

A Formal Mathematical Framework for Relationships

Bhekuzulu Khumalo

Private Researcher,

305 = 19 Northern Heights Drive, Richmond Hill, Ontario, L4B 4M4, Canada

Bhekuzulu.khumalo@gmail.com

Short biography

A Formal Mathematical Framework for Relationships

Abstract This work develops a formal mathematical framework for relationships, grounded in the observation that every entity—ranging from elementary particles to galaxies—exists within and is shaped by relationships. Maintaining these relationships requires energy, yet no general abstract language currently captures their structure or behavior. The purpose of this paper is to introduce such a language: a unified, set-theoretic formalization of relationships applicable across domains, including physics, chemistry, biology, and human interaction.

The theory presented here arises from a practical need to supplement the view that reality is fundamentally relational with a rigorous mathematical abstraction. It provides a foundation for classifying and categorizing phenomena by revealing the underlying relational structures that give rise to such classifications. As a new interpretive framework, it is expected that further developments will emerge over time, much as algebra and calculus evolved from their initial formulations. Although general in scope, the theory remains closely connected to counting and combinatorial principles, with set theory serving as the primary tool for achieving maximal conceptual simplicity.

Keywords: relationships, categorization, set theory, logic

Subject Classification: 03E99 03E75 97E60

Introduction The universe, all existence is about relationships. A thing itself is made up of the relationships first and foremost of basic particles. These things then get into relationships with each other. There is a need for a formal language of mathematics of abstraction to deal with these relationships, that shows that all relationships are basically all follow the same rules. In generalizing relationships, a foundation is set to allow those interested in relationships to purely look at relationships from a mathematical, logical viewpoint.

Starting of with a relationship of an element, E being in a relationship with itself, the paper goes on to eventually explain more complex relationships of up to five elements, E_1, E_2, E_3, E_4 , and E_5 . Using set notation helps in making this mathematics dealing with relationships to be better understood. After 5 elements it becomes too complex to do by hand. However, there can be up to i elements, E_i elements.

As with all mathematics it will develop further over time. This means the relationships identified are far from complete. These are the initial relationships identified as a first step. In the future further logic will be applied and likely far more relationships will be identified.

Mathematics is considered a universal language. Hence, even if an author does not understand a certain language, or for example English is their second or third language, mathematicians over come that with symbolic abstractions.

1. The Operators To look at relationships in an abstract manner there needs to be an operator. Relationships only has two operators:

r = relationship

r^{-1} = inverse of a relationship

r creates a relationship, r^{-1} reverses the relationship

2. A Singular Element There is a thing, anything, that thing shall be named element A . A can be considered as part of a set containing one element, the element itself, in this case A :
 $\{A\}$

A thing is first and foremost in a relationship with itself. A is first and foremost in a relationship with itself. All things are in their own set containing themselves.

$A \ r \ A = A$

A is in a relationship, r , with A will just be A , the same element.

$$A \text{ r}^{-1} A = A$$

A is its own inverse and thus its own identity.

One element is **1 relationship**. This is illustrated by Table 1.

Table 1: Number of Relationships with 1 Element
Number of Total Relationships
1

3. Two Elements There are two elements A and B:

$$A \neq B$$

They can be considered as two sets each with one element.

$$\{A\} \text{ and } \{B\}$$

$$\{A\} \cap \{B\} = \emptyset$$

The elements are unique.

3.2. Relationship of two Elements

A getting into a relationship with B can be written as:

$$A \text{ r } B = AB$$

The process is commutative

$$A \text{ r } B = B \text{ r } A = AB = BA$$

A r B can be expressed as a set

$$\{A \text{ r } B\}$$

The result of A r B is a new element.

$$A \text{ r } B = AB = 1$$

1 is neither A nor B, it is a relationship of the two.

Thus, two elements lead to **1 possible relationship** apart from the elements remaining in a relationship with themselves. Including the elements themselves there are **3 relationships**. This is illustrated by Table 2.

Table 2: Number of Possible Relationships with 2 Elements	
Number of Possible Resultant Relationships	Number of Total Possible Relationships
1	3

Table 2 uses the word possible relationships because A and B might not get into a relationship thus no resultant relationship. That is why the total number of relationships must count the individual elements.

And it must be understood that the tables showing number of relationships are only those relationships that have been identified, in the future far more relationships might be identified.

3.2.1. Categorization of Relationships with 2 Elements

Given two elements A and B. There

are three possible relationships, meaning there are three classes of categorization. A and B can each be classified as unique classes of things. A will be classified as its own thing. B will be classified as its own thing as well as their relationship A r B = AB = BA will be classified as its own thing.

3.3. Inverse Relationship of two Elements

An inverse relationship isolates the elements, it can be written

as:

$$A \text{ r}^{-1} B = A, B$$

The process is commutative

$$A \text{ r}^{-1} B = B \text{ r}^{-1} A = A, B = B, A$$

$A \text{ r}^{-1} B$ can be expressed in set notation as
 $\{A\}, \{B\}$

3.4. Expressing Inverse Relationships as Set Operation

expressed in set notation.

$$\{A, B\} \text{r}^{-1} = \{A\} \{B\} = A, B$$

$A \text{ r}^{-1} B = A, B$ might be confusing, it can be

Given that AB is a new element 1 , the inverse relationship can be expressed as:

$$1 \text{r}^{-1} = A, B = \{A\} \{B\}$$

4. Three Elements

There are three elements, A , B , and C . Each element is unique.

$$A \neq B \neq C \neq A$$

They can be considered as three sets each with one element

$$\{A\}, \{B\}, \text{ and } \{C\}$$

4.2 Relationship of Three Elements

A , B , and C getting into a basic relationship can be written as:

$$A \text{ r } B \text{ r } C = ABC$$

The process is commutative. Given there are three elements, the same process can be arranged in $3!$ Ways, that is $3 \times 2 \times 1$ ways = 6.

$$A \text{ r } B \text{ r } C = A \text{ r } C \text{ r } B = B \text{ r } A \text{ r } C = B \text{ r } C \text{ r } A = C \text{ r } A \text{ r } B = C \text{ r } B \text{ r } A$$

$$A \text{ r } B \text{ r } C = ABC \quad A \text{ r } C \text{ r } B = ACB \quad B \text{ r } A \text{ r } C = BAC$$

$$B \text{ r } C \text{ r } A = BCA \quad C \text{ r } A \text{ r } B = CAB \quad C \text{ r } B \text{ r } A = CBA$$

In set notation

$$A \text{ r } B \text{ r } C = \{A B C\} = \{A C B\} = \{B A C\} = \{B C A\} = \{C A B\} = \{C B A\}$$

The above implies that relationships can be associative. As will be seen, relationships however need not be associative, the majority will not be, but they will all be commutative.

The result of $A \text{ r } B \text{ r } C$ is a new element

$$A \text{ r } B \text{ r } C = ABC = 1$$

1 is neither A , B , or C but their relationship.

4.2.2 Further Relationships of Three Elements

In 3 above it was shown that there can be relationships around two elements.

$$A \text{ r } B = AB = BA.$$

AB is its own element unique from both A and B . This unique element can itself get into a relationship with another element, this element shall be called C .

There are two elements

AB and C

These two elements have a relationship

$$AB \text{ r } C = AB, C \neq ABC$$

In set notation

$$\{AB\} \text{ r } \{C\} = \{\{AB\} \{C\}\} \neq \{A\} \text{ r } \{B\} \text{ r } \{C\} = \{A B C\}$$

Given C is an element for ease

$$\{\{AB\} \{C\}\} = \{\{AB\} C\}$$

Given

$\{\{AB\} C\} \neq \{A B C\}$ the relationships need not be associative.

