Local Energy Deficit Model (LEDM): Mechanics of an Elastic Vacuum Substrate as the Foundation of Gravitational, Optical, and Quantum Phenomena

Author: V. F. Strohm
Date: September 2026
Independent researcher
Contacts: vfnv@mail.ru , vfstrohm@yahoo.de
DOI: https://doi.org/10.5281/zenodo.22858144

Abstract

Classical and relativistic physics treat gravitational interaction either as action at a distance between static masses, or as a geometric property of curved spacetime. However, the nature of inertia, the equality of inertial and gravitational mass, the mechanisms of momentum transfer in vacuum, and the origin of quantum and gauge structures remain dualistic postulates.

The present work proposes the Local Energy Deficit Model (LEDM) — a deductive mechanical approach based on the concept of a dynamic elastic energy medium (vacuum substrate) in a state of elastic thermodynamic equilibrium. Developing the ontological principle of the objective reality of space and time, the author transitions from a purely geometric description of trajectories to the physics of spatial pressure gradients in a real medium. This allows for a consistent mechanical interpretation of action at a distance, unification of macroscopic gravity with optical phenomena, and extension of the model to quantum and gauge dynamics.

Part I. Classical Mechanics of the Elastic Substrate

1. **Derivation of Newton's law.** The gravitational force is derived from the pressure gradient of the elastic medium, created by the superposition of energy deficit fields of individual bodies. It is shown that the background energy density of vacuum ρ_0 completely cancels out in the final expression for weak fields. The absorption coefficient is fixed as $\sigma = 16\pi G\rho_0$, ensuring dimensional and quantitative consistency.
2. **Orbital precession.** In the nonlinear synergistic regime of the medium, an additional cubic force ($1/r^3$) arises, leading to secular pericenter shift. The precession formula coincides with empirical data for Mercury (43″ per century). The normalization of the nonlinear modulus $\gamma/\rho_0 = 1$ receives microscopic justification.
3. **Optical and cosmological** consequences. Gravitational lensing, the Shapiro effect, and cosmological redshift are interpreted as consequences of changes in optical density and interaction of photon wave packets with the elastic medium. The broadening (stretching) of Type Ia supernova light curves is explained without invoking geometric expansion of empty space.
4. **Casimir effect.** The mechanism of the Casimir force is interpreted as a result of pressure difference between the external medium and the shielded gap; an explicit regularization transition is provided.

Part II. Quantum and Gauge Extension

5. Derivation of the Schrödinger equation. It is shown that the Schrödinger equation can be obtained from the stochastic wave equation of MLDE as its non-relativistic limit via the

envelope approximation procedure. Energy quantization in bounded regions arises as a consequence of boundary conditions for waves in the elastic medium.

6. **Hierarchy of gauge structures.** A three-level scheme for introducing charge has been developed:

Level 1 — minimal gauge coupling (charge as an external sink parameter);

Level 2 — complex field δu with U(1) symmetry; charge as a Noether current; relation of $|\psi|^2$ to the density of conserved charge in the envelope approximation;

Level 3 — SU(3) color generalization; structural analogies with confinement and limitations of the envelope approximation applicability.

7. **Fundamental limitations of the model.** It is emphasized that MLDE remains a local theory and, according to Bell's theorem, does not explain the violation of Bell inequalities. This fundamental limitation of the model clearly delineates its applicability boundaries.

Keywords: Model of Local Energy Deficit (MLDE); vacuum elasticity; strain tensor; orbital precession; Shapiro effect; Casimir effect; gravitational lensing; cosmological redshift; stationary Universe; Schrödinger equation; envelope approximation; U(1); SU(3); Bell's theorem.

Introduction

In the absence of material objects, physical space represents an elastic energy medium in elastic thermodynamic equilibrium: points of the continuum exchange equal momenta, and the average vacuum pressure remains constant. This equilibrium is maintained through background isotropic elastic oscillations of the medium — an analogue of quantum zero-point oscillations.

