

Estimating moment or kshana as a lowest
quantum time interval using the Heisenberg
uncertainty relation, within the context of
Compton effect and Fourier relationships for free
wave packets.

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Abstract

In this treatment, kshana or moment is used as a proposed lowest indivisible quantum of time, distinct from its philosophical origin and evaluated here as a physical hypothesis. Applying the Heisenberg uncertainty relation to particle localization through high-energy photon scattering in the Compton-effect framework, and Fourier relationships of free wave-packets estimated the lowest quantum time interval kshana or moment based on the statement of Maharishi Vyasa in his commentary on Patanjali Yog Darshan. The calculated lowest mean interval is $\Delta t_{min} = 1.93213300126795 \times 10^{-21}$ sec, which is 4.5 % lower than the previously reported value $2.0233249470 \times 10^{-21}$ sec, by the author. The value obtained from the Fourier relationships of free wave packets may be proposed as a lowest, indivisible quantum time kshana derived from the Heisenberg's uncertainty relation. This discrepancy may arise from the uncertainty relation itself; however, the difference between two values, one from uncertainty relation and the other value found previously by the author requires further investigation. Results should therefore be read as a theoretical estimate dependent on the chosen uncertainty relation and localization assumptions, and the discrepancy with earlier values requires further mathematical and experimental scrutiny. Kshana is experimentally measurable in future. There are possibilities of using it for time measurement at subatomic level in physical science that studies matter, energy, and the fundamental laws of nature.

Keywords: Quantum time Kshana, Krama, Viveka, Heisenberg uncertainty relation, Reduced Compton wavelength, Fourier relationship.

1 Introduction

The Heisenberg Uncertainty Principle, postulated by Werner Heisenberg in 1927, says that one cannot know the exact position of a particle (x) and its momentum (p) simultaneously. Better the knowledge about position, worse the knowledge about momentum and vice versa. The product of these two uncertainties must always be greater than, or at least equal to, some small numerical value [1].

Heisenberg [1] gave two derivations for position-momentum uncertainty relations. One based on the theory of Compton effect i.e., on localizing a particle through the scattering of a high-energy photon and other based on the Fourier relationships of free wave-packets. By considering the theory of the Compton effect, the precision of the determination of position x and momentum p connected by the uncertainty relation led to the product of uncertainties being of the order of magnitude of $\hbar$ [2]. A mathematically more general derivation based on the non-commutativity of x and p gave the result for the product of uncertainties greater than or equal to $\hbar/2$ [2].

The Heisenberg Uncertainty Principle imposes a fundamental limitation on operational possibilities in quantum mechanics. The core operational limitation is that the more precisely one property is measured, the less precisely the other can be known. That is, precision limit of where the particle is and the precision limit of how fast the particle is moving. There are three manifestations of principle. First, the widths of position and momentum distributions in any quantum state, second, the inaccuracies of any joint measurement of these quantities, and third, the inaccuracy of a measurement in one of the quantities and ensuing disturbance in the distribution of other quantity. These three kinds of uncertainty relations shown to be closely related [3].

Maharishi Patanjali in Yoga Drashan, third chapter, Sutra number 53, describes importance of concentration of mind on ‘Kshana’ and its ‘Krama’ [4]. Concentration of mind is, comprising the performance of Dharana, Dhyana, and Samadhi, or the last three stages in Yoga [5]. By focusing on ‘Kshana’ and its ‘krama’ one can achieve ‘Viveka.’ ‘Kshana’ or a ‘Moment’ regarded as a measure of time [5]. ‘Krama’ is the succession of such small unit of time and ‘Viveka’ is right judgment; the faculty of distinguishing and classifying things according to their real properties; the power of separating the invisible Spirit from the visible world; truth from untruth [5]. A person who concentrates on ‘Moment’ and its succession achieves ‘Viveka,’ experiences the distinction between spirit and matter, truth and untruth, reality and semblance or illusion, conscious and unconscious [5]. ‘Kshana’ is the lowest interval of time.

Following sections relates uncertainty relations to figure out such lowest quantum time ‘Kshana’ or ‘Moment’ defined by Maharishi Vyasa.

