

The life of Psi: how a wave function behaves during a measurement

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Abstract

An explanation of the quantum measurement process must do more than produce the Born Rule. Conventional quantum mechanics makes other specific predictions about intermediate states, which develop before the completion of a measurement. This paper's measurement model addresses that more general problem. For a detector with efficiency less than one, the model predicts the conventional quantum results almost exactly: an initial superposed state develops into a well-defined mixture of eigenstate and altered superposition. The minor departures of the model from conventional quantum mechanics could be revealed, in principle, in an optical interference experiment that is also described here. The measurement model assumes that the conversion into an eigenstate is not instantaneous but results from a random sequence of unitary perturbations arising from conventional quantum interactions with the microscopic components of the measuring device. A random walk model describes how wave functions change. And the model's success relies on small biases in transition probabilities which, however, could be expected of a device designed to measure a specific observable. The biases help to define a generalized Ornstein-Uhlenbeck equation for the distribution of quantum states, and it admits solutions that substantially replicate those of conventional quantum mechanics.

Background

"...[in] a complete physical description, statistical quantum theory would ... take an approximately analogous position to the statistical mechanics within the framework of classical mechanics ..." - [1]

No consensus has emerged for what happens to a wave function during a quantum measurement. Why don't the conventional laws of quantum mechanics explain the phenomenon? Is the transition made by a quantum object from superposed state to eigenstate instantaneous? If not, what are the intermediate states? Ref [2] proposed a physical mechanism and described a specific stochastic model embracing the idea of a sequential transition through intermediate states. If some such model can be validated experimentally, quantum mechanics will become understood as a deterministic theory, with a probabilistic aspect necessitated only by the random nature of interactions with the microscopic components comprising any macroscopic device.

Much discussion of the measurement issue has focused on whether the transition is mediated by a non-unitary, or even nonlinear transformation. Either answer is not even wrong: the premise of the question is invalid. The transition induced by a measurement on a quantum state is not a transformation at all, which is a mathematical mapping of single entities to single entities. The wave function change associated with a measurement proceeds from an initial state to any one of

a group of quantum states, and so should be understood as a mathematical *relation*. This paper suggests a model for that stochastic relation.

In the course of a measurement one normalized Hilbert space state vector ultimately becomes another, and if each interaction accords with the Schrödinger equation, then each is mediated by a unitary transformation. Because the product of unitary transformations is also unitary, the net result is a random transformation that obeys the rules of conventional quantum mechanics (CQM). Thus it is natural to attribute the transition to eigenstate to a sequence of more elementary perturbations, corresponding to encounters of the target quantum entity with the components of the measuring device. Unpredictable variations in such encounters, for example in the quantum states of the device components, impact parameters, or thermal effects can necessitate a probabilistic description of the process.

A *simple random walk* (SRW) model [2] of quantum measurements attempted to describe the process, to capture wave function behavior during a measurement. It assumed that during any measurement an initial superposed state stochastically transits a succession of intermediate quantum states, according to simple random walks executed by the squared magnitude I of any one of its component amplitudes. The model predicts the Born Rule: the result of a measurement is a probability distribution $P(I, t)$ for I that asymptotically approaches two δ functions:

$$P(I, t = \infty) = I_0 \delta(I - 1) + (1 - I_0) \delta(I) . \quad (1)$$

Here t is some measure of the progress of the measurement, monotonically related to time, and I_0 is the starting ($t = 0$) intensity value of the chosen component.

Importantly, CQM predicts a pair of δ functions for *interrupted* measurements as well. But if the measurement device is inefficient, as for a detector with absorbance $a < 1$, then the SRW model fails to reproduce the CQM result and so must be rejected¹. However, “Our body of knowledge advances with reports of things that did not work as well as those that did.” [3]. And in this case a simple modification of the SRW model produces an accurate approximation to standard quantum theory for *all* absorbance values.

If the measurement device slightly biases the random walk transition probabilities, then for interrupted measurements the model can generate two spikes coincident with CQM’s δ functions, but with a small nonzero width in one. Thus the revised model² described below produces the usual Born Rule for full quantum measurements, but also predicts only slight possible differences in snapshot experiments, in which the measurement process is examined before completion.

¹ "The great tragedy of Science — the slaying of a beautiful hypothesis by an ugly fact." - Thomas Henry Huxley.

² “Our doubts are traitors, And make us lose the good we oft might win by fearing to attempt.” *Measure for Measure*, W. Shakespeare.

The Bifurcating Convective (BCM) Measurement Model

The probability density function $P(I, t)$ describes the cumulative effect of all target/device interactions up to “time” t in the BCM model. The finite but small step size dt (called ϵ in the discrete model of [2]) of the random walks executed by any component intensity I of the target wave function corresponds roughly to the interval between interactions.

