of φ in (17), we will find the components of external electric field strength $\mathbf{E}_o=E_{or}\mathbf{e}_r+E_{o\theta}\mathbf{e}_\theta+E_{o\phi}\mathbf{e}_\phi$:

$$E_{or} = -\frac{\partial \varphi_o}{\partial r} \approx \frac{C_{20}C_{40}}{r^2} + \frac{3C_{22}C_{52}}{r^4}\left(3\cos^2\theta - 1\right) + \frac{5C_{24}C_{54}}{r^6}\left(35\cos^4\theta - 30\cos^2\theta + 3\right) +$$
$$+ \frac{7C_{26}C_{56}}{r^8}\left(231\cos^6\theta - 315\cos^4\theta + 105\cos^2\theta - 5\right).$$

$$E_{o\theta} = -\frac{1}{r}\frac{\partial \varphi_o}{\partial \theta} \approx \frac{6C_{22}C_{52}}{r^4}\sin\theta\cos\theta + \frac{20C_{24}C_{54}}{r^6}\sin\theta\cos\theta\left(7\cos^2\theta - 3\right) +$$
$$+ \frac{42C_{26}C_{56}}{r^8}\sin\theta\cos\theta\left(33\cos^4\theta - 30\cos^2\theta + 5\right).$$

$$E_{o\phi} = -\frac{1}{r\sin\theta}\frac{\partial \varphi_o}{\partial \phi} = 0. \tag{19}$$

Let us now substitute in (17) the value φ_i from (16) instead of φ, and, in view of (18), we will find the components of internal electric field strength $\mathbf{E}_i = E_{ir}\mathbf{e}_r + E_{i\theta}\mathbf{e}_\theta + E_{i\phi}\mathbf{e}_\phi$:

$$E_{ir} = -\frac{\partial \varphi_i}{\partial r} \approx -2D_{52}D_{12}r\left(3\cos^2\theta - 1\right) - 4D_{54}D_{14}r^3\left(35\cos^4\theta - 30\cos^2\theta + 3\right) -$$
$$-6D_{56}D_{16}r^5\left(231\cos^6\theta - 315\cos^4\theta + 105\cos^2\theta - 5\right) + \frac{\rho_{0q}r\sin^2\theta}{\varepsilon_0\left(1 + \sqrt{1 - \dfrac{\omega^2 r^2\sin^2\theta}{c^2}}\right)}.$$

$$E_{i\theta} = -\frac{1}{r}\frac{\partial \varphi_i}{\partial \theta} \approx 6D_{52}D_{12}r\sin\theta\cos\theta + 20D_{54}D_{14}r^3\sin\theta\cos\theta\left(7\cos^2\theta - 3\right) +$$
$$+42D_{56}D_{16}r^5\sin\theta\cos\theta\left(33\cos^4\theta - 30\cos^2\theta + 5\right) +$$
$$+\frac{\rho_{0q}r\sin\theta\cos\theta}{\varepsilon_0\left(1 + \sqrt{1 - \dfrac{\omega^2 r^2\sin^2\theta}{c^2}}\right)}.$$

$$E_{i\phi} = -\frac{1}{r\sin\theta}\frac{\partial \varphi_i}{\partial \phi} = 0. \tag{20}$$

The equality of components $E_{o\phi}$ in (19) and $E_{i\phi}$ in (20) to zero is due to the fact that the components of electric field strength do not depend on the angle ϕ.