2 Method

Maharishi Vyasa in his commentary on Maharishi Patanjali's Drashan defines and estimates the lowest time interval. According to him, kshana is the time taken by the indivisible fundamental particle to change its direction from east (previous X) to north (next position Y) as shown in Figure 1 [4, 6]. Elementary particles are the smallest known building blocks of the universe. Electrons are the most familiar elementary particles, but the Standard Model of physics, which describes the interactions of particles and almost all forces, recognizes 10 total elementary particles [7]. In the article a free electron is considered for finding the lowest time interval 'Kshana'.

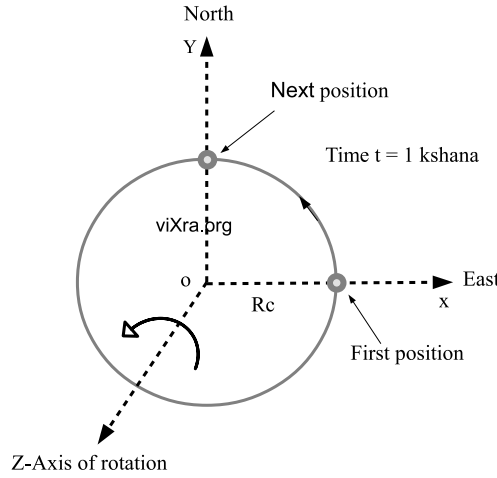


Figure 1: Thin disc model of spinning electron. Time 't' to change First position X to Next position Y or East to North is '1' kshana.

From Figure 1 the relativistic velocity v' of a spinning elementary particle electron as defined by Maharishi Vyasa [8], is–

$$v' = c' = \frac{\pi r_s}{2} = \frac{\pi \Lambda}{2} = 6.06577559166719 \times 10^{-13} \text{ m/kshana} \quad (1)$$

Where, c' m/kshana is the relativistic speed of light, and radius of electron r_s meter is the reduced Compton wavelength $\Lambda = \lambda_C/2\pi$ [9]. Where, λ_C is the Compton wavelength [10], and is–

$$\lambda_C = \frac{h}{m_0 c} \quad (2)$$

From Equation 2 we have–

$$\lambda_C = \frac{h}{m_0 n c'} \quad (3)$$

Where, m_0 , c' , and n is rest mass of spinning electron in Kg. , relativistic velocity of light in metre/kshana, and number of kshana in a second, respectively. Number of kshana in a second are–

$$n = \frac{c}{c'} \text{ kshana and } n = \frac{h}{m_0 c' \lambda_C} \quad (4)$$

Where, c m/sec is velocity of light. Substituting values for all constants h, m_0, c' , and λ_C in Equation 4 we have–

$$n = \frac{h}{m_0 c' \lambda_C} = 4.94235985809943575 \times 10^{20} \text{ kshana} \quad (5)$$

and $1 \text{ kshana} = 1/n = 2.0233249474160 \times 10^{-21} \text{ sec}$.

Where c is the speed of light and Λ is the reduced Compton wavelength.

Size of a Kshana and number of Kshana in a second can be determined as from Equations 1 and 4 as shown below–

$$1 \text{ kshana} = \frac{1}{n} = \frac{\pi \Lambda}{2c} \text{ second} \quad (6)$$

Substituting $\Lambda = 3.8615926764^{-13} \text{ metre}$, and $c = 2.99792458 \times 10^8 \text{ m/s}$ [9], we get $1 \text{ kshana} = 2.02332494690950 \times 10^{-21} \text{ sec}$ and $n = 4.9423598593366 \times 10^{20} \text{ kshana}$.

Verification of these results is done using the Heisenberg uncertainty principle as described in the following section.

Estimated time unit ‘Kshana’ or ‘Moment’ in seconds based on the first limitation of operational possibility i.e., the widths of the position and momentum distributions in any quantum state. Out of two derivations for position–momentum uncertainty relations i.e., one based on the theory of the Compton effect, and other based on the Fourier relationships of free wave-packets, we have taken first one which is based on the theory of Compton effect [2].