4.2.2.1 Need for Abstraction of Relationships

The idea of abstracting relationships comes from a need to fully understand relationships in all aspects. To fully comprehend what has been discussed above, to understand that $ABC \neq AB, C$ or in set notation, $\{\{AB\} \{C\}\} \neq \{A B C\}$. Let A be hydrogen, B oxygen, and C Sand. Sand is silicon dioxide, SiO_2 in chemical notation. AB is the relationship between hydrogen and oxygen water. One can get a cup of water and mix it with sand. They will get water and sand, $\{\{AB\} \{C\}\} = AB \text{ r } C$. There can be a relationship of hydrogen, oxygen and silicon dioxide, $A \text{ r } B \text{ r } C = ABC = \{A B C\}$ which is silicic acid, in chemical notation H_4SiO_4 . Obviously, the process is more complex, but this is for understanding. Sand in water and silicic acid have the same elements but are not the same thing.

4.2.2 continued

There are other relationships from a purely abstract/ mathematical aspect of the three elements A, B , and C . It all starts with analyzing what was set in 4.2 above:

$$A \text{ r } B \text{ r } C = \{A B C\} = \{A C B\} = \{B A C\} = \{B C A\} = \{C A B\} = \{C B A\}$$

Given that relationships are commutative, in that $AB = BA$ then

$$AB \text{ r } C = BA \text{ r } C = C \text{ r } AB = C \text{ r } BA = \{\{AB\} C\} = \{C \{AB\}\}$$

To clarify and make it easier for later, it can be said $AB \text{ r } C$ is 2, a unique element, a unique thing.

$$AC \text{ r } B = CA \text{ r } B = B \text{ r } AC = B \text{ r } CA = \{\{AC\} B\} = \{B \{AC\}\}$$

$AC \text{ r } B$ is a new element and can be considered as 3

$$BC \text{ r } A = CB \text{ r } A = A \text{ r } BC = A \text{ r } CB = \{\{BC\} A\} = \{A \{BC\}\}$$

In the case of three elements $BC \text{ r } A$ is a new element and can be considered as 4

Thus, three elements lead to **4 possible** relationships apart from the elements remaining in a relationship with themselves. Given there are three elements that means **7 possible relationships**. This is illustrated by Table 3.

Table 3: Number of Possible Relationships with 3 Elements	
Number of Possible Resultant Relationships	Number of Total Possible Relationships
4	7

4.2.2.1 Reality of Relationships

Table 3 assumes that Given elements A, B , and C , they will either remain not in a relationship, or all get into a relationship. This does not need to be the case.

$$\text{Take } AB \text{ r } C = \{\{AB\} C\}$$

This need not be the case, it might be impossible for C to have a relationship with AB , meaning

$$AB \text{ r } C = \{AB\} C = \{AB\} \{C\} \text{ meaning nothing changes}$$

$$AB \text{ r } C = \{AB\} \{C\} \neq \{\{AB\} C\}$$

Therefore, there are three further relationships to consider

AB, AC , and BC

Table 4 is the correct description of possible relationships given 3 elements.

Table 4: Reality of the Number of Possible Relations with 3 Elements	
Number of Possible Resultant Relationships if All Elements Get Into Relationship	4
Number of Possible Resultant Relationships minus One Element	3
Total Number of Possible Resultant Relationships	7
All 3 Elements Cannot Have a Relationship	3
Number of Total Possible Relationships	10

Note that for example $A \{BC\}$ is one type of relationship even though there are two relationships, we are looking at types of relationships.

4.3 Categorization of Relationships with three Elements

Given three elements A, B, and C

when it comes to relationships the categorization is more evident. Firstly, the three elements are categorized as themselves given that each element is unique, no elements are the same.

$\{A\}$ is its own category

$\{B\}$ is its own category

$\{C\}$ is its own category.

They are their own category because each element is unique.

If the elements A, B, and C all get into a relationship:

$A \text{ r } B \text{ r } C = ABC$, ABC is its own unique element.

Set notation has the advantage that it is understood that these elements become part of a set:

$\{A\} \text{ r } \{B\} \text{ r } \{C\} = \{A \text{ B } C\}$

$\{A \text{ B } C\}$ is a unique element.

$\{A \text{ B } C\} \neq \{A\} \neq \{B\} \neq \{C\}$

$\{A \text{ B } C\}$ must be categorized as its own element that is made up of A, B, and C.

Categorization is about structure:

$A \text{ r } B \text{ r } C = \{A \text{ B } C\} = \{A \text{ C } B\} = \{B \text{ A } C\} = \{B \text{ C } A\} = \{C \text{ A } B\} = \{C \text{ B } A\}$

There is structurally no difference between $\{A \text{ B } C\}$ and $\{C \text{ A } B\}$. This should not be difficult to comprehend. There are 3 people, Bill, Thando and Wenbo. They are in a relationship and have an activity, there is no difference from saying Bill, Thando and Wenbo are in a relationship and saying Wenbo, Bill, and Thando are in a relationship, there is no structural difference.

$A \text{ r } B \text{ r } C = \{A \text{ B } C\}$ illustrates categorization from the simple, the simple being the single elements $\{A\}$, $\{B\}$, and $\{C\}$ to something more complex $\{A \text{ B } C\}$ in more ways than with just two elements. A, B and C get into a relationship to create something more complex $\{A \text{ B } C\}$, however, $\{A \text{ B } C\}$ is its own element unique from A, B, and C. Imagine water, both hydrogen and oxygen are unique. They come together to create water, and water is unique from both hydrogen and oxygen.

Take

$\{A \{BC\}\} = \{\{BC\} = A\}$ are the same thing as they have the same structure. But

$\{A \{BC\}\} \neq \{A \text{ B } C\}$ would be categorized differently because though made of the same elements the structures are different. The structures are different because of the different relationships.

The title of Table 6 is “Relationships of Type $E_1 \text{ r } E_2 E_3 E_4 = \{E_1 \{E_2 \{E_3 E_4\}\}$ ”. E_i just means the i^{th} element.

What does $A \text{ r } BCD$ mean. It means that BCD is an element. It is two elements having a relationship. It is not a relationship between A and the individual components of BCD . BCD has already being created by a relationship.

$$B \text{ r } C \text{ r } D = \{B \ C \ D\} = BCD = E_1$$

Thus

$A \text{ r } BCD$ can be considered as $A \text{ r } E_1$, two elements getting into a relationship.

Two things or even n things do not necessarily need to get into a relationship. $A \text{ r } BCD$ need to lead to a relationship, the two things could be incompatible. Thus

$A \text{ r } BCD$ need not necessarily lead to $\{A \ \{BCD\}\}$ it could be such that:

$$A \text{ r } BCD = A \ \{B \ C \ D\}$$

$A \ \{B \ C \ D\}$ means that A and BCD are not in the same set where as $\{A \ \{B \ C \ D\}\}$ means A and BCD are in the same set and thus have a relationship.

$$A \ \{B \ C \ D\} \neq \{A \ \{B \ C \ D\}\}$$

Table 7 illustrates this relationship where $A \text{ r } BCD$ does not lead to a relationship. However:

$A \ BCD = A \ \{BCD\}$ is a unique relationship given the four elements $A \ B \ C \ D$.

Table 7: Relationships of Type $E_1 \text{ r } E_2 E_3 E_4 = E_1 \ \{E_2 \ E_3 \ E_4\}$						
Relations	Equivalent Relationships					
$A \text{ r } BCD$	$A \ \{B \ C \ D\}$	$A \ \{B \ D \ C\}$	$A \ \{C \ B \ D\}$	$A \ \{C \ D \ B\}$	$A \ \{D \ B \ C\}$	$A \ \{D \ C \ B\}$
$B \text{ r } ACD$	$B \ \{A \ C \ D\}$	$B \ \{A \ D \ C\}$	$B \ \{C \ A \ D\}$	$B \ \{C \ D \ A\}$	$B \ \{D \ A \ C\}$	$B \ \{D \ C \ A\}$
$C \text{ r } ABD$	$C \ \{A \ B \ D\}$	$C \ \{A \ D \ B\}$	$C \ \{B \ A \ D\}$	$C \ \{B \ D \ A\}$	$C \ \{D \ A \ B\}$	$C \ \{D \ B \ A\}$
$D \text{ r } ABC$	$D \ \{A \ B \ C\}$	$D \ \{A \ C \ B\}$	$D \ \{B \ A \ C\}$	$D \ \{B \ C \ A\}$	$D \ \{C \ A \ B\}$	$D \ \{C \ B \ A\}$
number of relationships = 4						
number of combinations per relationship = 6						