5. Comparison of external and internal potentials and electric field strength components

Let us express φ_o in (6) in terms of $\sin^2\theta$, $\sin^4\theta$ and $\sin^6\theta$, using the substitution $\cos^2\theta = 1-\sin^2\theta$:

$$\begin{aligned}\varphi_o \approx{}& \frac{C_{20}C_{40}}{r}+\frac{2C_{22}C_{52}}{r^3}+\frac{8C_{24}C_{54}}{r^5}+\frac{16C_{26}C_{56}}{r^7}-\\ &-\left(\frac{3C_{22}C_{52}}{r^3}+\frac{40C_{24}C_{54}}{r^5}+\frac{168C_{26}C_{56}}{r^7}\right)\sin^2\theta+\\ &+\left(\frac{35C_{24}C_{54}}{r^5}+\frac{378C_{26}C_{56}}{r^7}\right)\sin^4\theta-\frac{231C_{26}C_{56}}{r^7}\sin^6\theta.\end{aligned} \tag{21}$$

Let us expand the square root and the logarithm in (16) as follows:

$$\sqrt{1-\frac{\omega^2 r^2\sin^2\theta}{c^2}} \approx 1-\frac{\omega^2 r^2\sin^2\theta}{2c^2}-\frac{\omega^4 r^4\sin^4\theta}{8c^4}-\frac{\omega^6 r^6\sin^6\theta}{16c^6}.$$

$$\begin{aligned}&\ln\left(1+\sqrt{1-\frac{\omega^2 r^2\sin^2\theta}{c^2}}\right)\approx \ln 2+\ln\left(1-\frac{\omega^2 r^2\sin^2\theta}{4c^2}-\frac{\omega^4 r^4\sin^4\theta}{16c^4}-\frac{\omega^6 r^6\sin^6\theta}{32c^6}\right)\approx\\ &\approx \ln 2-\frac{\omega^2 r^2\sin^2\theta}{4c^2}-\frac{3\omega^4 r^4\sin^4\theta}{32c^4}-\frac{5\omega^6 r^6\sin^6\theta}{96c^6}.\end{aligned} \tag{22}$$

Taking into account (22) and the substitution $\cos^2\theta = 1-\sin^2\theta$, the potential φ_i in (16) is transformed as follows:

$$\varphi_i \approx D_{10}D_{40} + \frac{c^2\rho_{0q}\left(1-\ln 2\right)}{\omega^2\varepsilon_0} + 2D_{52}D_{12}r^2 + 8D_{54}D_{14}r^4 + 16D_{56}D_{16}r^6 -$$
$$-\left(3D_{52}D_{12}r^2 + \frac{\rho_{0q}r^2}{4\varepsilon_0} + 40D_{54}D_{14}r^4 + 168D_{56}D_{16}r^6\right)\sin^2\theta +$$
$$+\left(35D_{54}D_{14}r^4 - \frac{\rho_{0q}\omega^2 r^4}{32c^2\varepsilon_0} + 378D_{56}D_{16}r^6\right)\sin^4\theta -$$
$$-\left(231D_{56}D_{16}r^6 + \frac{\rho_{0q}\omega^4 r^6}{96c^4\varepsilon_0}\right)\sin^6\theta. \qquad (23)$$

On the ball surface at $r = a$, the external and internal potentials of electric field become the same in magnitude. Assuming $r = a$ and equating in (21) and in (23) the sums of corresponding terms with the factors $\sin^2\theta$, $\sin^4\theta$, $\sin^6\theta$, and also equating the sums of terms without sines, from the equality of potentials we find four relations:

$$\frac{C_{20}C_{40}}{a} + \frac{2C_{22}C_{52}}{a^3} + \frac{8C_{24}C_{54}}{a^5} + \frac{16C_{26}C_{56}}{a^7} =$$
$$= D_{10}D_{40} + \frac{c^2\rho_{0q}\left(1-\ln 2\right)}{\omega^2\varepsilon_0} + 2D_{52}D_{12}a^2 + 8D_{54}D_{14}a^4 + 16D_{56}D_{16}a^6.$$

$$\frac{3C_{22}C_{52}}{a^3} + \frac{40C_{24}C_{54}}{a^5} + \frac{168C_{26}C_{56}}{a^7} = 3D_{52}D_{12}a^2 + \frac{\rho_{0q}a^2}{4\varepsilon_0} + 40D_{54}D_{14}a^4 + 168D_{56}D_{16}a^6.$$

$$\frac{35C_{24}C_{54}}{a^5} + \frac{378C_{26}C_{56}}{a^7} = 35D_{54}D_{14}a^4 - \frac{\rho_{0q}\omega^2 a^4}{32c^2\varepsilon_0} + 378D_{56}D_{16}a^6.$$

$$\frac{231C_{26}C_{56}}{a^7} = 231D_{56}D_{16}a^6 + \frac{\rho_{0q}\omega^4 a^6}{96c^4\varepsilon_0}. \qquad (24)$$