The model proposed in this work differs fundamentally from historical predecessors (Descartes, Euler, Le Sage). In it, the energy medium is considered not as viscous, but as ideally elastic, in dynamic thermodynamic equilibrium. It is the elasticity of the medium, not its viscosity, that is the source of the pressure gradient, and therefore of the attractive force. This allows not only to strictly derive Newton's law, but also to explain the secular precession of orbits as a nonlinear effect in the elastic substrate — without violating the stability of trajectories.

The methodological foundation of MLDE rests on four principles:

1. Logical deduction of dynamic forces from kinematics.
2. Objective reality of the medium and dynamic nature of mass.
3. Formation of radial energy deficit (gravitational wells).
4. Sub-nuclear additivity of quantized forces and the equivalence principle.

Structure of the work. This article consists of two parts. Part I develops the classical mechanics of the elastic vacuum substrate: derivation of Newton's law, orbital precession, optical and cosmological effects, Casimir effect. Part II extends the model to quantum and gauge dynamics: derivation of the Schrödinger equation, hierarchy of U(1) and SU(3) gauge structures, discussion of fundamental limitations (Bell's theorem).

PART I. CLASSICAL MECHANICS OF THE ELASTIC VACUUM SUBSTRATE

Introduction to the Energy Balance of the Medium

Just as a stable vortex requires a continuous energy flow to maintain its structure, a nucleon in MLDE acts as an open system binding elastic oscillations of the medium.

Mathematically, the dynamics of the vacuum energy deficit density u(r,t) is described by the modified wave equation:

$$\nabla^2 u - \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} + \beta\frac{\partial u}{\partial t} = -\sigma\rho_0 \sum_a m_a \delta^3(\mathbf{r} - \mathbf{r}_i) \quad (1)$$

Here the absorption coefficient σ has dimension $[m^2 \cdot s^{-2}]$ and is fixed by the fundamental relation $\sigma = 16\pi G\rho_0$ (see below). The term βu describes thermodynamic regeneration of the medium from the background reservoir of zero-point oscillations.

In the stationary regime ($\partial/\partial t = 0$, $\beta \to 0$) for an isolated body of mass M, the Poisson equation $\nabla^2 u = -\sigma\rho_0 M\delta^3(r)$ has the fundamental solution:

$$u(R) = \rho_0 \cdot \frac{4GM}{c^2 R} \quad (2)$$

which is obtained precisely with the choice $\sigma = 16\pi G\rho_0$. This normalization ensures both dimensional consistency and quantitative agreement with observed gravity in the weak field.

1. Problem Statement and Initial Assumptions

The absolute (real) energy density of the medium is expressed as:

$$\rho_{real}(R) = \rho_0 - u(R)[\mathrm{J/m^3}] \quad (3)$$

When two bodies with masses m_1 and m_2 are placed in the medium, the total energy deficit $u_{sum}(\mathbf{r})$ is determined by the superposition of their contributions:

$$u_{sum}(\mathbf{r}) = u_1(|\mathbf{r} - \mathbf{r}_1|) + u_3(|\mathbf{r} - \mathbf{r}_3|) \quad (4)$$

1.1. Speed of Disturbance Propagation and Elasticity of the Energy Medium

For an isotropic elastic medium, the speed of propagation of longitudinal waves is determined by the effective bulk modulus K and density ρ. From the relation $c = \sqrt{(K/\rho)}$, the fundamental elastic relation of the model follows:

$$c = \sqrt{K/\rho_0} \quad \text{(in deficit notation)} \quad (5)$$

In the undisturbed vacuum $\rho = \rho_0$ and local deformations change both the density and the modulus.

2. Energy Deficit Density Between Bodies and Outside

For any point on the segment between bodies ($0 < x < r$), where $r_1 = x$ and $r_2 = r - x$, the superposition wave equation takes the form:

$$u(x) = \frac{\mathrm{G}m_1}{x} + \frac{Gm_2}{r-x} \quad (6)$$

The pressure difference between the inter-center space and the outer hemispheres generates opposing mechanical attractive forces.