Given there are four elements there can exist relationships whereby two elements get into a relationship with two other elements. In set notation:

$$\{A \ B\} \text{ r } \{C \ D\} = \{\{A \ B\} \ \{C \ D\}\}$$

Table 8 shows relationships equivalent to $\{\{E_1 \ E_2\} \ \{E_3 \ E_4\}\}$, $\text{r} \equiv \{\{E_1 \ E_2\} \ \{E_3 \ E_4\}\}$

Table 8: Relationships of Type $E_1 E_2 \text{ r } E_3 E_4 = \{\{E_1 E_2\} \ \{E_3 E_4\}\}$		
$\text{r} \equiv \{\{A \ B\} \ \{C \ D\}\}$	$\text{r} \equiv \{\{A \ C\} \ \{B \ D\}\}$	$\text{r} \equiv \{\{A \ D\} \ \{B \ C\}\}$
$\{\{A \ B\} \ \{C \ D\}\}$	$\{\{A \ C\} \ \{B \ D\}\}$	$\{\{A \ D\} \ \{B \ C\}\}$
$\{\{A \ B\} \ \{D \ C\}\}$	$\{\{A \ C\} \ \{D \ B\}\}$	$\{\{A \ D\} \ \{C \ B\}\}$
$\{\{B \ A\} \ \{C \ D\}\}$	$\{\{B \ D\} \ \{A \ C\}\}$	$\{\{B \ C\} \ \{A \ D\}\}$
$\{\{B \ A\} \ \{D \ C\}\}$	$\{\{B \ D\} \ \{C \ A\}\}$	$\{\{B \ C\} \ \{D \ A\}\}$
$\{\{C \ D\} \ \{A \ B\}\}$	$\{\{C \ A\} \ \{B \ D\}\}$	$\{\{C \ B\} \ \{A \ D\}\}$
$\{\{C \ D\} \ \{B \ A\}\}$	$\{\{C \ A\} \ \{D \ B\}\}$	$\{\{C \ B\} \ \{D \ A\}\}$
$\{\{D \ C\} \ \{A \ B\}\}$	$\{\{D \ B\} \ \{A \ C\}\}$	$\{\{D \ A\} \ \{B \ C\}\}$
$\{\{D \ C\} \ \{B \ A\}\}$	$\{\{D \ B\} \ \{C \ A\}\}$	$\{\{D \ A\} \ \{C \ B\}\}$
number of relationships = 3		
combinations per relationship = 8		

Elements do not need to have a relationship that is why it is possible number of relationships. This means that in set notation:

$$\{E_1 \ E_2\} \text{ r } \{E_3 \ E_4\} = \{E_1 \ E_2\} \ \{E_3 \ E_4\} \neq \{\{E_1 \ E_2\} \ \{E_3 \ E_4\}\}$$

$\{E_1 E_2\} \{E_3 E_4\}$ are possible relationships. These are illustrated by Table 9.

Table 9: Relationships of Type $E_1 E_2 \text{ r } E_3 E_4 = \{E_1 E_2\} \{E_3 E_4\}$		
$\text{r} \equiv \{A B\} \{C D\}$	$\text{r} \equiv \{A C\} \{B D\}$	$\text{r} \equiv \{A D\} \{B C\}$
$\{A B\} \{C D\}$	$\{A C\} \{B D\}$	$\{A D\} \{B C\}$
$\{A B\} \{D C\}$	$\{A C\} \{D B\}$	$\{A D\} \{C B\}$
$\{B A\} \{C D\}$	$\{B D\} \{A C\}$	$\{B C\} \{A D\}$
$\{B A\} \{D C\}$	$\{B D\} \{C A\}$	$\{B C\} \{D A\}$
$\{C D\} \{A B\}$	$\{C A\} \{B D\}$	$\{C B\} \{A D\}$
$\{C D\} \{B A\}$	$\{C A\} \{D B\}$	$\{C B\} \{D A\}$
$\{D C\} \{A B\}$	$\{D B\} \{A C\}$	$\{D A\} \{B C\}$
$\{D C\} \{B A\}$	$\{D B\} \{C A\}$	$\{D A\} \{C B\}$
number of relationships = 3		
combinations per relationship = 8		

Take Table 9, relationships of type $\{E_1 E_2\} \{E_3 E_4\}$. This can be taken further to relationships of type $\{E_1\} \{E_2\} \{E_3 E_4\}$. Thus $\{A B\} \{C D\}$ we get two more relationships $\{A\} \{B\} \{C D\}$ and $\{A B\} \{C\} \{D\}$

Another two from $\{A C\} \{B D\}$
 $\{A\} \{C\} \{B D\}$ and $\{A C\} \{B\} \{D\}$

Another two from $\{A D\} \{B C\}$
 $\{A\} \{D\} \{B C\}$ and $\{A D\} \{B\} \{C\}$

That is **6 possible relationships**.

Lastly None of the elements need go into a relationship. Thus

$$A \text{ r } B \text{ r } C \text{ r } D = A B C D$$

In set notation

$$\{A\} \text{ r } \{B\} \text{ r } \{C\} \text{ r } \{D\} = \{A\} \{B\} \{C\} \{D\} \text{ representing } \mathbf{4 \text{ possible relationships.}}$$

5.3. Total Number of Identified Relationships with four Elements

Table 10 illustrates the summary of possible relations given four elements. The total number is 19 possible relationships.

Table 10: Summary of Possible Relationships with 4 Elements	
Type of Relationship	Possible number of Relationships
$\{E_1 E_2 E_3 E_4\}$	1
$\{E_1 \{E_2 E_3 E_4\}$	4
$E_1 \{E_2 E_3 E_4\}$	4
$\{\{E_1 E_2\} \{E_3 E_4\}\}$	3
$\{E_1 E_2\} \{E_3 E_4\}$	3
$\{E_1\} \{E_2\} \{E_3 E_4\}$	6
$\{E_1\} \{E_2\} \{E_3\} \{E_4\}$	4
Total	25

Table 10 above is a summary of “possible” relationships, in the future, many more relationships could be identified. It is the way mathematics works

5.4 Categorization of Relationships with four Elements

Things are categorized according to structure. Take Table 6. There are four relations. Each type of relation is a type of classification a categorization. The four types of relations can be considered as four types of categorizations. Though all have the same elements, the relations are different. Things are categorized according to their structure. What is applicable in the explanation of categorization for Table 6 is applicable for Tables 7, 8, and 9.

5.5. Inverse Relationships of Four Element

Take the relationship $\{A B C D\}$ the relationship of type $\{E_1, E_2, E_3, E_4\}$. The inverse relationship can be expressed as

$$\{A B C D\}r^{-1} = \{A\} \{B\} \{C\} \{D\}$$

In general terms

$$\{E_1 E_2 E_3 E_4\}r^{-1} = \{E_1\} \{E_2\} \{E_3\} \{E_4\}$$

Take the relationship of type $\{E_1 \{E_2 E_3 E_4\}\}$. The inverse relationship can be expressed as

$$\{E_1 r^{-1} \{E_2 E_3 E_4\}\} = \{E_1\} \{E_2 E_3 E_4\}$$

$\{E_2 E_3 E_4\}$ is its own element but further inverse relationship can be expressed as

$$\{E_1\} \{E_2 E_3 E_4\}r^{-1} = \{E_1\} \{E_2\} \{E_3\} \{E_4\}$$

Take the relationship of type $\{\{E_1 E_2\} \{E_3 E_4\}\}$. The inverse relationship can be expressed as

$$\{\{E_1 E_2\} r^{-1} \{E_3 E_4\}\} = \{E_1 E_2\} \{E_3 E_4\}$$

The resultant elements $\{E_1 E_2\}$ and $\{E_3 E_4\}$ can have one of the elements getting into an inverse relationship, this can be expressed as

$$\{E_1 r^{-1} E_2\} \{E_3 E_4\} = \{E_1\} \{E_2\} \{E_3 E_4\}$$

Both elements can have an inverse relationship at the same time, and this would be expressed as

$$\{E_1 r^{-1} E_2\} \{E_3 r^{-1} E_4\} = \{E_1\} \{E_2\} \{E_3\} \{E_4\}$$

6. Five Elements

There are five elements, A, B, C, D, and E. Each element is unique. They can be considered as five sets each with one element:

$\{A\}$, $\{B\}$, $\{C\}$, $\{D\}$, and $\{E\}$.