Between times t and $t + dt$, the random walk executed by I is assumed to increase by dI with probability $u(I)$ and to decrease by dI with probability $d(I)$. (In the SRW model these are taken to be equal.) This leads to an equation for the evolution of P :

$$P(I, t + dt) = d(I)P(I + dI, t) + u(I)P(I - dI, t), \quad (0 < I < 1). \quad (2)$$

Expanding the terms in Equation (2) and using Taylor’s theorem produces the partial differential equation:

$$\frac{\partial P(I, t)}{\partial t} = - \frac{\partial(v(I)P(I, t))}{\partial I} + \frac{\partial^2 P(I, t)}{\partial I^2} D, \quad 0 < I < 1, \quad (3a)$$

$$\text{with } D = \lim_{dI, dt \rightarrow 0} \frac{(dI)^2}{2dt} \quad \text{and} \quad v(I) = \frac{2D}{dI}(u(I) - d(I)). \quad (3b)$$

(The limiting form in Equation (3b) keeps the variance of the random walk nonzero and finite.) Equation (3a) is the one-dimensional Fokker-Planck equation, which describes, among other behaviors, the diffusion of a gas that also drifts (advects) with velocity v at coordinate value I . Because dI is small, the form for the velocity field in Equation (3b) implies that v can have large values with only minuscule differences in the up and down transition rates u and d . Yet those differences prove critical in determining the life of a wave function during a measurement. The model with $v \neq 0$ is called the Bifurcating Convection Measurement model, with convection meant to convey the action of both advection and diffusion.

Next we invoke known quantum properties of measuring devices: they never alter a pure eigenstate of the observable that they are designed to measure, and they never produce an eigenstate that is absent from an initial superposition. Therefore,

$$d(1) = u(0) = 0, \quad (4)$$

and so the points $I = 0, 1$ represent probability sinks. Once I has walked to either of those values, it stays there. This is reflected in a form for P that can include δ functions at the endpoints with amplitudes that can grow with t . Also, by definition

$$u(1) = d(0) = 0. \quad (5)$$

If $u = d$ exactly, then the velocity term in Equation (3a) is absent, and P develops according to pure diffusion, such as illustrated in Figure 1, unless there are enough interactions that some trajectories reach the endpoints, as in Figure 2.³ There they accumulate and in the limit $t \rightarrow \infty$ all probability becomes concentrated in δ functions at the endpoints (represented by blue spikes). That limit corresponds to a full measurement and generally exhibits a rightmost δ function scale of I_0 , with the left one at $1 - I_0$. This is the Born Rule of CQM, which is successfully predicted by the SRW model of [2].

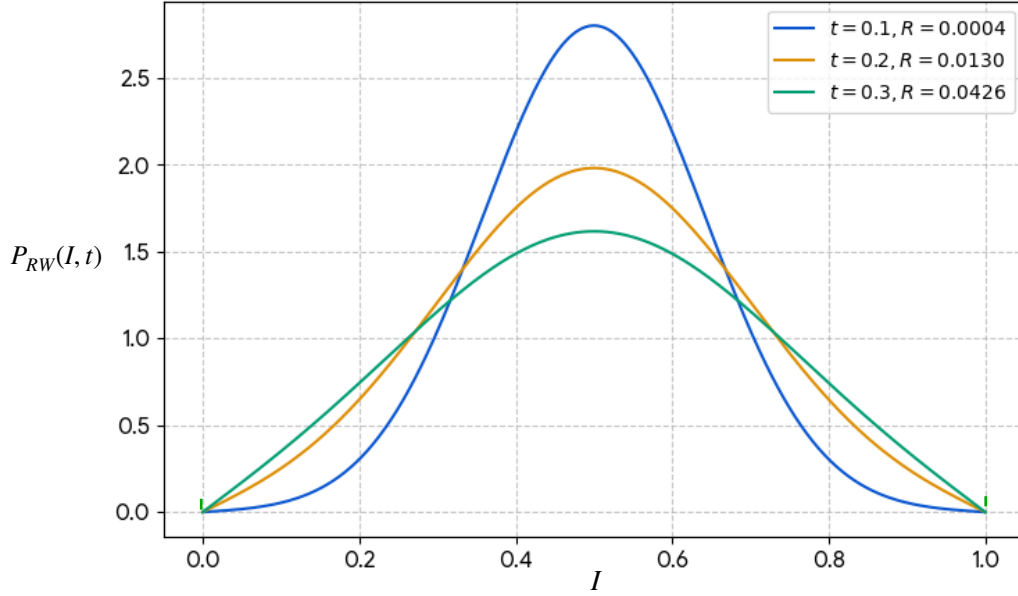


Figure 1. $I_0 = 1/2$ distributions of component intensities for 3 degrees of target/background interactions in the SRW model. R is the scale of the δ functions at $I = 0,1$.

However, the SRW model fails to agree with CQM for interrupted measurements (i.e., for some intermediate values of t), which behave according to an extended version of the Born Rule. That constraint describes interactions with detectors with less than 100% efficiency, for example photodetectors that absorb only a fraction of incident light. It will be shown that consistency with this extended Born Rule of CQM requires at least some small differences in up- and down- transition rates, u and d .

