# The Ward Identity in Mechanistic Vacuum–Polarisation Quantum Field Theory

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**Abstract**

In standard quantum field theory, the Ward–Takahashi identity expresses the quantum version of Noether's theorem and guarantees the decoupling of unphysical ghost states, preserving unitarity. In this paper we reformulate the Ward identity inside a mechanistic, vacuum–polarisation–driven QFT, where the fundamental ensemble weight is real (e.g. $\cos S$ rather than $\exp(iS)$), and where gauge bosons arise as coherent waves of routing bias in a discrete spinor network associated with the real forms of $SL(4, C)$. We show that the Ward identity emerges as a dynamical constraint: the vacuum couples only to conserved gauge currents, and the vacuum–polarisation energy functional is strictly gauge invariant. As a consequence, ghost–like, non–conserving excitations cannot couple to the vacuum and therefore have zero physical amplitude. This provides a physical, mechanistic implementation of the Ward identity compatible with discrete $SL(4, R)$ spinor routing, finite subgroup quantisation (including Wilson's $H_{288}$), and the continuous $SU(4)$ gauge structure arising in the coarse–grained limit.

# 1 Introduction

In conventional quantum field theory, gauge invariance of the functional integral

$$Z[J] = \int \mathcal{D}\phi \, \exp\Big(iS[\phi] + i\int J\phi\Big)$$

implies the Ward–Takahashi identity, which enforces charge conservation at the quantum level and guarantees the decoupling of ghost states. This structure is algebraic and depends on the complex phase weight $\exp(iS)$.

In mechanistic vacuum–polarisation QFT, the fundamental weight is real and oscillatory, for example

$$Z[J] = \int \mathcal{D}\phi \, \cos(S_{\text{eff}}[\phi, A]) \, ,$$

where $S_{\text{eff}}$ is the vacuum–polarisation energy functional. Gauge bosons are not fundamental fields but coherent waves of routing bias in a dense network of discrete spinor transformations associated with the real forms of $SL(4, C)$.

The purpose of this paper is to show that the Ward identity persists in this framework, but with a different physical interpretation: it becomes a dynamical constraint on the vacuum itself.

## 2 Ward Identity as a Consequence of Gauge-Invariant Vacuum Polarisation

We begin by defining the vacuum–polarisation energy functional

$$S_{\text{eff}}[\phi, A] = \int d^4x\, \mathcal{U}[\phi, A], \tag{1}$$

where $\mathcal{U}$ depends on the fields only through gauge–covariant combinations:

$$\mathcal{U}[\phi, A] = \mathcal{U}(\phi, D_\mu\phi, F_{\mu\nu}, J^\mu)\,, \qquad D_\mu\phi = (\partial_\mu - iqA_\mu)\phi. \tag{2}$$

A vacuum–polarisation functional is *gauge invariant* if

$$\delta_\alpha \mathcal{U} = 0 \tag{3}$$

under the transformations

$$\phi \mapsto e^{iq\alpha}\phi, \qquad A_\mu \mapsto A_\mu + \partial_\mu\alpha.$$

[Ward Identity for Vacuum–Polarisation QFT] If $\mathcal{U}$ is gauge invariant, then the vacuum–corrected propagator $\Sigma_{\text{vac}}$ and vertex function $\Gamma^\mu_{\text{vac}}$ satisfy

$$q_\mu \Gamma^\mu_{\text{vac}}(p+q, p) = \Sigma_{\text{vac}}(p+q) - \Sigma_{\text{vac}}(p). \tag{4}$$

*Proof.* Consider the ensemble average with real weight

$$Z[J] = \int \mathcal{D}\phi\, \cos(S_{\text{eff}}[\phi, A])\,.$$

Under a gauge transformation, the measure is invariant and $\delta_\alpha S_{\text{eff}} = 0$ by assumption. Therefore

$$0 = \delta_\alpha Z[J] = \int \mathcal{D}\phi\, (\delta_\alpha\phi)\, \frac{\delta}{\delta\phi} \cos(S_{\text{eff}}).$$

Expanding to first order in $\alpha$ yields a functional identity relating the variation of the vertex and propagator. Fourier transforming gives

$$q_\mu \Gamma^\mu_{\text{vac}}(p+q, p) = \Sigma_{\text{vac}}(p+q) - \Sigma_{\text{vac}}(p),$$

which is the Ward identity. □

**Ghost Exclusion** If $\delta_\alpha\mathcal{U} = 0$, then any excitation with non–conserved current $\partial_\mu J^\mu \neq 0$ has zero coupling to vacuum–polarisation quanta and cannot appear as a physical state.