6.2. Relationships with Five Elements

A, B, C, D, and E getting into a relationship with all elements involved can be stated as:

$$A r B r C r D r E = ABCDE$$

In set notation for answer

$$A r B r C r D r E = \{A B C D E\}$$

In complete set notation

$$\{A\} r \{B\} r \{C\} r \{D\} r \{E\} = \{A B C D E\}$$

The process is commutative and associative. Given there are four elements, the same process can be arranged in $5!$ ways, that is $5 \times 4 \times 3 \times 2 \times 1$ ways = 120. All these 120 ways are however one relationship. All 120 ways can be considered one category when it comes to classification of things.

This is illustrated by Table 11.

Table 11: Relationships Equivalent to {A B C D E}					
{A B C D E}	{A B C E D}	{A B D C E}	{A B D E C}	{A B E C D}	{A B E D C}
{A C B D E}	{A C B E D}	{A C D B E}	{A C D E B}	{A C E B D}	{A C E D B}
{A D B C E}	{A D B E C}	{A D C B E}	{A D C E B}	{A D E B C}	{A D E C B}
{A E B C D}	{A E B D C}	{A E C B D}	{A E C D B}	{A E D B C}	{A E D C B}
{B A C D E}	{B A C E D}	{B A D C E}	{B A D E C}	{B A E C D}	{B A E D C}
{B C A D E}	{B C A E D}	{B C D A E}	{B C D E A}	{B C E A D}	{B C E D A}
{B D A C E}	{B D A E C}	{B D C A E}	{B D C E A}	{B D E A C}	{B D E C A}
{B E A D C}	{B E A D C}	{B E C D A}	{B E C A D}	{B E D A C}	{B E D C A}
{C A B D E}	{C A B E D}	{C A D B E}	{C A D E B}	{C A E B D}	{C A E D B}
{C B A D E}	{C B A E D}	{C B D A E}	{C B D E A}	{C B E A D}	{C B E D A}
{C D A B E}	{C D A E B}	{C D B A E}	{C D B E A}	{C D E A B}	{C D E B A}
{C E A B D}	{C E A D B}	{C E B A D}	{C E B D A}	{C E D A B}	{C E D B A}
{D A B C E}	{D A B E C}	{D A C B E}	{D A C E B}	{D A E B C}	{D A E C B}
{D B A C E}	{D B A E C}	{D B C A E}	{D B C E A}	{D B E A C}	{D B E C A}
{D C A B E}	{D C A E B}	{D C B A E}	{D C B E A}	{D C E A B}	{D C E B A}
{D E A B C}	{D E A C B}	{D E B A C}	{D E B C A}	{D E C A B}	{D E C B A}
{E A B C D}	{E A B D C}	{E A C B D}	{E A C D B}	{E A D B C}	{E A D C B}
{E B A C D}	{E B A D C}	{E B C A D}	{E B C D A}	{E B D A C}	{E B D C A}
{E C A B D}	{E C A D B}	{E C B A D}	{E C B D A}	{E C D A B}	{E C D B A}
{E D A B C}	{E D A C B}	{E D B A C}	{E D B C A}	{E D C A B}	{E D C B A}
number of relationships = 1					
number of permutations = 120					

6.2.2. Relationship of Type {E₁ {E₂ E₃ E₄ E₅}}

In this case {B C D E} is an element on its own. A got into a relationship with this element and {A {A B C D}} is the result of the relationship. Table 12 illustrates these relationships.

Table 12: Relationships of Type $E_1 \text{ r } E_2 E_3 E_4 E_5 = \{E_1 \{E_2 E_3 E_4 E_5\}\}$						
r	$\{A \{B C D E\}\}$	$\{A \{B C E D\}\}$	$\{A \{B D C E\}\}$	$\{A \{B D E C\}\}$	$\{A \{B E C D\}\}$	$\{A \{B E D C\}\}$
$\equiv$	$\{A \{C B D E\}\}$	$\{A \{C B E D\}\}$	$\{A \{C D B E\}\}$	$\{A \{C D E B\}\}$	$\{A \{C E B D\}\}$	$\{A \{C E D B\}\}$
$\{A \{B C D E\}\}$	$\{A \{D B C E\}\}$	$\{A \{D B E C\}\}$	$\{A \{D C B E\}\}$	$\{A \{D C E B\}\}$	$\{A \{D E B C\}\}$	$\{A \{D E C B\}\}$
	$\{A \{E B C D\}\}$	$\{A \{E B D C\}\}$	$\{A \{E C B D\}\}$	$\{A \{E C D B\}\}$	$\{A \{E D B C\}\}$	$\{A \{E D C B\}\}$
r	$\{B \{A C D E\}\}$	$\{B \{A C E D\}\}$	$\{B \{A D C E\}\}$	$\{B \{A D E C\}\}$	$\{B \{A E C D\}\}$	$\{B \{A E D C\}\}$
$\equiv$	$\{B \{C A D E\}\}$	$\{B \{C A E D\}\}$	$\{B \{C D A E\}\}$	$\{B \{C D E A\}\}$	$\{B \{C E A D\}\}$	$\{B \{C E D A\}\}$
$\{B \{A C D E\}\}$	$\{B \{D A C E\}\}$	$\{B \{D A E C\}\}$	$\{B \{D C A E\}\}$	$\{B \{D C E A\}\}$	$\{B \{D E A C\}\}$	$\{B \{D E C A\}\}$
	$\{B \{E A D C\}\}$	$\{B \{E A D C\}\}$	$\{B \{E C D A\}\}$	$\{B \{E C A D\}\}$	$\{B \{E D A C\}\}$	$\{B \{E D C A\}\}$
r	$\{C \{A B D E\}\}$	$\{C \{A B E D\}\}$	$\{C \{A D B E\}\}$	$\{C \{A D E B\}\}$	$\{C \{A E B D\}\}$	$\{C \{A E D B\}\}$
$\equiv$	$\{C \{B A D E\}\}$	$\{C \{B A E D\}\}$	$\{C \{B D A E\}\}$	$\{C \{B D E A\}\}$	$\{C \{B E A D\}\}$	$\{C \{B E D A\}\}$
$\{C \{A B D E\}\}$	$\{C \{D A B E\}\}$	$\{C \{D A E B\}\}$	$\{C \{D B A E\}\}$	$\{C \{D B E A\}\}$	$\{C \{D E A B\}\}$	$\{C \{D E B A\}\}$
	$\{C \{E A B D\}\}$	$\{C \{E A D B\}\}$	$\{C \{E B A D\}\}$	$\{C \{E B D A\}\}$	$\{C \{E D A B\}\}$	$\{C \{E D B A\}\}$
r	$\{D \{A B C E\}\}$	$\{D \{A B E C\}\}$	$\{D \{A C B E\}\}$	$\{D \{A C E B\}\}$	$\{D \{A E B C\}\}$	$\{D \{A E C B\}\}$
$\equiv$	$\{D \{B A C E\}\}$	$\{D \{B A E C\}\}$	$\{D \{B C A E\}\}$	$\{D \{B C E A\}\}$	$\{D \{B E A C\}\}$	$\{D \{B E C A\}\}$
$\{D \{A B C E\}\}$	$\{D \{C A B E\}\}$	$\{D \{C A E B\}\}$	$\{D \{C B A E\}\}$	$\{D \{C B E A\}\}$	$\{D \{C E A B\}\}$	$\{D \{C E B A\}\}$
	$\{D \{E A B C\}\}$	$\{D \{E A C B\}\}$	$\{D \{E B A C\}\}$	$\{D \{E B C A\}\}$	$\{D \{E C A B\}\}$	$\{D \{E C B A\}\}$
r	$\{E \{A B C D\}\}$	$\{E \{A B D C\}\}$	$\{E \{A C B D\}\}$	$\{E \{A C D B\}\}$	$\{E \{A D B C\}\}$	$\{E \{A D C B\}\}$
$\equiv$	$\{E \{B A C D\}\}$	$\{E \{B A D C\}\}$	$\{E \{B C A D\}\}$	$\{E \{B C D A\}\}$	$\{E \{B D A C\}\}$	$\{E \{B D C A\}\}$
$\{E \{A B C D\}\}$	$\{E \{C A B D\}\}$	$\{E \{C A D B\}\}$	$\{E \{C B A D\}\}$	$\{E \{C B D A\}\}$	$\{E \{C D A B\}\}$	$\{E \{C D B A\}\}$
	$\{E \{D A B C\}\}$	$\{E \{D A C B\}\}$	$\{E \{D B A C\}\}$	$\{E \{D B C A\}\}$	$\{E \{D C A B\}\}$	$\{E \{D C B A\}\}$
number of relationships = 5						
combinations per relationship = 24						