Unlike the SRW model, even for an interrupted measurement CQM produces two δ -functions, only one of which is guaranteed to lie at $I = 1$ (and whose scale denotes the absorption amount). Figure 3 illustrates an example of the extended Born Rule, which is more fully explicated in an illustrative experiment discussed below. Later the BCM model will be shown capable of predict-

³ Closed-form solutions to the continuum form of SRW were used to generate Figures 1 and 2 [4].

ing narrow distributions whose peaks coincide exactly with CQM's δ functions. The BCM model's principal differences from CQM are: (1) a minor and potentially detectable width of the left distribution, and (2) a small current of probability connecting the two modes of the distribution.

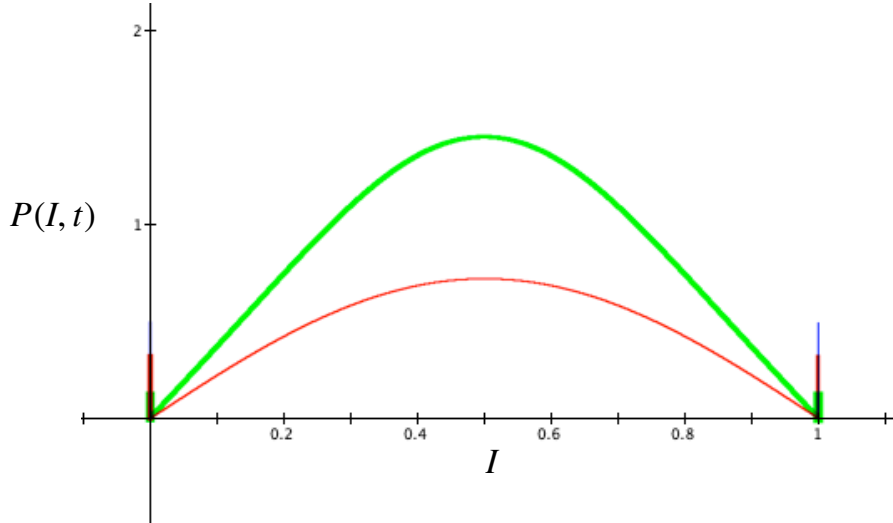


Figure 2. Three $I_0 = 1/2$ distributions of component intensities, whose continuum parts approach half a sinusoid cycle as the number of interactions increases, but ultimately reduce to only a pair of δ functions (in blue).

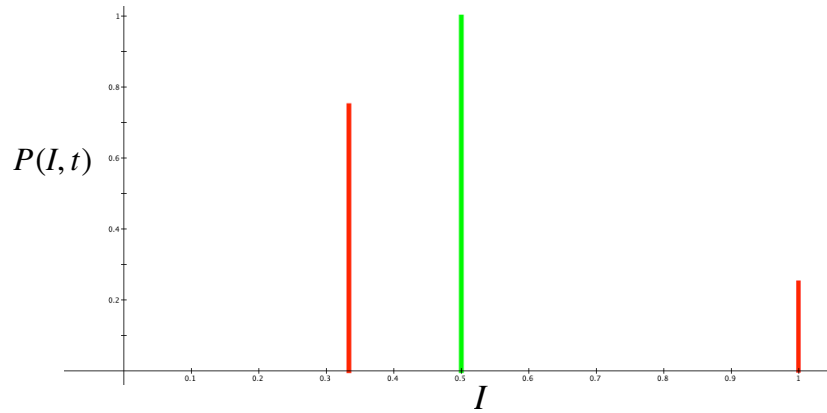


Figure 3. An initial optical quantum state characterized by $I_0 = 1/2$ (green) interacts with a measuring device that absorbs only half the polarized photons, producing a bifurcated distribution (red) of quantum states. CQM predicts a pair of such δ -functions for each $a > 0$.

An expositive experiment

Ref [5] proposed experiments involving spin- $\frac{1}{2}$ particles to test the SRW model. The required equipment, idealized arrangements of Stern-Gerlach devices, were hypothetical, apparently not existing in any laboratory and with little prospect of being realized. This paper proposes a more realizable analogous experiment (see Figure 4), based on conventional optics, for comparing BCM versus CQM models. It is also a convenient vehicle for illustrating the extended Born Rule.

A laser produces monochromatic linearly polarized light propagating through a Mach-Zehnder interferometer (MZI). The initial angle θ_0 of linear polarization relative to the horizontal is an adjustable parameter of the experiments. The laser light first encounters a polarizing beam splitter that reflects vertically polarized light and transmits any horizontally polarized component. The test sample in the lower path consists of some light-absorbing material, with absorbance a .