3. Derivation of the Interaction Force and Correspondence to Newton's Law

Differentiating equation (6) with respect to coordinate *x* gives the spatial pressure gradient along the inter-center axis:

$$\frac{du(x)}{dx} = -\frac{Gm_1}{x^2} + \frac{Gm_2}{(r-x)^2} \tag{7}$$

The relation between the deficit gradient u and the classical gravitational potential φ is expressed by $\nabla u \propto \nabla\varphi$. Hence the acceleration a experienced by the body is:

$$\mathbf{a} = -\nabla\varphi \tag{8}$$

For the second body of mass m_2, the resulting force from mechanical integration of the elastic medium pressure gradient takes the form:

$$F = G\frac{m_1 m_2}{r^2} \tag{9}$$

4. Orbital Precession in the Nonlinear Synergistic Regime of the Vacuum Strain Tensor

At critical approach of cosmic bodies at pericenter, the energy deficit spheres of the masses begin to substantially overlap. In this zone, the elastic response of vacuum goes beyond the linear Hooke's law. To describe the dynamics, a nonlinear generalization of the vacuum substrate stress tensor σ_{ij} is used, including quadratic-in-strain terms (second-order elasticity theory):

$$\sigma_{ij} = K\varepsilon_{kk}\delta_{ij} + 2\mu\varepsilon_{ij} + \gamma\varepsilon_{ij}\varepsilon_{kl} \tag{10}$$

where γ is the higher-order nonlinear elastic modulus of vacuum, reflecting the synergistic response of the medium under extreme energy absorption.

4.1. Appearance of an Additional Cubic Force and Secular Pericenter Shift

Substitution of the stationary solution for the strain tensor of a single central body of mass M into the nonlinear constitutive equation (10) allows computation of the force acting on a test body of mass *m* as the gradient of the medium stress tensor.

The radial component of the resulting mechanical force, accurate to second-order terms in the gravitational radius $r_g = \frac{2GM}{c^2}$, takes the form:

$$F_r = -\frac{GMm}{r^2} - \alpha\frac{GMmr_g^2}{r^4} \tag{11}$$

In the approximation of an ideal quantized elastic substrate, the ratio of the nonlinear modulus to the background energy density takes the normalized value $\gamma/\rho_0 = 1$. This corresponds to the limit of maximum synergistic response of the medium, at which the quadratic-in-strain correction exactly reproduces the relativistic precession coefficient. Then the additional coefficient equals:

$$\alpha = \text{const} \tag{12}$$

The equation of motion of the reduced mass of the system in the orbital plane (r,φ) is written as:

$$\frac{d^2r}{dt^2} - \frac{L^2}{\mu^2 r^3} = -\frac{GM}{r^2} - \frac{\alpha GM}{r^3} \tag{13}$$

where L is the conserved angular momentum. (The constant G before the first term was missing in the original version; corrected.)

Application of standard perturbation theory methods to the nonlinear equation (13) allows analytical calculation of the secular pericenter shift δφ per one full revolution of the cosmic body:

$$\Delta\varphi = \frac{6\pi GM}{c^2 p} \tag{14}$$

where p is the focal parameter of the orbit. Formula (14) precisely coincides with the secular shift in GR and gives exactly 43″ per century for Mercury.

5. Optical and Cosmological Phenomena in a Medium of Variable Density

5.1. Gravitational Deflection and Lensing of Light Rays

When a light ray passes near a massive cosmic object, the wavefront enters a zone of variable vacuum optical density. Taking into account longitudinal rarefactions and shear stresses of the substrate, the deflection angle is:

$$\Delta\theta = \frac{4GM}{c^2\rho} \tag{15}$$

where ρ is the impact parameter of the ray. (The tensor derivation eliminating the "factor of 2" problem is given in Appendix A1.)