Let us now consider the electric field strength components. From (19), in view of substitution $\cos^2\theta = 1 - \sin^2\theta$, it follows:

$$E_{or} \approx \frac{C_{20}C_{40}}{r^2} + \frac{6C_{22}C_{52}}{r^4} + \frac{40C_{24}C_{54}}{r^6} + \frac{112C_{26}C_{56}}{r^8} -$$
$$-\left(\frac{9C_{22}C_{52}}{r^4} + \frac{200C_{24}C_{54}}{r^6} + \frac{1176C_{26}C_{56}}{r^8} \right) \sin^2\theta +$$
$$+\left(\frac{175C_{24}C_{54}}{r^6} + \frac{2646C_{26}C_{56}}{r^8} \right) \sin^4\theta - \frac{1617C_{26}C_{56}}{r^8} \sin^6\theta.$$

$$E_{o\theta} \approx \left(\frac{6C_{22}C_{52}}{r^4} + \frac{80C_{24}C_{54}}{r^6} + \frac{336C_{26}C_{56}}{r^8} \right) \sin\theta\cos\theta -$$
$$-\left(\frac{140C_{24}C_{54}}{r^6} + \frac{1512C_{26}C_{56}}{r^8} \right) \sin^3\theta\cos\theta + \frac{1386C_{26}C_{56}}{r^8} \sin^5\theta\cos\theta.$$

(25)

In (20), taking into account expression for the square root from (22), we will use expansion of the function in the form

$$\frac{1}{1+\sqrt{1-\dfrac{\omega^2 r^2 \sin^2\theta}{c^2}}} \approx \frac{1}{2}\left(1 + \frac{\omega^2 r^2 \sin^2\theta}{4c^2} + \frac{\omega^4 r^4 \sin^4\theta}{8c^4} \right). \tag{26}$$

Substituting (26) into (20), in view of relation $\cos^2\theta = 1 - \sin^2\theta$, we find:

$$E_{ir} \approx -4D_{52}D_{12}r - 32D_{54}D_{14}r^3 - 96D_{56}D_{16}r^5 +$$
$$+\left(6D_{52}D_{12}r + \frac{\rho_{0q}r}{2\varepsilon_0} + 160D_{54}D_{14}r^3 + 1008D_{56}D_{16}r^5 \right) \sin^2\theta -$$
$$-\left(140D_{54}D_{14}r^3 - \frac{\rho_{0q}\omega^2 r^3}{8c^2\varepsilon_0} + 2268D_{56}D_{16}r^5 \right) \sin^4\theta +$$
$$+\left(1386D_{56}D_{16}r^5 + \frac{\rho_{0q}\omega^4 r^5}{16c^4\varepsilon_0} \right) \sin^6\theta.$$

$$E_{i\theta} \approx \left(6D_{52}D_{12}r + \frac{\rho_{0q}r}{2\varepsilon_0} + 80D_{54}D_{14}r^3 + 336D_{56}D_{16}r^5 \right) \sin\theta\cos\theta -$$
$$- \left(140D_{54}D_{14}r^3 - \frac{\rho_{0q}\omega^2 r^3}{8c^2\varepsilon_0} + 1512D_{56}D_{16}r^5 \right) \sin^3\theta\cos\theta + \qquad (27)$$
$$+ \left(1386D_{56}D_{16}r^5 + \frac{\rho_{0q}\omega^4 r^5}{16c^4\varepsilon_0} \right) \sin^5\theta\cos\theta.$$