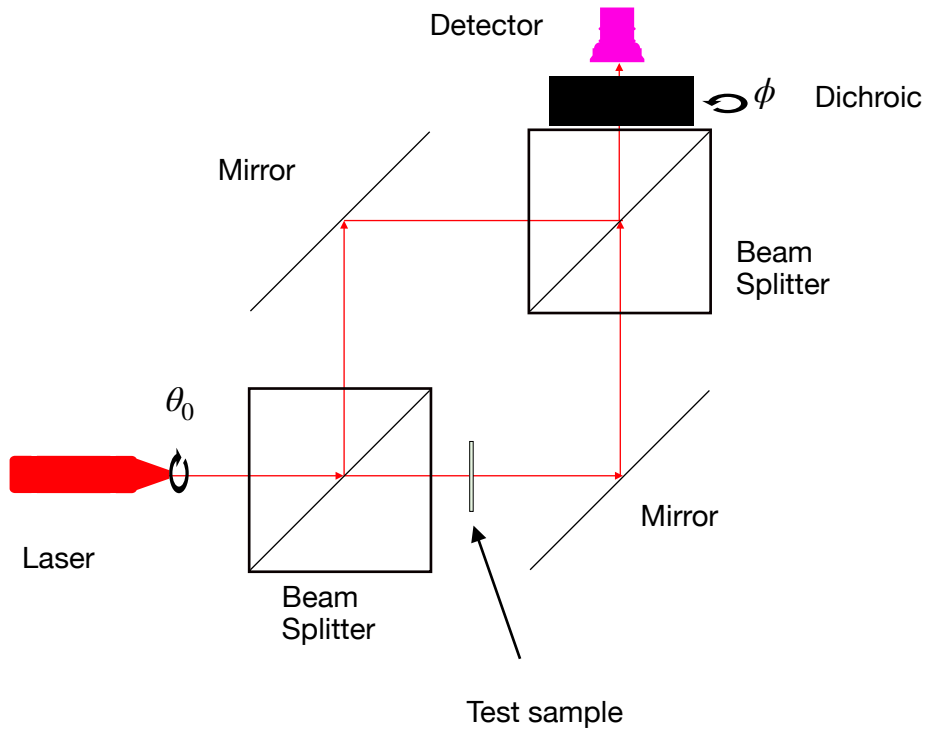


Figure 4. Laser light with selectable linear polarization angle θ_0 is divided into two components by a polarization-based beam splitter. The horizontal component passes through an absorbing sample before both beams are recombined. Photons then pass through a rotatable dichroic filter, whose nominal orientation ($\phi = 0$) transmits only horizontally polarized light. The photo-detector measures output intensity.

An incident linearly polarized photon at angle θ is forced into either a vertical or horizontal output eigenstate (i.e. measured) if the test sample is fully absorbing. Or such a measurement can be truncated, if the absorbing material of the sample is not perfectly efficient. When $a = 1$ the sample makes a measurement, by absorbing a photon, with probability based on the Born Rule. For $0 < a < 1$ the combination of beam splitter and test sample executes an interrupted measurement of the horizontal component of polarized light, enabling a more detailed comparison of model (SRW or BCM) versus CQM predictions about the process.

The two beams are recombined using mirrors and a second beam splitter, and then traverse a rotatable dichroic element, such as tourmaline, whose nominal orientation ($\phi = 0$) transmits only horizontally polarized light. Finally, a photo-detector records the transmitted light.

The laser and beam splitter produce photons with spatially separated entangled polarization states. We call the magnitude squared of the photon's initial quantum amplitude for horizontal polarization its (horizontal) intensity

$$I_0 = \cos^2\theta. \quad (6)$$

With the usual wave function normalization, the intensity of the vertical component is $1 - I_0$. We denote the general photon state

$$| I \rangle = \sqrt{I} | H \rangle + \sqrt{1-I} | V \rangle \quad (7)$$

by the value I of its horizontal (H) component's intensity. (Each polarization undergoes an odd number of π phase shifts from reflections, whose effects therefore cancel on recombination. Zero relative phase is otherwise assumed to be maintained by MZI adjustments during the measurement, and so phase is irrelevant to this discussion.)

Conventional quantum predictions

In CQM a perfectly absorbing test sample ($a = 1$) in Figure 4 would effect a measurement and so would change the initial superposed quantum state in one of two ways. With probability I_0 it would convert the self-entangled photon into one with a purely horizontal polarization and absorb it. Alternatively, with probability $1 - I_0$ it would destroy the horizontal component, but leave a vertically polarized photon appearing in the upper arm of the interferometer.

A similar type of bifurcation also occurs when $a < 1$. In that case a photon in the initial state $| I_0 \rangle$ injected by the laser is absorbed by the test sample with probability

$$a I_0 \quad (\text{absorbed by sample as } | H_{abs} \rangle). \quad (8)$$

Otherwise, it survives the test sample with probability $(1 - a)I_0$, in which case its amplitude for surviving in the $|H\rangle$ state is $\sqrt{(1 - a)I_0}$. Similarly, the amplitude in the upper arm for the photon to be in the $|V\rangle$ state is $\sqrt{1 - I_0}$. Because there are two paths to the final beam splitter, which recombines the states spatially, the conditional amplitude for the state after traversing the spectrometer is

$$\sqrt{(1 - a)I_0} |H\rangle + \sqrt{1 - I_0} |V\rangle, \quad (9)$$

whose squared norm $1 - aI_0$ reflects the “surviving” probability after some possible absorption by the test sample. In the notation of Equation (7), the normalized form of Expression (9) is

$$\left| \frac{(1 - a)I_0}{1 - aI_0} \right\rangle \quad (10)$$

Thus CQM predicts that the MZI produces a bimodal distribution of quantum states, consisting of two δ functions:

$$R\delta(I - 1) \text{ and } (1 - R)\delta(I - (I_0 - R)/(1 - R)), \quad \text{with } R = aI_0. \quad (11)$$