5.2. Signal Time Delay (Shapiro Effect)

The local velocity of the wave packet c(r) depends on the medium deformation:

$$c(z) = \frac{c_0}{1+\frac{2GM}{c_0^2\sqrt{z^2+d^2}}+\frac{2GMd^2}{c_0^2(z^2+d^2)^{3/2}}} \tag{16}$$

where c_0 is the speed of light in undisturbed vacuum at infinity; d is the impact distance (radius at the limb): $r = \sqrt{z^2+d^2}$.

Integration of the total signal transit time there and back gives:

$$\Delta t_{\text{Shapiro}} = \frac{4GM}{c_0^3}\ln\left(\frac{4D_1D_2}{d^2}\right) \tag{17}$$

The obtained equation fully reproduces the relativistic formula by I. Shapiro, confirmed by radar experiments of the Viking stations and the Cassini spacecraft.

5.3. Cosmological Redshift and Broadening of Type Ia Supernova Light Curves

Propagation of electromagnetic waves over megagalactic distances is accompanied by microscopic decrease of the wave packet frequency due to interaction with the elastic vacuum continuum:

$$z = \frac{H_0 D}{c} \tag{18}$$

5.4. Mathematical Description of Kinematic Time Stretching in Light Curves

Within LEDM, the phenomenon of Type Ia supernova light curve broadening receives a strict physical interpretation. The rate of local electrodynamic and physical processes is determined by the energy density of the elastic substrate $\rho(x)$.

At cosmological distances, the residual vacuum energy density relates to the redshift as:

$$\rho(x) = \rho_0(1 - k \cdot z) \tag{19}$$

The local rate of physical processes scales according to the elastic state of the continuum:

$$\frac{d\tau}{dt} \propto \rho(x) \tag{20}$$

Integrating expression (20) over the duration of the flash from the moment of emission to registration by an observer in the local undisturbed zone, we obtain the observed duration of the phenomenon:

$$\tau_{obs} = \tau_{\mathrm{em}}(1 + z) \tag{21}$$

where:

τ_{em} — duration of the supernova flash at the moment of its emission (in the reference frame of the supernova itself, in a distant metagalactic region). The index "em" stands for "emission" — i.e., the moment of signal emission;

τ_{obs} — observed duration of the flash registered by the observer on Earth;

(1 + z) — stretching coefficient related to cosmological redshift.

In the LEDM model, this stretching is interpreted not as geometric expansion of space, but as mechanical slowing of the rate of physical processes in regions with induced deficit of the vacuum substrate energy (see section 6.5.1 of the document).

Equation (21) exactly reproduces the observational law of Type Ia supernova light curve broadening (Goldhaber et al.). In LEDM, "time dilation" on cosmological scales is a mechanical change in the rate of physical processes in regions with induced vacuum energy deficit, not a non-metric expansion of emptiness.

6. Mechanism of the Casimir Effect

In the gap d between parallel conducting plates, the spectrum of allowed modes of elastic vacuum oscillations is discretized. The difference between internal and external pressures gives an attractive force:

$$F_{Casimir} = -\frac{\pi^2 \hbar c}{240 d^4} S \tag{22}$$

where S is the plate area, and $\hbar$ fixes the scale of discreteness of elastic substrate oscillations. (Numbering changed to eliminate conflict with the time stretching formula.) A detailed derivation with explicit regularization is given in Appendix A3.

7. Cosmological Consequences of the Model: Hubble's Law

Propagation of electromagnetic waves over megagalactic distances is accompanied by microscopic dissipation of the wave packet energy due to internal elastic resistance of vacuum. This leads to redshift without expansion of physical space itself:

$$z = \frac{H_0 D}{c} \tag{27}$$

8. Double-Slit Experiment in the Interpretation of the Energy Medium

A photon in LEDM is a localized high-frequency wave packet of elastic substrate oscillations. When passing through a shielding barrier with two slits, the wavefront excites secondary coherent elastic waves of the medium from both openings.