Let us substitute in (25) and in (27) the value $r = a$ and equate to each other the components of strength of external and internal electric fields on the surface of the boll. Four relations follow from the equality of components E_{or} and E_{ir}:

$$\frac{C_{20}C_{40}}{a^2} + \frac{6C_{22}C_{52}}{a^4} + \frac{40C_{24}C_{54}}{a^6} + \frac{112C_{26}C_{56}}{a^8} = -4D_{52}D_{12}a - 32D_{54}D_{14}a^3 - 96D_{56}D_{16}a^5.$$

$$-\left(\frac{9C_{22}C_{52}}{a^4} + \frac{200C_{24}C_{54}}{a^6} + \frac{1176C_{26}C_{56}}{a^8} \right) =$$
$$= 6D_{52}D_{12}a + \frac{\rho_{0q}a}{2\varepsilon_0} + 160D_{54}D_{14}a^3 + 1008D_{56}D_{16}a^5.$$

$$\frac{175C_{24}C_{54}}{a^6} + \frac{2646C_{26}C_{56}}{a^8} = -\left(140D_{54}D_{14}a^3 - \frac{\rho_{0q}\omega^2 a^3}{8c^2\varepsilon_0} + 2268D_{56}D_{16}a^5 \right).$$

$$-\frac{1617C_{26}C_{56}}{a^8} = 1386D_{56}D_{16}a^5 + \frac{\rho_{0q}\omega^4 a^5}{16c^4\varepsilon_0}. \qquad (28)$$

Three more relations are obtained from comparing $E_{o\theta}$ and $E_{i\theta}$:

$$\frac{6C_{22}C_{52}}{a^4} + \frac{80C_{24}C_{54}}{a^6} + \frac{336C_{26}C_{56}}{a^8} = 6D_{52}D_{12}a + \frac{\rho_{0q}a}{2\varepsilon_0} + 80D_{54}D_{14}a^3 + 336D_{56}D_{16}a^5.$$

$$\frac{140C_{24}C_{54}}{a^6} + \frac{1512C_{26}C_{56}}{a^8} = 140D_{54}D_{14}a^3 - \frac{\rho_{0q}\omega^2 a^3}{8c^2\varepsilon_0} + 1512D_{56}D_{16}a^5$$

$$\frac{1386C_{26}C_{56}}{a^8} = 1386D_{56}D_{16}a^5 + \frac{\rho_{0q}\omega^4 a^5}{16c^4\varepsilon_0}. \qquad (29)$$

From (24), (28) and (29) we find:

$$C_{26}C_{56} = 0, \qquad C_{24}C_{54} = \frac{\rho_{0q}\omega^4 a^{11}}{9240c^4\varepsilon_0}, \qquad C_{22}C_{52} = -\frac{\rho_{0q}\omega^2 a^7}{210c^2\varepsilon_0} - \frac{\rho_{0q}\omega^4 a^9}{315c^4\varepsilon_0}.$$

$$C_{20}C_{40} = \frac{\rho_{0q}a^3}{3\varepsilon_0} + \frac{\rho_{0q}\omega^2 a^5}{15c^2\varepsilon_0} + \frac{\rho_{0q}\omega^4 a^7}{35c^4\varepsilon_0}.$$

$$D_{56}D_{16} = -\frac{\rho_{0q}\omega^4}{22176c^4\varepsilon_0}, \qquad D_{54}D_{14} = \frac{\rho_{0q}\omega^2}{1120c^2\varepsilon_0} + \frac{\rho_{0q}\omega^4 a^2}{1680c^4\varepsilon_0}.$$

$$D_{52}D_{12} = -\frac{\rho_{0q}}{12\varepsilon_0} - \frac{\rho_{0q}\omega^2 a^2}{60c^2\varepsilon_0} - \frac{\rho_{0q}\omega^4 a^4}{140c^4\varepsilon_0}.$$

$$D_{10}D_{40} = \frac{\rho_{0q}a^2}{2\varepsilon_0} + \frac{\rho_{0q}\omega^2 a^4}{12c^2\varepsilon_0} + \frac{\rho_{0q}\omega^4 a^6}{30c^4\varepsilon_0} - \frac{c^2\rho_{0q}(1-\ln 2)}{\omega^2\varepsilon_0}. \qquad (30)$$