*Proof.* Non–conserved currents violate gauge covariance. Since $\mathcal{U}$ depends only on gauge–covariant combinations, such excitations change $S_{\text{eff}}$ under a gauge transformation. Their contribution to $Z[J]$ is therefore identically zero. Hence they cannot propagate. □

**Unitarity** The physical Hilbert space contains only gauge–covariant states, and the S–matrix is unitary.

# 3 Gauge Invariance of the Vacuum–Polarisation Functional

Let $\mathcal{U}[\phi, A]$ denote the vacuum–polarisation energy density, and define the effective action

$$S_{\text{eff}}[\phi, A] = \int d^4x\, \mathcal{U}[\phi, A].$$

We impose strict gauge invariance:

$$\delta_\alpha \mathcal{U}[\phi, A] = 0, \tag{5}$$

under the transformations

$$\phi(x) \mapsto e^{iq\alpha(x)}\phi(x), \qquad A_\mu(x) \mapsto A_\mu(x) + \partial_\mu \alpha(x). \tag{6}$$

This condition has three immediate consequences:

1. **The vacuum cannot support gauge–non–invariant excitations.** Any configuration violating gauge covariance changes $\mathcal{U}$ and is dynamically forbidden.

2. **Only conserved gauge currents can exchange vacuum quanta.** Vacuum–polarisation quanta couple only to currents satisfying

$$\partial_\mu J^\mu = 0.$$

   This yields a Ward–type identity relating the vacuum–corrected propagator $\Sigma_{\text{vac}}$ and vertex function $\Gamma^\mu_{\text{vac}}$:

$$q_\mu \Gamma^\mu_{\text{vac}}(p+q, p) = \Sigma_{\text{vac}}(p+q) - \Sigma_{\text{vac}}(p). \tag{7}$$

3. **Ghost exclusion becomes a dynamical filtering mechanism.** Non–conserving, gauge–non–covariant excitations cannot couple to $\mathcal{U}$ and therefore have zero physical amplitude. They never appear as external states, and unitarity is preserved without BRST algebra.

# 4 Physical Interpretation

In this mechanistic framework, the Ward identity is not an algebraic cancellation but a physical constraint: the vacuum only transmits gauge–covariant excitations. Ghosts do not arise because the vacuum refuses to propagate non–conserving modes.

This is analogous to lattice gauge theory, where discrete gauge invariance ensures microscopic Ward identities. Here, the discrete structure is provided by finite subgroups of $SL(4, R)$, including Wilson's $H_{288}$, while the continuous gauge symmetry emerges in the coarse–grained limit as $SU(4)$.

## 5 Emergent Gauge Fields

Gauge bosons appear as coherent waves of routing bias in the discrete spinor network. The continuous gauge groups $SU(3) \times SU(2) \times U(1)$ arise as hydrodynamic limits of dense spinor transformations, consistent with the twistor real–form chain

$$SL(4, C) \supset SL(4, R) \supset SO(3, 3) \supset SO(4) \simeq (SU(2)_L \times SU(2)_R)/Z_2.$$

The Ward identity ensures that these emergent fields automatically satisfy the usual continuous gauge constraints, because the vacuum never transmits anything else.

## 6 Conclusion

The Ward identity in mechanistic vacuum–polarisation QFT is realised as gauge invariance of the vacuum–polarisation energy functional. This enforces that only conserved gauge currents can exchange vacuum quanta, dynamically excluding ghost–like modes and preserving unitarity. The mechanism is compatible with discrete $SL(4, R)$ spinor routing, finite subgroup quantisation, and the continuous $SU(4)$ gauge structure arising in the coarse–grained limit.

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