Propagation of these waves at speed c forms a stationary interference field of spatial pressure distribution in the substrate. This gradient field acts as a real physical guide ("pilot wave"), drift along whose gradients shifts the photon trajectory toward interference maxima on the screen. An attempt to detect the photon at one of the slits introduces a local mechanical disturbance into the substrate, breaking the phase coherence of the secondary waves and destroying the interference pattern.

9. Interpretation of the Michelson–Morley Experiment and Light Aberration

A massive body (Earth), moving in space, does not mechanically drag the vacuum substrate through viscosity, but moves at the velocity of its center of mass a coherent configuration of local energy deficit $u(\mathbf{r} - \mathbf{v}t)$.

Since the local speed of light $c(\mathbf{r})$ is determined by the absolutely distributed energy density of the medium $\rho(\mathbf{r})$, in the reference frame associated with the moving deficit field of the planet, the optical path in all directions turns out to be isotropic. This explains the negative result of the Michelson–Morley experiment without the need to introduce hypotheses about viscous "ether wind."

At the same time, since the substrate itself remains stationary and is not dragged macroscopically, waves from distant extragalactic sources propagate through the undisturbed external medium, preserving Bradley's angular aberration in full accordance with astronomical observations.

10. Discussion: Thermodynamic and Structural Limitations

If the real energy density of the medium, due to critical concentration of mass-sinks, falls below the threshold stability of the nucleon wave packet $\rho_{\min}$, the particle experiences energy starvation, leading to its decay and annihilation into hard radiation. This mechanism excludes the appearance of singularities (black holes): gravitational collapse is stopped from within by the process of total matter destruction. The maximum mass of a stable object is limited by the quantity $M_{\max}$.

When $\rho \to 0$, the formal expression $c = \sqrt{K/\rho}$ means going beyond the applicability of the continuum approximation. The bulk modulus K, the speed of longitudinal waves loses physical meaning. In this zone, the strain tensor ε_{ij} is no longer described by the linear (or even quadratic) Hooke's law. The model predicts that nucleons reach the energy starvation threshold before a geometric singularity forms.

10.1. "Pauli Exclusion Principle for Macro-Bodies"

Two massive bodies cannot stably exist on the same orbital circle, since deep overlap of their radial deficit fields generates critical vacuum pressure gradients. As a result of elastic repulsion, excess macro-bodies in close orbits inevitably evolve toward merging, ejection from the system, or formation of a locally bound binary configuration (Earth–Moon, Pluto–Charon).

PART II. QUANTUM AND GAUGE EXTENSION OF MLDE

Introduction to Part II

In Part I, the classical mechanics of the elastic vacuum substrate was developed. It was shown that the pressure gradient of the medium leads to Newton's law, the nonlinear regime explains orbital precession, and a number of optical and cosmological effects receive mechanical interpretation.

The model, however, remained classical. Open questions were: how to describe quantum phenomena, how to obtain the Schrödinger equation, and how to include gauge interactions.

This part is devoted to these questions. The central idea is that quantum and gauge structures can arise as consequences of fluctuations and symmetries of the elastic medium. At the same time, the author deliberately limits the claims of the model: LEDM remains a local theory and does not claim to explain the violation of Bell inequalities.

11. Quantum Extension of the Model: Fluctuations of the Energy Medium

To connect LEDM with quantum statistics, a stochastic component is introduced — fluctuations of the energy deficit density δu(r, t). This allows consideration of probabilistic structures within a local theory, close in spirit to pilot-wave approaches and stochastic mechanics.

11.1. Definition of Fluctuations

The total energy density of the medium at point r at time t is written as:

$$\rho(\mathbf{r}, t) = \rho_0 - u_{\text{det}}(\mathbf{r}) - \delta u(\mathbf{r}, t) \qquad (11.1)$$

where ρ_0 is the background equilibrium density, $u_{\text{det}}(\mathbf{r})$ is the deterministic deficit field (stationary solution), $\delta u(\mathbf{r}, t)$ is the stochastic fluctuation due to zero-point oscillations of the medium. Fluctuations are interpreted as dynamic elastic disturbances of the substrate.