3 Results

3.1 Heisenberg uncertainty relation based on the theory of the Compton effect i.e., on localizing a particle through the scattering of a high-energy photon and a kshana

Let us combine the Heisenberg’s uncertainty principle, the Compton scattering based on the Position-Momentum/Time-Energy uncertainties, and the reduced Compton Wavelength [11]. Heisenberg’s uncertainty principle is usually taken to express a limitation of operational possibilities imposed by quantum mechanics

[3], and if the Heisenberg uncertainty principle holds for the Compton electron, then we determined the value of a kshana as shown below. Heisenberg position momentum inequality is–

$$\Delta x \Delta P \geq \hbar \quad (7)$$

Setting a lowest limit on the Δx similar to kshana which is exceedingly small and which cannot be divided further. we can write–

$$\Delta x \Delta P = \hbar \quad (8)$$

It is important to note the equality in Equation 8, compared to the order of magnitude in Equation 7 [2]. It is worth to note that the more accurately we know the position i.e., the smaller Δx is, the less accurately we know the momentum i.e., the larger Δp is; and vice versa.

Similarly, the time-energy uncertainty relation is as shown below–

$$\Delta t \Delta E \geq \hbar \quad (9)$$

Where, Δt , and ΔE are time and energy uncertainties, respectively. This shows that a precise energy determination can be reached only through a corresponding uncertainty in the time [2].

Let Δt_{min} , and ΔE_{max} be minimum and maximum uncertainty in time and energy, respectively. Then, from time and energy, uncertainty principle (Equation 9), we have–

$$\Delta t_{min} \times \Delta E_{max} \geq \hbar \quad (10)$$

According to Maharishi Vyasa quantum time unit kshana being extremely low-est and cannot be further divided. There is no time unit lower than the kshana. Therefore, Equation 10 is rewritten as shown below–

$$\Delta t_{min} \times \Delta E_{max} = \hbar \quad (11)$$

From position and momentum uncertainty principle (Equation 5), we have–

$$\Delta x_{min} \times \Delta P_{max} = \hbar \quad (12)$$

$$\Delta P_{max} = \frac{\hbar}{\Delta x_{min}} \quad (13)$$

But we know that ΔP_{max} is:-

$$\Delta E_{max} = \frac{(\Delta P_{max})^2}{2m_0} \quad (14)$$

From Equations 6, 8, and 9, we have–

$$\Delta t_{min} \times \frac{(\Delta P_{max})^2}{2m_0} = \hbar \quad (15)$$

$$\Delta t_{min} \times \left(\frac{\hbar}{\Delta x_{min}} \right)^2 \frac{1}{2m_0} = \hbar \quad (16)$$

$$\Delta t_{min} = \frac{2m_0 \Delta x_{min}^2}{\hbar} \quad (17)$$

The Compton wavelength gives the smallest radius within which the mass of a particle may be localized due to quantum effects [12]. Therefore, assuming widths of position equal to the Compton radius, i.e., $\Delta x_{min} = r_s$, where r_s is radius of electron (Figure 1) and is equal to reduced Compton wavelength $\Lambda = \lambda_C/2\pi$. λ_C is the Compton wavelength (Equation 1). Assuming that Δt_{min} is exceedingly small, indivisible quanta of time kshana, and is the lowest limit in time. We have–

$$\Delta t_{min} = \frac{2m_0 r_s^2}{\hbar} \quad (18)$$

$$\Delta t_{min} = \frac{m_0 \lambda_C^2}{2\pi^2 \hbar} \quad (19)$$

Keeping for $\lambda_C = h/m_0 c$, and $\hbar = h/2\pi$, we get:-

$$\Delta t_{min} = \frac{2\hbar}{m_0 c^2} \quad (20)$$

Right side of Equation 20 for Δt_{min} , all are constants. Hence, Δt_{min} is a constant and indivisible time which is termed as a ‘Kshana.’ Substituting data of m_0 , c , and $\hbar$ [9], we have–

$$\Delta t_{min} = 2.5761773333048 \times 10^{-21} \text{ sec} \quad (21)$$

Thus, Δt_{min} is exceedingly small, indivisible quanta of time, and is slightly large as compared to the value of a kshana (Equation 3) which is also found earlier by the author. Number of kshana in a second is given as $n = 1/\Delta t_{min} = 3.88172035534757 \times 10^{20}$ kshana which is smaller than the previously determined value by the author [13, 14] (Equation 3).