The first reflects $|H_{abs}\rangle$ (see Expression (8)), and the second reflects the surviving state in Expression (10). We call Expression (11) the *extended* Born Rule. Figure 3 illustrates the distribution of I for the example $I_0 = 1/2$ (diagonal input polarization) and $a = 1/2$. As $a \rightarrow 1$, the left δ function moves toward $I = 0$ and shrinks to intensity $1 - I_0$, while the one at $I = 1$ grows to intensity I_0 . Notice that the SRW model shapes in Figures 1 and 2 can never develop into any distributions mimicking CQM’s δ functions, except in the limit of large t .

The superposed state in Expression (9) makes a linear polarization angle (see Figure 5)

$$\alpha = \tan^{-1} \sqrt{\frac{(1 - I_0)}{(1 - a)I_0}}. \quad (12)$$

The amplitude for the dichroic to transmit such a photon is $\cos(\phi - \alpha)$, so that the total probability that a photon is photo-detected according to CQM (in particular, the extended Born Rule) is

$$C(\phi) = (1 - aI_0)\cos^2(\phi - \alpha) = [\cos\phi\sqrt{(1 - a)I_0} + \sin\phi\sqrt{1 - I_0}]^2. \quad (13)$$

Notice that if I_0 and $1 - I_0$ are interpreted as (normalized) radiation intensities in classical electromagnetic wave theory, then α is the polarization angle of the output wave, and $C(\phi)$ is its in-

tensity along the dichroic's transmission axis. In this sense CQM usually agrees with classical wave theory⁴.

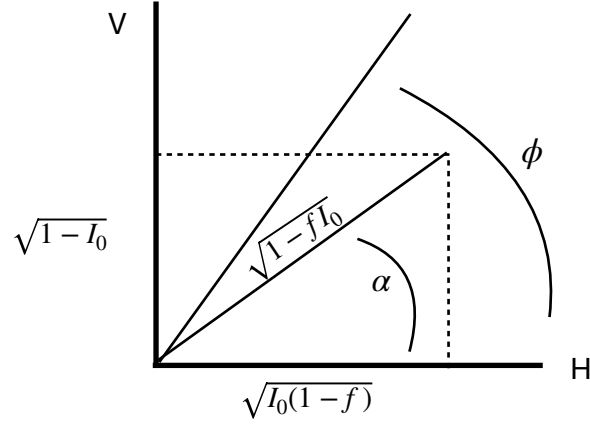


Figure 5. The geometry of polarization and its relationship to the dichroic transmission angle ϕ .

A solution for the BCM model

For many applications of the Fokker-Planck equation, an initial δ function $P(I, 0) = \delta(I - I_0)$ spreads as t advances according to diffusion (governed by the 2nd-derivative term in Equation (3a)) and also translates according to the velocity field $v(I)$. However, for some velocity fields, e.g. $v(I) = k(I - I_0)$, the initial δ function can split into separate left- and right-moving spikes. This possibility suggests a path within the Fokker-Planck formalism for explaining the extended Born rule.

If the velocity difference on opposite sides of I_0 is not large (v is small compared to D)⁵, then diffusion can fill in the gap between putative spikes and result in a unimodal distribution. Here we assume that the transition probabilities u and d prompted by a measurement device can generate dynamics for P residing at least partially in the “advective-dominant” regime, meaning that velocity-field effects can swamp diffusion. According to Equation (3b), even minute differences in transition rates can imply large velocity fields, and this can lead to a bifurcated solution. In this paper’s BCM model, as a target increasingly interacts with the measuring device, a probability current flows advectively into a sink at $I = 1$, while simultaneously a spike moves by a kind of compressed diffusion toward the sink at $I = 0$.

⁴ For an exception, see [6].

⁵ The quantity D/v is $O(1/Pe)$, where the structural Péclet number. Pe is used to describe transport phenomena [7].

Now for the general measurement mechanism we construct a specific mathematical model whose solution nearly replicates the extended Born Rule. The distribution for the intensity I of a target interacting with a measurement device is assumed to obey a generalized form of Ornstein-Uhlenbeck (OU) equation [8], which is itself a generalization of the Fokker-Planck equation. In OU models the velocity field depends on the mean value μ of the probability distribution for I :

$$\mu = \langle I \rangle = \frac{1}{M} \int_0^1 I P_c(I, t) dI, \quad (14)$$

with

$$M = \int_0^1 P_c(I, t) dI. \quad (15)$$

P_c is the continuum part of the probability distribution, not yet absorbed into the sinks at $I = 0, 1$.