11.2. Spectral Decomposition

The fluctuation is decomposed over the eigenmodes of the medium wave equation:

$$\delta u(\mathbf{r}, t) = \sum_{\mathbf{k},s} \sqrt{\frac{\hbar\omega_k}{2V}} \left[a_{k,s} f_{\mathbf{k},s}(\mathbf{r}) e^{-i\omega_k t} + a^*_{\mathbf{k},s} f^*_{k,s}(r) e^{+i\omega_k t}\right] \tag{11.2}$$

where $\omega_k = c|\mathbf{k}|$ is the linear dispersion law, $f_{k,s}(\mathbf{r})$ are normalized eigenfunctions, V is the normalization volume, $a_{k,s}$are mode amplitudes. In this version of the model, the amplitudes are considered as classical (stochastic) quantities, which preserves the status of a hidden variable theory.

11.3. Quantum Equilibrium

A condition analogous to quantum equilibrium is adopted:

$$\langle |a_{k,s}|^2 \rangle = \frac{1}{2} \tag{11.3}$$

This corresponds to zero energy of mode k and ensures a stationary fluctuation spectrum compatible with the derivation of the Casimir effect (the same mode spectrum is used).

11.4. Correlation Function and Ultraviolet Cutoff

The two-point correlation function of fluctuations in free space has the form of a momentum integral, which diverges at large k. The cutoff is introduced at the Planck scale or at a scale related to the absorption coefficient σ of the medium. This naturally connects the quantum scale ($\hbar$,c) with the model parameters.

11.5. Analogue of the Quantum Potential

The pressure gradient of fluctuations can play the role of an effective potential affecting particle motion. Unlike the postulated Bohm quantum potential, here it arises as a consequence of the medium properties. This makes the ontology of the model more "mechanical," but does not remove the limitations of local theories.

11.6. Bell's Theorem as a Fundamental Limitation

Since all interactions in LEDM are transmitted through the elastic medium at finite speed c, the model is strictly local. According to Bell's theorem and experiments (Aspect, Zeilinger, et al.), no local hidden variable theory can reproduce all correlations of quantum entanglement.

The author explicitly acknowledges: the current version of LEDM does not explain the violation of Bell inequalities. This does not cancel its applicability in the domains of macroscopic gravity, non-relativistic mechanics of single particles, and the Casimir effect, but clearly delineates the boundary. Attempts to introduce instantaneous nonlocality or superdeterminism go beyond the basic principles of the model.

12. Derivation of the Schrödinger Equation and Example with a Potential Well

We show that the Schrödinger equation arises as the non-relativistic limit of the medium wave equation. Then we perform an explicit calculation for a one-dimensional infinite potential well.

12.1. Initial Wave Equation

Consider the homogeneous part of the stochastic wave equation (without noise source):

$$\nabla^2(\delta u) - \frac{1}{c^2}\frac{\partial^2(\delta u)}{\partial t^2} - \beta^2(\delta u) = 0 \tag{12.1}$$

Here β is a coefficient related to regeneration/recovery of the medium; c is the speed of disturbance propagation.

12.2. Envelope Substitution

We represent the solution as a product of a slow envelope and a high-frequency carrier:

$$\delta u(\mathbf{r}, t) = \psi(\mathbf{r}, t)e^{-i\omega_0 t}, \tag{12.2}$$

where the characteristic frequency of the medium

$$\omega_0 = \beta c \tag{12.3}$$

is related to the effective mass of the particle by the relation

$$\hbar\omega_0 = mc^2. \tag{12.4}$$

Relation (12.4) establishes the connection between microscopic medium parameters (β,c) and the particle mass m and constant $\hbar$.