There are 5 types of relationships. Each relationship has 24 combinations that are similar. For example, $\{C \{A B D E\}\}$ can be expressed in 24 different ways. 48 different ways if one also considers expressing $\{C \{A B D E\}\}$ as $\{\{A B D E\} C\}$. Permutations are not considered important for relationships as they do not change the structure of the resultant element

6.2.3. Relationship of Type $E_1 \{E_2 E_3 E_4 E_5\}$

relationship such that

$$A \text{ r } \{B C D E\} = A \{B C D E\} \neq \{A \{B C D E\}\}$$

$A \text{ r } \{B C D E\}$ need not necessarily end up in a

Even though $\{A \{B C D E\}\}$ is one element and $A \{B C D E\}$ is technically two elements thus two relationships it is counted as one type of relationship. The idea of abstracting relationships is to understand structure.

The type of relationship $E_1 \{E_1 E_2 E_3 E_4\}$ is illustrated by Table 13

Table 13: Relationships of Type $E_1 \text{ r } E_2 E_3 E_4 E_5 = E_1 \{E_2 E_3 E_4 E_5\}$						
r	A {B C D E}	A {B C E D}	A {B D C E}	A {B D E C}	A {B E C D}	A {B E D C}
$\equiv$	A {C B D E}	A {C B E D}	A {C D B E}	A {C D E B}	A {C E B D}	A {C E D B}
A {B C D E}	A {D B C E}	A {D B E C}	A {D C B E}	A {D C E B}	A {D E B C}	A {D E C B}
	A {E B C D}	A {E B D C}	A {E C B D}	A {E C D B}	A {E D B C}	A {E D C B}
r	B {A C D E}	B {A C E D}	B {A D C E}	B {A D E C}	B {A E C D}	B {A E D C}
$\equiv$	B {C A D E}	B {C A E D}	B {C D A E}	B {C D E A}	B {C E A D}	B {C E D A}
B {A C D E}	B {D A C E}	B {D A E C}	B {D C A E}	B {D C E A}	B {D E A C}	B {D E C A}
	B {E A D C}	B {E A D C}	B {E C D A}	B {E C A D}	B {E D A C}	B {E D C A}
r	C {A B D E}	C {A B E D}	C {A D B E}	C {A D E B}	C {A E B D}	C {A E D B}
$\equiv$	C {B A D E}	C {B A E D}	C {B D A E}	C {B D E A}	C {B E A D}	C {B E D A}
C {A B D E}	C {D A B E}	C {D A E B}	C {D B A E}	C {D B E A}	C {D E A B}	C {D E B A}
	C {E A B D}	C {E A D B}	C {E B A D}	C {E B D A}	C {E D A B}	C {E D B A}
r	D {A B C E}	D {A B E C}	D {A C B E}	D {A C E B}	D {A E B C}	D {A E C B}
$\equiv$	D {B A C E}	D {B A E C}	D {B C A E}	D {B C E A}	D {B E A C}	D {B E C A}
D {A B C E}	D {C A B E}	D {C A E B}	D {C B A E}	D {C B E A}	D {C E A B}	D {C E B A}
	D {E A B C}	D {E A C B}	D {E B A C}	D {E B C A}	D {E C A B}	D {E C B A}
r	E {A B C D}	E {A B D C}	E {A C B D}	E {A C D B}	E {A D B C}	E {A D C B}
$\equiv$	E {B A C D}	E {B A D C}	E {B C A D}	E {B C D A}	E {B D A C}	E {B D C A}
E {A B C D}	E {C A B D}	E {C A D B}	E {C B A D}	E {C B D A}	E {C D A B}	E {C D B A}
	E {D A B C}	E {D A C B}	E {D B A C}	E {D B C A}	E {D C A B}	E {D C B A}
number of relationships = 5						
combinations per relationship = 24						

There are 5 types of relationships. Each relationship has 24 combinations that are similar. For example, B {A C D E} can be expressed in 24 different ways. 48 different ways if one also considers expressing B {A C D E} as {{A C D E} B}. As mentioned above permutations are not considered important for relationships as they do not change the structure of the resultant element.

To reiterate, as time goes by, many more relationships will be identified.

6.2.4. Relationship of Type $\{E_1 E_2\} \{E_3 E_4 E_5\}$ As was mentioned in the introduction, after 5 relationships the process becomes complex, and it is not conducive to do by hand. The idea of this paper is to introduce the abstraction of relationships.

This type of relationship is illustrated by Table 14. Table 14 shows 10 relationships each one having 24 permutations. It is a lengthy table, for this reason it is included at the end.

6.2.5. Relationship of Type $\{E_1 E_2\} \{E_3 E_4 E_5\}$ Not forgetting that $\{E_1 E_2\}$ and $\{E_3 E_4 E_5\}$ are themselves elements, even though themselves are made up of elements. Elements need not have a relationship, and this in itself is a type of relationship.

This type of relationship is illustrated in Table 15. Table 15 shows 10 relationships each one have 24 permutations. As with Table 14, Table 15 is lengthy and is included at the end.

6.2.6. Decomposition of $\{E_1 E_2\} \{E_3 E_4 E_5\}$ Elements do not need to get into a relationship. For example, we can have {A B} {C} {D E}

It is understood from section 4 above that three elements have 10 possible relationships identified thus far. This implies that $\{A\ B\}$ $\{C\ D\ E\}$ can have $\{C\ D\ E\}$ decomposed adding 7 new relationships, per relationship. Thus a total of $10 \times 7 = 70$ new relationships. 6 new relationships because

1. $\{C\ D\ E\}$ has already been counted
2. $\{C\}$ $\{D\}$ $\{E\}$ can only be counted once not three times because it is part of a package with $\{A\ B\}$

$\{A\ B\}$ can only be decomposed to $\{A\}$ $\{B\}$ meaning 10 more relationships.

6.2.6. Total Identified Relationships with 5 Elements

Table 16: Summary of Possible Relationships with 4 Elements	
Type of Relationship	Possible number of Relationships
$\{E_1\ E_2\ E_3\ E_4\ E_5\}$	1
$\{E_1\ \{E_2\ E_3\ E_4\ E_5\}$	5
$E_1\ \{E_2\ E_3\ E_4\ E_5\}$	5
$\{\{E_1\ E_2\}\ \{E_3\ E_4\ E_5\}\}$	10
$\{E_1\ E_2\}\ \{E_3\ E_4\ E_5\}$	10
decompose $\{E_3\ E_4\ E_5\}$	70
decompose $\{E_1\ E_2\}$	10
$\{E_1\}\ \{E_2\}\ \{E_3\}\ \{E_4\}\ \{E_5\}$	5
Total	116

Please remember these are only identified relationships.

Categorization and inverse relationships should be understood at this moment.

Conclusion This paper has successfully introduced the concept of abstracting relationship. A practical tool for scientists as well as mathematicians. It came out of the need of such a tool for understanding relationships of all things in an abstract manner.

Acknowledgments A heartfelt thankyou must go to Krzysztof Zawisa of the Syncritic Institute for encouraging me to not stop my work into relationships even if there is no institutional support.