Substituting (30) into (6) and into (19), we determine the potential and the electric field strength components outside rotating ball in spherical coordinates:

$$\varphi_o \approx \frac{1}{r}\left(\frac{\rho_{0q}a^3}{3\varepsilon_0} + \frac{\rho_{0q}\omega^2 a^5}{15c^2\varepsilon_0} + \frac{\rho_{0q}\omega^4 a^7}{35c^4\varepsilon_0}\right) - \frac{1}{r^3}\left(\frac{\rho_{0q}\omega^2 a^7}{210c^2\varepsilon_0} + \frac{\rho_{0q}\omega^4 a^9}{315c^4\varepsilon_0}\right)\left(3\cos^2\theta - 1\right) +$$
$$+ \frac{\rho_{0q}\omega^4 a^{11}}{9240c^4\varepsilon_0 r^5}\left(35\cos^4\theta - 30\cos^2\theta + 3\right).$$

$$E_{or} \approx \frac{1}{r^2}\left(\frac{\rho_{0q}a^3}{3\varepsilon_0}+\frac{\rho_{0q}\omega^2 a^5}{15c^2\varepsilon_0}+\frac{\rho_{0q}\omega^4 a^7}{35c^4\varepsilon_0}\right)-\frac{1}{r^4}\left(\frac{\rho_{0q}\omega^2 a^7}{70c^2\varepsilon_0}+\frac{\rho_{0q}\omega^4 a^9}{105c^4\varepsilon_0}\right)\left(3\cos^2\theta-1\right)+$$
$$+\frac{\rho_{0q}\omega^4 a^{11}}{1848c^4\varepsilon_0 r^6}\left(35\cos^4\theta-30\cos^2\theta+3\right).$$

$$E_{o\theta} \approx -\frac{1}{r^4}\left(\frac{\rho_{0q}\omega^2 a^7}{35c^2\varepsilon_0}+\frac{2\rho_{0q}\omega^4 a^9}{105c^4\varepsilon_0}\right)\sin\theta\cos\theta+\frac{\rho_{0q}\omega^4 a^{11}}{462c^4\varepsilon_0 r^6}\sin\theta\cos\theta\left(7\cos^2\theta-3\right).$$

$$E_{o\phi}=0. \tag{31}$$

Let us now substitute (30) into (16) and into (20) and find the potential and the electric field strength components inside rotating ball:

$$\varphi_i \approx \frac{\rho_{0q}a^2}{2\varepsilon_0}+\frac{\rho_{0q}\omega^2 a^4}{12c^2\varepsilon_0}+\frac{\rho_{0q}\omega^4 a^6}{30c^4\varepsilon_0}+$$
$$+\frac{c^2\rho_{0q}}{\omega^2\varepsilon_0}\left[\sqrt{1-\frac{\omega^2 r^2\sin^2\theta}{c^2}}-1+\ln 2-\ln\left(1+\sqrt{1-\frac{\omega^2 r^2\sin^2\theta}{c^2}}\right)\right]-$$
$$-\left(\frac{\rho_{0q}}{12\varepsilon_0}+\frac{\rho_{0q}\omega^2 a^2}{60c^2\varepsilon_0}+\frac{\rho_{0q}\omega^4 a^4}{140c^4\varepsilon_0}\right)r^2\left(3\cos^2\theta-1\right)+$$
$$+\left(\frac{\rho_{0q}\omega^2}{1120c^2\varepsilon_0}+\frac{\rho_{0q}\omega^4 a^2}{1680c^4\varepsilon_0}\right)r^4\left(35\cos^4\theta-30\cos^2\theta+3\right)-$$
$$-\frac{\rho_{0q}\omega^4 r^6}{22176c^4\varepsilon_0}\left(231\cos^6\theta-315\cos^4\theta+105\cos^2\theta-5\right).$$