In the BCM model the measurement device reacts to an incident target intensity A by exciting transition probabilities, which in turn define the equation's velocity field. The target distribution is initially represented by a δ function at $I = I_0$, which evolves as an OU process. P_c continually develops in the model as a narrow spike derived from the initial δ function, so that its location remains nearly identical to its mean value. Therefore, we treat the peak location A as equal to the mean value:

$$A = \langle I \rangle = \frac{1}{M} \int_0^1 I P_c(I, t) dI. \quad (16)$$

In BCM the spike propagates slowly to the left, so that for most of the process, the sink at $I = 0$ remains empty, but the right one accumulates to a value R , as probability bleeds into it from the spike. Thus, the conservation of total probability becomes

$$M + R = 1 \quad (17)$$

until the process terminates, when accumulation L in the left sink replaces M in the equation. R represents the probability that the measurement device has converted the target into an eigenstate.

With the definition

$$\sigma^2 = \langle (I - A)^2 \rangle = \frac{1}{M} \int_0^1 P_c(I, t) (I - A)^2 dI, \quad (18)$$

the BCM model is Equation (3a), with the velocity term

$$v_A(I) =: -k(I - A) + \lambda[(I - A)^2 - \sigma^2] . \quad (19)$$

The solution P_c nearly replicates CQM's distribution function. The positive constants k , λ , D are arbitrary, except for relative size constraints

$$\lambda \gg k \gg 1 \gg D . \quad (20)$$

(The case $\lambda = 0$ corresponds to the standard OU process.) A large value of k keeps the spike at $I \approx A$ compressed as it creeps via diffusion to the left at velocity $\approx -\lambda\sigma^2$. Intuitively, the time it takes diffusion to spread a distance σ is σ^2/D , while for compression it is $|\sigma/v| = \sigma/k\sigma$ (from Equation (19)), so that the variance can be estimated by balancing them

$$\sigma^2 \approx D/k \ll 1 . \quad (21)$$

(See the Appendix for a more rigorous derivation⁶, which also considers extra constraints on the parameters in Expression (20).)

A device measures a given observable by presenting transition rates (Equation (3b)) to a quantum target whose corresponding intensity I has the value A . And so the transition rates ultimately determine the velocity field in Equation (19).

An initial δ function $P(I, 0) = \delta(I - I_0)$ (for which, therefore, $A(t = 0) = I_0$), developing according to Equation (3a) with the velocity field in Equation (19), experiences a negative velocity $-\lambda\sigma^2$, which competes with diffusion. And any small probability that wins by escaping to the right of A is quickly swept into the right sink (λ is the largest parameter).

Simultaneously, the remaining distribution keeps the form of an extremely narrow spike concentrated at $I = A$ and migrates slowly toward the left sink with velocity $\approx -\lambda\sigma^2$ (Equation (21)). Consequently, as the process progresses the measurement device continually resets the velocity field in Equation (19), adjusting the value of A to accommodate the new approximate δ function presented to it. Then the sweeping process repeats until, if the measurement is completed, the value of A reaches zero. At all intermediate points, the double-spike feature seen in CQM is produced, one peak at $I = A$, the other at the sink $I = 1$.⁷

Importantly, the BCM model velocity form in Equation (19) implies CQM's extended Born Rule, Expression (11).

⁶ Hence, the variance term in Equation (19) can be rewritten in terms of model parameters.

⁷ The process that Equation (3a) with (19) describes is analogous to negatively charged ions inserted at some point I_0 into a tube containing a rapidly moving fluid. An external electric field slowly pulls them against the current, but occasionally an electron is stripped off and the neutral particle is swept into a sink.

To see why, first note that

$$\langle v \rangle = 0 , \quad (22)$$

which follows from the definitions in Equations (19), (18), (14) and (16). Next, from Equations (15), (3a), and (22), the rate of probability loss from the continuum equals the current

$$J = vP_c - D \frac{\partial P_c}{\partial I} \quad (23)$$

into the right sink

$$\dot{R} = -\dot{M} = - \int_0^1 \frac{\partial P_c(I, t)}{\partial t} dI = \int_0^1 \frac{\partial J(1, t)}{\partial I} dI = -D \frac{\partial P_c(1, t)}{\partial I} , \quad (24)$$

which follows because the current into the left sink is negligible for most of the active process, and because $P_c \rightarrow 0$ at any sink.

Then, from Equations (16), (3a), (15), and (24)

$$\frac{d(MA)}{dt} = \int_0^1 IdI \left[-\frac{\partial(vP_c)}{\partial I} + D \frac{\partial^2 P_c}{\partial I^2} \right] = D \frac{\partial P_c(1, t)}{\partial I} = \dot{M} \quad (25)$$

which follows from integration by parts and the above arguments, along with Equations (22) and (24). The solution to the differential Equation (25)

$$A = \frac{I_0 - 1 + M}{M} = \frac{I_0 - R}{1 - R} \quad (26)$$

follows from the initial conditions: $M(t = 0) = 1$, $A(t = 0) = I_0$ along with Equation (17). Thus, the characteristic law from CQM, Equation (11) (as well as the Born Rule (1) when $R \rightarrow I_0$), is reproduced by the BCM model.

For completeness we note that the above type of analysis applies to any multi-state system, not just the two-component photon.