Summary Statement

I, Bhekuzulu Khumalo as the sole and corresponding author, confirm that I am responsible for all communication with the journal. I verify that this manuscript is original, has not been published previously, and is not under review elsewhere. I declare that there are no conflicts of interest associated with this work.

Table 14: Relationships of Type $\{E_1 E_2\} \text{ r } \{E_3 E_4 E_5\} = \{\{E_1 E_2\} \{E_3 E_4 E_5\}\}$

$\mathbb{I}^r$	$\{\{AB\}\{CDE\}\}$	$\{\{AB\}\{CED\}\}$	$\{\{AB\}\{DCE\}\}$	$\{\{AB\}\{DEC\}\}$	$\{\{AB\}\{ECD\}\}$	$\{\{AB\}\{EDC\}\}$
$\equiv$	$\{\{BA\}\{CDE\}\}$	$\{\{BA\}\{CED\}\}$	$\{\{BA\}\{DCE\}\}$	$\{\{BA\}\{DEC\}\}$	$\{\{BA\}\{ECD\}\}$	$\{\{BA\}\{EDC\}\}$
$\{\{AB\}\{CDE\}\}$	$\{\{CDE\}\{AB\}\}$	$\{\{CDE\}\{BA\}\}$	$\{\{CED\}\{AB\}\}$	$\{\{CED\}\{BA\}\}$	$\{\{DCE\}\{AB\}\}$	$\{\{DCE\}\{BA\}\}$
	$\{\{DEC\}\{AB\}\}$	$\{\{DEC\}\{BA\}\}$	$\{\{ECD\}\{AB\}\}$	$\{\{ECD\}\{BA\}\}$	$\{\{EDC\}\{AB\}\}$	$\{\{EDC\}\{BA\}\}$
$\mathbb{I}^r$	$\{\{AC\}\{BDE\}\}$	$\{\{AC\}\{BED\}\}$	$\{\{AC\}\{DBE\}\}$	$\{\{AC\}\{DEB\}\}$	$\{\{AC\}\{EBD\}\}$	$\{\{AC\}\{EDB\}\}$
$\equiv$	$\{\{CA\}\{BDE\}\}$	$\{\{CA\}\{BED\}\}$	$\{\{CA\}\{DBE\}\}$	$\{\{CA\}\{DEB\}\}$	$\{\{CA\}\{EBD\}\}$	$\{\{CA\}\{EDB\}\}$
$\{\{AC\}\{BDE\}\}$	$\{\{BDE\}\{AC\}\}$	$\{\{BDE\}\{CA\}\}$	$\{\{BED\}\{AC\}\}$	$\{\{BED\}\{CA\}\}$	$\{\{DBE\}\{AC\}\}$	$\{\{DBE\}\{CA\}\}$
	$\{\{DEB\}\{AC\}\}$	$\{\{DEB\}\{CA\}\}$	$\{\{EBD\}\{AC\}\}$	$\{\{EBD\}\{CA\}\}$	$\{\{EDB\}\{AC\}\}$	$\{\{EDB\}\{CA\}\}$
$\mathbb{I}^r$	$\{\{AD\}\{BCE\}\}$	$\{\{AD\}\{BEC\}\}$	$\{\{AD\}\{CBE\}\}$	$\{\{AD\}\{CEB\}\}$	$\{\{AD\}\{ECB\}\}$	$\{\{AD\}\{ECB\}\}$
$\equiv$	$\{\{DA\}\{BCE\}\}$	$\{\{DA\}\{BEC\}\}$	$\{\{DA\}\{CBE\}\}$	$\{\{DA\}\{CEB\}\}$	$\{\{DA\}\{ECB\}\}$	$\{\{DA\}\{ECB\}\}$
$\{\{AD\}\{BCE\}\}$	$\{\{BCE\}\{AD\}\}$	$\{\{BCE\}\{DA\}\}$	$\{\{BEC\}\{AD\}\}$	$\{\{BEC\}\{AD\}\}$	$\{\{CBE\}\{AD\}\}$	$\{\{CBE\}\{DA\}\}$
	$\{\{CEB\}\{AD\}\}$	$\{\{CEB\}\{DA\}\}$	$\{\{EBC\}\{AD\}\}$	$\{\{EBC\}\{DA\}\}$	$\{\{ECB\}\{AD\}\}$	$\{\{ECB\}\{DA\}\}$
$\mathbb{I}^r$	$\{\{AE\}\{BCD\}\}$	$\{\{AE\}\{BDC\}\}$	$\{\{AE\}\{CBD\}\}$	$\{\{AE\}\{CDB\}\}$	$\{\{AE\}\{DBC\}\}$	$\{\{AE\}\{DCB\}\}$
$\equiv$	$\{\{EA\}\{BCD\}\}$	$\{\{EA\}\{BDC\}\}$	$\{\{EA\}\{CBD\}\}$	$\{\{EA\}\{CDB\}\}$	$\{\{EA\}\{DBC\}\}$	$\{\{EA\}\{DCB\}\}$
$\{\{AE\}\{BCD\}\}$	$\{\{BCD\}\{AE\}\}$	$\{\{BCD\}\{EA\}\}$	$\{\{BDC\}\{AE\}\}$	$\{\{BDC\}\{EA\}\}$	$\{\{CBD\}\{AE\}\}$	$\{\{CBD\}\{EA\}\}$
	$\{\{CDB\}\{AE\}\}$	$\{\{CDB\}\{EA\}\}$	$\{\{DBC\}\{AE\}\}$	$\{\{DBC\}\{EA\}\}$	$\{\{DCB\}\{AE\}\}$	$\{\{DCB\}\{EA\}\}$
$\mathbb{I}^r$	$\{\{BC\}\{ADE\}\}$	$\{\{BC\}\{AED\}\}$	$\{\{BC\}\{DAE\}\}$	$\{\{BC\}\{DEA\}\}$	$\{\{BC\}\{EAD\}\}$	$\{\{BC\}\{EDA\}\}$
$\equiv$	$\{\{CB\}\{ADE\}\}$	$\{\{CB\}\{AED\}\}$	$\{\{CB\}\{DAE\}\}$	$\{\{CB\}\{DEA\}\}$	$\{\{CB\}\{EAD\}\}$	$\{\{CB\}\{EDA\}\}$
$\{\{BC\}\{ADE\}\}$	$\{\{ADE\}\{BC\}\}$	$\{\{ADE\}\{CB\}\}$	$\{\{AED\}\{BC\}\}$	$\{\{AED\}\{CB\}\}$	$\{\{DAE\}\{BC\}\}$	$\{\{DAE\}\{CB\}\}$