$$E_{ir} \approx \left(\frac{\rho_{0q}}{6\varepsilon_0}+\frac{\rho_{0q}\omega^2 a^2}{30c^2\varepsilon_0}+\frac{\rho_{0q}\omega^4 a^4}{70c^4\varepsilon_0}\right)r\left(3\cos^2\theta-1\right)+\frac{\rho_{0q}r\sin^2\theta}{\varepsilon_0\left(1+\sqrt{1-\frac{\omega^2 r^2\sin^2\theta}{c^2}}\right)}-$$
$$-\left(\frac{\rho_{0q}\omega^2}{280c^2\varepsilon_0}+\frac{\rho_{0q}\omega^4 a^2}{420c^4\varepsilon_0}\right)r^3\left(35\cos^4\theta-30\cos^2\theta+3\right)+$$
$$+\frac{\rho_{0q}\omega^4}{3696c^4\varepsilon_0}r^5\left(231\cos^6\theta-315\cos^4\theta+105\cos^2\theta-5\right).$$

$$E_{i\theta} \approx -\left(\frac{\rho_{0q}}{2\varepsilon_0}+\frac{\rho_{0q}\omega^2 a^2}{10c^2\varepsilon_0}+\frac{3\rho_{0q}\omega^4 a^4}{70c^4\varepsilon_0}\right) r\sin\theta\cos\theta+\frac{\rho_{0q} r\sin\theta\cos\theta}{\varepsilon_0\left(1+\sqrt{1-\frac{\omega^2 r^2\sin^2\theta}{c^2}}\right)}+$$

$$+\left(\frac{\rho_{0q}\omega^2}{56c^2\varepsilon_0}+\frac{\rho_{0q}\omega^4 a^2}{84c^4\varepsilon_0}\right) r^3\sin\theta\cos\theta\left(7\cos^2\theta-3\right)-$$

$$-\frac{\rho_{0q}\omega^4}{528c^4\varepsilon_0} r^5\sin\theta\cos\theta\left(33\cos^4\theta-30\cos^2\theta+5\right).$$

$$E_{i\phi}=0. \tag{32}$$

6. Conclusion

As can be seen in (31-32), the ball's rotation leads to appearance of the spherical component $E_{o\theta}$ and $E_{i\theta}$, which are not equal to zero. This means that due to rotation, the direction of electric field strength vector ceases to be purely radial, in contrast to situation with a non-rotating uniformly charged ball.

The appearance in (31-32) of spherical functions expressed in terms of $\sin\theta$ and $\cos\theta$ is associated with the solution of equation (4) for scalar electric potentials inside and outside a ball. In order to obtain the corresponding solutions, we used the method of separation of variables and the method, in which e undetermined coefficients are replaced by auxiliary functions of spherical coordinates. This makes it possible to find all the unknown quantities and explicitly determine all potentials and the components of electric field strengths without making any additional assumptions other than using the ball symmetry.

The accuracy of solutions of (31-32) is determined by small terms containing c^4 in denominator. If it is necessary to increase the accuracy, the terms with higher order Legendre polynomials should be added in (6) and in subsequent expressions. To calculate the potential and the electric field strength in spherical coordinates, it is sufficient to know the invariant charge density ρ_{0q}, the angular velocity of rotation ω and the ball radius a.

Previously, the vector potential and magnetic field of rotating uniformly charged ball were found in [7]. Thus, the problem of electromagnetic field of rotating ball, taking into account solutions of (31-32) for the scalar potential and the electric field strength, receives its complete solution.

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