3.2 Heisenberg uncertainty relation based on Fourier relationships of free wave-packets and a kshana.

Calculations based on the Fourier relationships of free wave-packets the time-energy uncertainty relation is as shown below–

$$\Delta t \Delta E \geq \frac{\hbar}{2} \quad (22)$$

Where, Δt , and ΔE are time and energy uncertainties, respectively. This shows that a precise energy determination can be reached only through a corresponding uncertainty in the time [2]. This says that the product of these two uncertainties must always be greater than, or at least equal to, some small numerical value $\hbar/2$.

Let Δt_{min} be lowest, and ΔE_{max} be maximum uncertainty in time and energy, respectively. Then, from Equation 22, we have–

$$\Delta t_{min} \times \Delta E_{max} = \frac{\hbar}{2} \quad (23)$$

and from position-momentum uncertainty principle, we have–

$$\Delta x_{min} \times \Delta P_{max} = \frac{\hbar}{2} \quad (24)$$

$$\Delta P_{max} = \frac{\hbar}{2\Delta x_{min}} \quad (25)$$

But,

$$\Delta E_{max} = \frac{(\Delta P_{max})^2}{2m_0} = \frac{\hbar^2}{4m_0\Delta x_{min}^2} \quad (26)$$

From Equations 23, and 26, we get–

$$\Delta t_{min} \times \frac{\hbar^2}{4m_0\Delta x_{min}^2} = \frac{\hbar}{2} \quad (27)$$

$$\Delta t_{min} = \frac{2m_0\Delta x_{min}^2}{\hbar} \quad (28)$$

$$\Delta t_{min} = \frac{2m_0r_s^2}{\hbar} \quad (29)$$

For $\Delta x_{min} = r_s$, where r_s is radius of electron (Figure 1) and is equal to reduced Compton wavelength. Again, assuming that Δt_{min} is exceedingly small, indivisible quanta of time kshana, we have–

$$\Delta t_{min} = \frac{m_0\lambda_C^2}{4\pi^2\hbar} \quad (30)$$

Again, keeping for $\lambda_C = h/m_0c$, and $\hbar = h/2\pi$, we get–

$$\Delta t_{min} = \frac{\hbar}{m_0c^2} \quad (31)$$

Again, right side of Equation 31 for Δt_{min} , all are constants. Hence, Δt_{min} is a constant and indivisible time which is termed as a kshana. Substituting for m_0 , c , and $\hbar$ [9], we have–

$$\Delta t_{min} = 1.2880886666524 \times 10^{-21} \text{ sec} \quad (32)$$

Thus, the small, indivisible quanta of time Kshana is $\Delta t_{min} = 1.2880886666524 \times 10^{-21} \text{ sec}$ is lower than the value of a Kshana which was found earlier by author [15].

4 Discussion

Time Kshana determined from Heisenberg's uncertainty relation, based on the Compton effect theory involving the localization of a particle through the scattering of a high-energy photon, is twice the value obtained from the Fourier relationships of free wave packets. Mean of these two results $1.93213300126795 \times 10^{-21}$ is 4.5% below the previously estimated value $2.023324947 \times 10^{-21}$ sec. The lowest time interval, $\Delta t_{min} = 1.288088666524 \times 10^{-21}$ sec, obtained from the Fourier relationships of free wave packets is 36% lower, and based on the Compton effect theory involving the localization of a particle through the scattering of a high-energy photon $\Delta t_{min} = 2.5761773333048 \times 10^{-21}$ sec is 27% higher than previously determined value.

The value obtained from the Fourier relationships of free wave packets may be proposed as a lowest, indivisible quantum time Kshana derived from the Heisenberg's uncertainty relation which is lower than the previously reported value of Kshana $2.023324947 \times 10^{-21}$ sec.

Quantum time unit Kshana being exceedingly small, indivisible just like fundamental particle, is experimentally measurable time in future. Since, Kshana is even smaller than the experimentally observed zeptosecond birth time delay of photoelectrons in photoionization which has been measured [16]. There are possibilities of using it at subatomic level in physics. It is not mere theoretical concept.

5 Conclusion

This discrepancy may arise from the uncertainty relation itself; however, the difference between two values one from uncertainty relation and the other value found previously by the author requires further investigation.

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