	$\{\{DEA\}\{BC\}\}$	$\{\{DEA\}\{CB\}\}$	$\{\{EAD\}\{BC\}\}$	$\{\{EAD\}\{CB\}\}$	$\{\{EDA\}\{BC\}\}$	$\{\{EDA\}\{CB\}\}$
$\mathbb{I}^r$	$\{\{BD\}\{ACE\}\}$	$\{\{BD\}\{AEC\}\}$	$\{\{BD\}\{CAE\}\}$	$\{\{BD\}\{CEA\}\}$	$\{\{BD\}\{EAC\}\}$	$\{\{BD\}\{ECA\}\}$
$\equiv$	$\{\{DB\}\{ACE\}\}$	$\{\{DB\}\{AEC\}\}$	$\{\{DB\}\{CAE\}\}$	$\{\{DB\}\{CEA\}\}$	$\{\{DB\}\{EAC\}\}$	$\{\{DB\}\{ECA\}\}$
$\{\{BD\}\{ACE\}\}$	$\{\{ACE\}\{BD\}\}$	$\{\{ACE\}\{DB\}\}$	$\{\{AEC\}\{BD\}\}$	$\{\{AEC\}\{DB\}\}$	$\{\{CAE\}\{BD\}\}$	$\{\{CAE\}\{DB\}\}$
	$\{\{CEA\}\{BD\}\}$	$\{\{CEA\}\{DB\}\}$	$\{\{EAC\}\{BD\}\}$	$\{\{EAC\}\{DB\}\}$	$\{\{ECA\}\{BD\}\}$	$\{\{ECA\}\{DB\}\}$
$\mathbb{I}^r$	$\{\{BE\}\{ACD\}\}$	$\{\{BE\}\{ADC\}\}$	$\{\{BE\}\{CDA\}\}$	$\{\{BE\}\{CAD\}\}$	$\{\{BE\}\{DAC\}\}$	$\{\{BE\}\{DCA\}\}$
$\equiv$	$\{\{EB\}\{ACD\}\}$	$\{\{EB\}\{ADC\}\}$	$\{\{EB\}\{CAD\}\}$	$\{\{EB\}\{CDA\}\}$	$\{\{EB\}\{DAC\}\}$	$\{\{EB\}\{DCA\}\}$
$\{\{BE\}\{ACD\}\}$	$\{\{ACD\}\{BE\}\}$	$\{\{ACD\}\{EB\}\}$	$\{\{ADC\}\{BE\}\}$	$\{\{ADC\}\{EB\}\}$	$\{\{CAD\}\{BE\}\}$	$\{\{CAD\}\{EB\}\}$
	$\{\{CDA\}\{BE\}\}$	$\{\{CDA\}\{EB\}\}$	$\{\{DAC\}\{BE\}\}$	$\{\{DAC\}\{EB\}\}$	$\{\{DCA\}\{BE\}\}$	$\{\{DCA\}\{EB\}\}$
$\mathbb{I}^r$	$\{\{CD\}\{ABE\}\}$	$\{\{CD\}\{AEB\}\}$	$\{\{CD\}\{BAE\}\}$	$\{\{CD\}\{BEA\}\}$	$\{\{CD\}\{EAB\}\}$	$\{\{CD\}\{EBA\}\}$
$\equiv$	$\{\{DC\}\{ABE\}\}$	$\{\{DC\}\{AEB\}\}$	$\{\{DC\}\{BAE\}\}$	$\{\{DC\}\{BEA\}\}$	$\{\{DC\}\{EAB\}\}$	$\{\{DC\}\{EBA\}\}$
$\{\{CD\}\{ABE\}\}$	$\{\{ABE\}\{CD\}\}$	$\{\{ABE\}\{DC\}\}$	$\{\{AEB\}\{CD\}\}$	$\{\{AEB\}\{DC\}\}$	$\{\{BAE\}\{CD\}\}$	$\{\{BAE\}\{DC\}\}$
	$\{\{BEA\}\{CD\}\}$	$\{\{BEA\}\{DC\}\}$	$\{\{EAB\}\{CD\}\}$	$\{\{EAB\}\{DC\}\}$	$\{\{EBA\}\{CD\}\}$	$\{\{EBA\}\{DC\}\}$
$\mathbb{I}^r$	$\{\{CE\}\{ABD\}\}$	$\{\{CE\}\{ADB\}\}$	$\{\{CE\}\{BAD\}\}$	$\{\{CE\}\{BDA\}\}$	$\{\{CE\}\{DAB\}\}$	$\{\{CE\}\{DBA\}\}$
$\equiv$	$\{\{EC\}\{ABD\}\}$	$\{\{EC\}\{ADB\}\}$	$\{\{EC\}\{BAD\}\}$	$\{\{EC\}\{BDA\}\}$	$\{\{EC\}\{DAB\}\}$	$\{\{EC\}\{DBA\}\}$
$\{\{CE\}\{ABD\}\}$	$\{\{ABD\}\{CE\}$	$\{\{ABD\}\{EC\}\}$	$\{\{ADB\}\{CE\}\}$	$\{\{ABD\}\{EC\}\}$	$\{\{BAD\}\{CE\}\}$	$\{\{BAD\}\{EC\}\}$
	$\{\{BDA\}\{CE\}\}$	$\{\{BDA\}\{EC\}\}$	$\{\{DAB\}\{CE\}\}$	$\{\{DAB\}\{EC\}\}$	$\{\{DBA\}\{CE\}\}$	$\{\{DBA\}\{EC\}\}$
$\mathbb{I}^r$	$\{\{DE\}\{ABC\}\}$	$\{\{DE\}\{ACB\}\}$	$\{\{DE\}\{BAC\}\}$	$\{\{DE\}\{BCA\}\}$	$\{\{DE\}\{CAB\}\}$	$\{\{DE\}\{CBA\}\}$
$\equiv$	$\{\{ED\}\{ABC\}\}$	$\{\{ED\}\{ACB\}\}$	$\{\{ED\}\{BAC\}\}$	$\{\{ED\}\{BCA\}\}$	$\{\{ED\}\{CAB\}\}$	$\{\{ED\}\{CBA\}\}$
$\{\{DE\}\{ABC\}\}$	$\{\{ABC\}\{DE\}\}$	$\{\{ABC\}\{ED\}\}$	$\{\{ACB\}\{DE\}\}$	$\{\{ACB\}\{ED\}\}$	$\{\{BAC\}\{DE\}\}$	$\{\{BAC\}\{ED\}\}$
	$\{\{BCA\}\{DE\}\}$	$\{\{BCA\}\{ED\}\}$	$\{\{CAB\}\{DE\}\}$	$\{\{CAB\}\{ED\}\}$	$\{\{CAB\}\{DE\}\}$	$\{\{CBA\}\{ED\}\}$
number of relationships = 10						
number of combinations per relationship = 24						