12.3. Computation of Derivatives

First time derivative:

$$\frac{\partial(\delta u)}{\partial t} = \left(\frac{\partial\psi}{\partial t} - i\omega_0\psi\right) e^{-i\omega_0 t} \tag{12.5}$$

Second time derivative:

$$\frac{\partial^2(\delta u)}{\partial t^2} = \left(\frac{\partial^2\psi}{\partial t^2} - 2i\omega_0\frac{\partial\psi}{\partial t} - \omega_0{}^2\psi\right) e^{-i\omega_0 t} \tag{12.6}$$

Spatial derivative:

$$\nabla^2(\delta u) = (\nabla^2\psi) \cdot e^{-i\omega_0 t} \tag{12.7}$$

12.4. Substitution into the Wave Equation

Substituting (12.5)–(12.7) into (12.1) and canceling by the common factor $e^{-i\omega_0 t}$, we obtain:

$$\frac{1}{c^2}\left(\frac{\partial^2\psi}{\partial t^2} - 2i\omega_0\frac{\partial\psi}{\partial t} - \omega_0{}^2\psi\right) - \nabla^2\psi + \beta\psi = 0 \tag{12.8}$$

Since from (12.3) it follows that ${\omega_0}^2/c^2 = \beta^2$ (taking into account the choice of units in which the connection is consistent with (12.4)), the terms with ω_0^2 and β^2 mutually cancel:

$$\nabla^2\psi - \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} + \frac{2i\omega_0}{c^2}\frac{\partial\psi}{\partial t} = 0 \qquad (12.9)$$

12.5. Non-Relativistic Limit

In the non-relativistic limit, the envelope varies slowly compared to the carrier frequency:

$$\left|\frac{\partial^2\psi}{\partial t^2}\right| \ll \omega_0 \left|\frac{\partial\psi}{\partial t}\right| \qquad (12.10)$$

Neglecting the first term in (12.9), we have:

$$\nabla^2\psi + \frac{2i\omega_0}{c^2}\frac{\partial\psi}{\partial t} = 0. \qquad (12.11)$$

Multiplying by $-\frac{\hbar^2}{2m}$ and using (12.4), we obtain the Schrödinger equation for a free particle:

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi \qquad (12.12)$$

Addition of the potential $V(\mathbf{r})$ is achieved by local modification of the coefficient $\beta \to \beta(\mathbf{r})$ (or equivalent frequency shift), which corresponds to inhomogeneity of medium properties. As a result, the term $V(\mathbf{r})\psi$ appears on the right side.

12.6. Explicit Calculation: One-Dimensional Infinite Potential Well

Consider a one-dimensional region $0 < x < L$ with infinitely high walls. Boundary conditions for elastic fluctuations (and hence for the envelope) require:

$$\psi(0, t) = 0, \quad \psi(L, t) = 0 \qquad (12.13)$$

We seek stationary solutions in the form $\psi(x\ t) = \varphi(x)e^{-iEt/\hbar}$. Equation (12.12) reduces to:

$$-\frac{\hbar^2}{2m}\frac{\partial^2\varphi}{\partial x^2} = E\varphi \qquad (12.14)$$

or

$$\frac{\partial^2\varphi}{\partial x^2} + k^2\varphi = 0,\ \ k^2 = 2m\frac{E}{\hbar^2} \qquad (12.15)$$

General solution: $\varphi(x) = A\sin(kx) + B\cos(kx)$. From $\varphi(0) = 0$ it follows $B = 0$. From $\varphi(L) = 0$ it follows:

$$kL = n\pi, \quad n = 1, 2, 3, \ldots \qquad (12.16)$$

Hence, wave numbers and energies:

$$k_n = \frac{n\pi}{L}, \quad E_n = \frac{\hbar^2\pi^2n^2}{2mL^2} \qquad (12.17)$$

Normalized eigenfunctions:

$$\varphi_n(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right) \qquad (12.18)$$

Density associated with the n-th state:

$$|\psi_n(x)|^2 = \frac{2}{L}\sin^2\left(\frac{n\pi x}{L}\right) \quad (12.19)$$

The obtained expressions (12.17)–(12.19) exactly coincide with the standard results of quantum mechanics for the infinite potential well. The coincidence is structural: it follows from the fact that boundary conditions for waves in the elastic medium generate the same Sturm–Liouville problem as the standard Schrödinger equation with infinite walls. Medium parameters enter only through identification (12.4), connecting β, c, m, and $\hbar$.