Finally, note that for uninterrupted interactions with macroscopic devices not designed to measure a particular observable, asymmetry in the transition probabilities might not occur, and hence $v = 0$ is possible. In that case the Ornstein-Uhlenbeck equation reduces to a description of pure diffusion, but it still ends in an eigenstate $I = 1$ or 0 (coincidentally obeying the Born Rule) as in Figure 2 (blue spikes), thus explaining the process commonly called decoherence.

Measurement differences

A general measurement model can produce a density operator with multiple components:

$$\rho(t) = \int_0^1 P(I, t) |I\rangle \langle I| dI . \quad (27)$$

CQM's density operator is defined by the two components of Equation (11), while a BCM model dresses the left δ function with a finite-width distribution, and includes a small probability current streaming from $I = A$ to $I = 1$. Thus, the two models can be distinguished experimentally, at least in principle.

For example if, in accordance with CQM principles, the dichroic in Figure 4 is rotated to $\alpha + \pi/2$ (see Equation (12)) to block any output (e.g., the left spike in Figure 3), then the BCM and CQM models might be distinguishable. Even with errors in estimating the absorbance value, CQM predicts perfect blocking at some angle (which could be scanned), while BCM admits leakage at all angles except for $a = 0, 1$.

In practice, the leakage could be small⁸, and the present analysis does not quantify expected differences. Also, many other choices of velocity field might reproduce the extended Born Rule, with their associated error predictions. Finally, many practical effects would considerably complicate such an experiment, which would probably also entail at least a cryogenic version or analog of Figure 4.

In conclusion, confirming the BCM model experimentally might prove difficult, but so might falsifying it. Its attractiveness comes from its reliance on a simple credible physical mechanism, a sequence of normal quantum interactions.

Implications for applications

“... it would be a poor theory of computation that never addressed issues of how the output is obtained from the input” [9]

Validation of the BCM model could have immediate impact on some nascent quantum-based technologies. For example, the main technical problem in quantum computing is noise, that is, the interaction of qubits with the macroscopic environment. Confirmation of the BCM model could provide necessary insights for understanding the noise mechanism.

⁸ Solutions to the BCM model appear to reproduce CQM exactly in the limit $D \rightarrow 0$.

Also, quantum network designs assume that CQM laws of physics make eavesdropping impossible to hide. This idea might be compromised, if the stochastic nature of quantum mechanics is more of an inconvenience than a fundamental limitation.

Finally it should be noted that the idea that the transition from superposition to eigenstate is not instantaneous is not new. It has been shown that “the measurement process causes the superposition to collapse gradually (although the whole process is over in about a millionth of a second)... Also ... (such) work might be used to improve error correction in quantum computers, given that weak measurements could in principle allow errors to be detected in quantum states without destroying those states in the process.” [10].

Implications for the foundations of quantum mechanics

Several explanations of the quantum measurement process have been proposed. For at least the leading contenders, “... no predictions ... differ from those of standard quantum mechanics” [11]. By contrast, the BCM model replaces the δ functions of conventional quantum mechanics with slightly different and realizable distributions, representing a potentially falsifiable contrast.

The BCM model marries ordinary quantum principles with a randomness assumption. Its mundane physics hypothesis, repeated independent random interactions of a simple quantum system with the components comprising any detection device, is analogous in statistical mechanics to the ergodic hypothesis or a random walk model for diffusion in dilute gases.

The particular stochastic model (Equation (19)) suggested here is more contrived than derived, but represents an existence proof: The proposed physical mechanism admits at least one mathematical model that reproduces not only the Born Rule, but the extended version dictated by conventional quantum mechanics. The BCM model is surely more palatable than drastic revisions of the Schrödinger equation or metaphysical excursions to parallel universes. And many other models consistent with CQM properties can be devised using the same physical mechanism. As for the most physics-appropriate mathematical model, this paper frames no hypotheses. As Wolfgang Pauli wrote in a letter to George Gamow: “Only technical details are missing ...”.⁹

If the BCM mechanism is correct, then measurements do not create reality. Nor is it necessary to conclude that “reality is multi-valued” [9]. Archetypical physical descriptors associated with the particulars of human evolution like shape, position, speed, etc., emerge from a more fundamental reality described by the wave function. And there is, after all, no stark divide between the classical and quantum worlds.

⁹ But in that case, on an otherwise empty page.

Appendix

The following argues that with appropriate constraints on the parameters, Equation (21) is an accurate estimate of the spike variance σ^2 . This is not essential to the main thesis of the paper, but demonstrates a sufficient condition for the BCM model to replicate CQM.

The Ornstein-Uhlenbeck Equation (3a) with the Equation (19) definition allows a more precise analysis of the variance in Equation (18), whose time derivative yields

$$(\dot{\sigma}^2) = \frac{1}{M} \int_0^1 \left[\frac{-\partial(P_c v)}{\partial I} + D \frac{\partial^2 P_c}{\partial I^2} \right] (I - A)^2 dI + \frac{\dot{M}}{M}. \quad (\text{A1})$$

In the last term, a factor $\sigma^2 - (1 - A)^2$ has been replaced by -1 , which is a valid approximation at all times unless the initial state is nearly indistinguishable from an eigenstate.