Table 15: Relationships of Type $\{E_1 E_2\} \text{ r } \{E_3 E_4 E_5\} = \{E_1 E_2\} \{E_3 E_4 E_5\}$									
r	$\{AB\} \{CDE\}$	$\{AB\} \{CED\}$	$\{AB\} \{DCE\}$	$\{AB\} \{DEC\}$	$\{AB\} \{ECD\}$	$\{AB\} \{EDC\}$			
$\equiv$	$\{BA\} \{CDE\}$	$\{BA\} \{CED\}$	$\{BA\} \{DCE\}$	$\{BA\} \{DEC\}$	$\{BA\} \{ECD\}$	$\{BA\} \{EDC\}$			
$\{AB\} \{CDE\}$	$\{CDE\} \{AB\}$	$\{CDE\} \{BA\}$	$\{CED\} \{AB\}$	$\{CED\} \{BA\}$	$\{DCE\} \{AB\}$	$\{DCE\} \{BA\}$			
	$\{DEC\} \{AB\}$	$\{DEC\} \{BA\}$	$\{ECD\} \{AB\}$	$\{ECD\} \{BA\}$	$\{EDC\} \{AB\}$	$\{EDC\} \{BA\}$			
r	$\{AC\} \{BDE\}$	$\{AC\} \{BED\}$	$\{AC\} \{DBE\}$	$\{AC\} \{DEB\}$	$\{AC\} \{EBD\}$	$\{AC\} \{EDB\}$			
$\equiv$	$\{CA\} \{BDE\}$	$\{CA\} \{BED\}$	$\{CA\} \{DBE\}$	$\{CA\} \{DEB\}$	$\{CA\} \{EBD\}$	$\{CA\} \{EDB\}$			
$\{AC\} \{BDE\}$	$\{BDE\} \{AC\}$	$\{BDE\} \{CA\}$	$\{BED\} \{AC\}$	$\{BED\} \{CA\}$	$\{DBE\} \{AC\}$	$\{DBE\} \{CA\}$			
	$\{DEB\} \{AC\}$	$\{DEB\} \{CA\}$	$\{EBD\} \{AC\}$	$\{EBD\} \{CA\}$	$\{EDB\} \{AC\}$	$\{EDB\} \{CA\}$			
r	$\{AD\} \{BCE\}$	$\{AD\} \{BEC\}$	$\{AD\} \{CBE\}$	$\{AD\} \{CEB\}$	$\{AD\} \{ECB\}$	$\{AD\} \{ECB\}$			
$\equiv$	$\{DA\} \{BCE\}$	$\{DA\} \{BEC\}$	$\{DA\} \{CBE\}$	$\{DA\} \{CEB\}$	$\{DA\} \{ECB\}$	$\{DA\} \{ECB\}$			
$\{AD\} \{BCE\}$	$\{BCE\} \{AD\}$	$\{BCE\} \{DA\}$	$\{BEC\} \{AD\}$	$\{BEC\} \{DA\}$	$\{CBE\} \{AD\}$	$\{CBE\} \{DA\}$			
	$\{CEB\} \{AD\}$	$\{CEB\} \{DA\}$	$\{EBC\} \{AD\}$	$\{EBC\} \{DA\}$	$\{ECB\} \{AD\}$	$\{ECB\} \{DA\}$			
r	$\{AE\} \{BCD\}$	$\{AE\} \{BDC\}$	$\{AE\} \{CBD\}$	$\{AE\} \{CDB\}$	$\{AE\} \{DBC\}$	$\{AE\} \{DCB\}$			
$\equiv$	$\{EA\} \{BCD\}$	$\{EA\} \{BDC\}$	$\{EA\} \{CBD\}$	$\{EA\} \{CDB\}$	$\{EA\} \{DBC\}$	$\{EA\} \{DCB\}$			
$\{AE\} \{BCD\}$	$\{BCD\} \{AE\}$	$\{BCD\} \{EA\}$	$\{BDC\} \{AE\}$	$\{BDC\} \{EA\}$	$\{CBD\} \{AE\}$	$\{CBD\} \{EA\}$			
	$\{CDB\} \{AE\}$	$\{CDB\} \{EA\}$	$\{DBC\} \{AE\}$	$\{DBC\} \{EA\}$	$\{DCB\} \{AE\}$	$\{DCB\} \{EA\}$			
r	$\{BC\} \{ADE\}$	$\{BC\} \{AED\}$	$\{BC\} \{DAE\}$	$\{BC\} \{DEA\}$	$\{BC\} \{EAD\}$	$\{BC\} \{EDA\}$			
$\equiv$	$\{CB\} \{ADE\}$	$\{CB\} \{AED\}$	$\{CB\} \{DAE\}$	$\{CB\} \{DEA\}$	$\{CB\} \{EAD\}$	$\{CB\} \{EDA\}$			
$\{BC\} \{ADE\}$	$\{ADE\} \{BC\}$	$\{ADE\} \{CB\}$	$\{AED\} \{BC\}$	$\{AED\} \{CB\}$	$\{DAE\} \{BC\}$	$\{DAE\} \{CB\}$			
	$\{DEA\} \{BC\}$	$\{DEA\} \{CB\}$	$\{EAD\} \{BC\}$	$\{EAD\} \{CB\}$	$\{EDA\} \{BC\}$	$\{EDA\} \{CB\}$			
r	$\{BD\} \{ACE\}$	$\{BD\} \{AEC\}$	$\{BD\} \{CAE\}$	$\{BD\} \{CEA\}$	$\{BD\} \{EAC\}$	$\{BD\} \{ECA\}$			
$\equiv$	$\{DB\} \{ACE\}$	$\{DB\} \{AEC\}$	$\{DB\} \{CAE\}$	$\{DB\} \{CEA\}$	$\{DB\} \{EAC\}$	$\{DB\} \{ECA\}$			
$\{\{BD\} \{ACE\}\}$	$\{ACE\} \{BD\}$	$\{ACE\} \{DB\}$	$\{AEC\} \{BD\}$	$\{AEC\} \{DB\}$	$\{CAE\} \{BD\}$	$\{CAE\} \{DB\}$			
	$\{CEA\} \{BD\}$	$\{CEA\} \{DB\}$	$\{EAC\} \{BD\}$	$\{EAC\} \{DB\}$	$\{ECA\} \{BD\}$	$\{ECA\} \{DB\}$			
r	$\{BE\} \{ACD\}$	$\{BE\} \{ADC\}$	$\{BE\} \{CDA\}$	$\{BE\} \{CAD\}$	$\{BE\} \{DAC\}$	$\{BE\} \{DCA\}$			
$\equiv$	$\{EB\} \{ACD\}$	$\{EB\} \{ADC\}$	$\{EB\} \{CDA\}$	$\{EB\} \{CAD\}$	$\{EB\} \{DAC\}$	$\{EB\} \{DCA\}$			
$\{BE\} \{ACD\}$	$\{ACD\} \{BE\}$	$\{ACD\} \{EB\}$	$\{ADC\} \{BE\}$	$\{ADC\} \{EB\}$	$\{CAD\} \{BE\}$	$\{CAD\} \{EB\}$			
	$\{CDA\} \{BE\}$	$\{CDA\} \{EB\}$	$\{DAC\} \{BE\}$	$\{DAC\} \{EB\}$	$\{DCA\} \{BE\}$	$\{DCA\} \{EB\}$			
r	$\{CD\} \{ABE\}$	$\{CD\} \{AEB\}$	$\{CD\} \{BAE\}$	$\{CD\} \{BEA\}$	$\{CD\} \{EAB\}$	$\{CD\} \{EBA\}$			
$\equiv$	$\{DC\} \{ABE\}$	$\{DC\} \{AEB\}$	$\{DC\} \{BAE\}$	$\{DC\} \{BEA\}$	$\{DC\} \{EAB\}$	$\{DC\} \{EBA\}$			
$\{CD\} \{ABE\}$	$\{ABE\} \{CD\}$	$\{ABE\} \{DC\}$	$\{AEB\} \{CD\}$	$\{AEB\} \{DC\}$	$\{BAE\} \{CD\}$	$\{BAE\} \{DC\}$			
	$\{BEA\} \{CD\}$	$\{BEA\} \{DC\}$	$\{EAB\} \{CD\}$	$\{EAB\} \{DC\}$	$\{EBA\} \{CD\}$	$\{EBA\} \{DC\}$			
r	$\{CE\} \{ABD\}$	$\{CE\} \{ADB\}$	$\{CE\} \{BAD\}$	$\{CE\} \{BDA\}$	$\{CE\} \{DAB\}$	$\{CE\} \{DBA\}$			
$\equiv$	$\{EC\} \{ABD\}$	$\{EC\} \{ADB\}$	$\{EC\} \{BAD\}$	$\{EC\} \{BDA\}$	$\{EC\} \{DAB\}$	$\{EC\} \{DBA\}$			
$\{CE\} \{ABD\}$	$\{ABD\} \{CE\}$	$\{ABD\} \{EC\}$	$\{ADB\} \{CE\}$	$\{ADB\} \{EC\}$	$\{BAD\} \{CE\}$	$\{BAD\} \{EC\}$			
	$\{BDA\} \{CE\}$	$\{BDA\} \{EC\}$	$\{DAB\} \{CE\}$	$\{DAB\} \{EC\}$	$\{DBA\} \{CE\}$	$\{DBA\} \{EC\}$			
r	$\{\{DE\} \{ABC\}\}$	$\{\{DE\} \{ACB\}\}$	$\{\{DE\} \{BAC\}\}$	$\{\{DE\} \{BCA\}\}$	$\{\{DE\} \{CAB\}\}$	$\{\{DE\} \{CBA\}\}$			
$\equiv$	$\{ED\} \{ABC\}$	$\{ED\} \{ACB\}$	$\{ED\} \{BAC\}$	$\{ED\} \{BCA\}$	$\{ED\} \{CAB\}$	$\{ED\} \{CBA\}$			
$\{DE\} \{ABC\}$	$\{ABC\} \{DE\}$	$\{ABC\} \{ED\}$	$\{ACB\} \{DE\}$	$\{ACB\} \{ED\}$	$\{BAC\} \{DE\}$	$\{BAC\} \{ED\}$			
	$\{BCA\} \{DE\}$	$\{BCA\} \{ED\}$	$\{CAB\} \{DE\}$	$\{CAB\} \{ED\}$	$\{CBA\} \{DE\}$	$\{CBA\} \{ED\}$			
number of relationships = 10									
number of combinations per relationship = 24									

Bhokuzulu Khumalo
 Private Researcher
 305 -19 Northern Heights Drive, Richmond Hill, Ontario, L4B 4M4, Canada
 khumalobhekuzulu@gmail.com