After expanding the integrand and using Equation (19), several partial integrations yield

$$(\dot{\sigma}^2) = -2k\sigma^2 + 2D + 2\lambda \langle (I - A)^3 \rangle + \frac{\dot{M}}{M}. \quad (\text{A2})$$

Equation (A2) relies on the vanishing of P_c at the sink and its concentration around $I = A$. It also uses the fact that the probability current J_1 at the right sink is given by $-D \frac{\partial P_c(1,t)}{\partial I} = -\dot{M}$.

We will argue that the last two terms in Equation (A2) can be neglected if some extra constraints consistent with Expression (20) are imposed. Then the differential equation is solved by

$$\sigma^2 = \frac{D}{k} (1 - e^{-2kt}), \quad (\text{A3})$$

and so Equation (21) applies after a brief relaxation period.

The cubic term in Equation (A2) can be neglected if

$$\langle (I - A)^3 \rangle \ll k\sigma^2/\lambda, \quad (\text{A4})$$

which is consistent with Approximation (21) if the third normalized central moment $\langle (I - A)^3 \rangle / \sigma^3$ is $O(1)$, and the condition

$$r =: \lambda\sigma/k \approx \lambda D^{1/2}/k^{3/2} \ll 1 \quad (\text{A5})$$

is satisfied. A good indication that the moment condition holds is the behavior of a truncated Gaussian, which is the approximate form of the probability spike in the BCM model. Its computable third central moment is negative for all truncations, but never less than -0.3 , i.e. $O(1)$. Regardless, the inequality in Equation (A5) can always be achieved by making D small enough.

By introducing another minor constraint, ignoring the final term in Equation (A2) will be shown consistent with Equation (21). To see this note that $\dot{M}$ is the rate at which probability diffuses across the right barrier at x_0 in Figure 6, and it can be kept small if the curvature $\sim k$ is great enough. Demonstrating this requires examining the rate at which a probability element dM leaves the spike at $x = 0$ ($x =: I - A$) and enters the high velocity domain past $x_0 \approx k/\lambda$.

The mean time T for a trip over a distance from x to x_0 is described by the “backward Fokker-Planck (or Kolmogorov) equation”:

$$-1 = v(x)T'(x) + DT''(x), \quad (\text{A6})$$

with

$$v(x) = -kx + D[x^2 - \sigma^2] \quad (\text{and } \sigma^2 =: D/k). \quad (\text{A7})$$

Equation (A6) can be solved for T' by use of an exponential integrating factor, and then for T by integrating again, with the result

$$T(x) = \frac{1}{D} \int_x^{x_0} dz e^{V(z)/D} \int_{-\infty}^z dy e^{-V(y)/D} \quad (0 \leq x \leq x_0), \quad (\text{A8})$$

where V is an “effective drift potential” defined as any solution to:

$$V'(x) = -v(x), \quad x \in [0, x_0]. \quad (\text{A9})$$

A detailed calculation using the approximate method of steepest descents to evaluate both integrals, along with the constraints of Equation (20) produces:

$$T(0) \approx (2\pi/k)e^{1/6r^2}. \quad (\text{A10})$$

Therefore, in view of Equation (A5), a large value of k implies a long time of “first passage” $T(0)$. This is the mean time for dM to travel to the high positive-velocity region and be swept into the right sink. Therefore, the probability mass in the spike can change slowly compared to its variance. Its relative rate of change is

$$\frac{|\dot{M}|}{M} \approx \frac{1}{T(0)} \approx \frac{k}{2\pi} e^{-\frac{1}{6r^2}}. \quad (\text{A11})$$

The condition for ignoring the final term of Equation (A2) is then

$$\frac{k}{2\pi} e^{-\frac{1}{6r^2}} < < 2D, \quad (\text{A12})$$

which can be satisfied consistently with Expression (20). Thus, the constraints in (A5) and (A12) can insure that $\sigma^2 \approx D/k$.

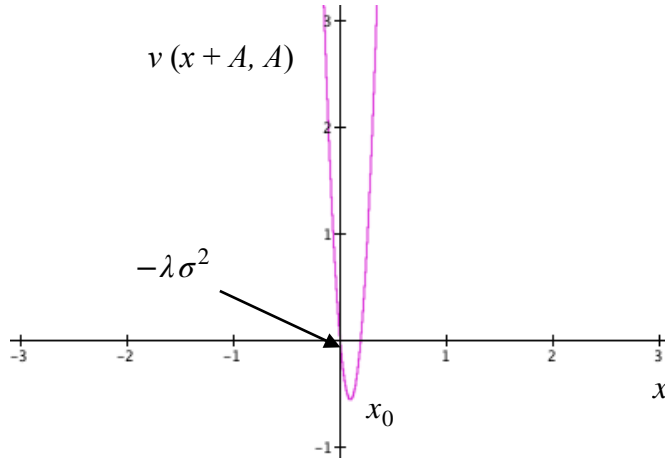


Figure 6. A spike at $x = I - A = 0$ drifts slowly leftward and remains narrow because of the restoring force k . It also sheds probability dx , which proceeds to the positive velocity region $x > x_0$ with a low probability, when the bi-directional diffusion strength D is small.

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