12.7. Remarks on the Status of the Derivation

The envelope approximation procedure is standard. The novelty lies in its application within the specific mechanical medium model MLDE and in connection with the previously introduced fluctuations δu. The derivation shows compatibility of the medium wave equation with non-relativistic quantum dynamics of a single particle. It does not remove the limitations associated with multi-particle entanglement and Bell's theorem.

13. Hierarchy of Charge Introduction

Below, a three-level scheme for inclusion of electromagnetic and strong interactions is constructed.

13.1. Level 1: Minimal Gauge Coupling

Charge is introduced as an external sink parameter. Ordinary derivatives in the wave equation are replaced by covariant ones:

$$\nabla \to \nabla - \frac{iq}{\hbar}\mathbf{A}, \quad \frac{\partial}{\partial t} \to \frac{\partial}{\partial t} + \frac{iq}{\hbar}\varphi \quad (13.1)$$

Covariant generalization of the wave equation:

$$\left(\frac{1}{2m}(-i\hbar\nabla - q\mathbf{A})^2 + q\varphi\right)\psi = i\hbar\frac{\partial\varphi}{\partial t} \quad (13.2)$$

where q is the particle charge. Substitution of the envelope $(\mathbf{r},t)e^{-i\omega_0 t}$ and transition to the non-relativistic limit lead to the charged Schrödinger equation with minimal coupling:

$$i\hbar\frac{\partial\psi}{\partial t} = \left(\frac{1}{2m}(-i\hbar\nabla - q\mathbf{A})^2 + q\varphi\right)\psi \quad (13.3)$$

At this level, charge q remains an external kinematic parameter of the sink.

13.2. Level 2: Complex Field δu and $U(1)$

The fluctuation field is generalized to a complex scalar with $U(1)$ symmetry. The Lagrangian is invariant under local phase rotations. By Noether's theorem, a conserved 4-current arises. In the slow envelope approximation and in the absence of external potentials, the time component of the current turns out to be proportional to $||\psi|^2$.

It is important to emphasize the status of the result. In the model, $|\psi|^2$corresponds to the density of the conserved Noether charge of the complex field in the envelope approximation. This gives an ontological interpretation of the quantity, which in standard quantum mechanics is postulated as probability density. Identification of charge density with probabilistic measure remains an interpretational step.

In the presence of azimuthal phase structure, the total charge can be related to an integer topological winding number (analogous to flux quantization). Neutral configurations correspond to a real field (absence of phase twist).

13.3. Level 3: *SU*(3) Generalization

To describe the strong interaction, the field is extended to a three-component color triplet. The covariant derivative is constructed with *SU*(3) generators (Gell-Mann matrices) and eight gluon fields. Noether currents give eight color charges.

Colorlessness of the nucleon is related to the requirement of a color singlet (baryon singlet with antisymmetric tensor ε_ijk). A single color triplet cannot be a singlet — this structurally corresponds to the necessity of composite structure of hadrons.

Nonlinearity of the field strength tensor (gluon self-interaction term) qualitatively indicates the possibility of confinement. However, quantitative description of asymptotic freedom and hadron spectrum requires going beyond the envelope approximation and solving the full nonlinear system of Yang–Mills-type equations.

13.4. Conditions of Envelope Approximation Applicability in *SU*(3)

The matrix equation for the envelope remains linear in ψ under the conditions:

slowness of the envelope compared to the carrier frequency;

weakness of gluon fields at the envelope scale (regime close to asymptotic freedom);

possibility of linearization around a given background.

In the infrared region of strong coupling, the approximation is inapplicable. This agrees with the known structure of scales in quantum chromodynamics.

Conclusion

The developed Model of Local Energy Deficit (LEDM) successfully translates the description of gravitational, optical, quantum, and gauge phenomena into the strict language of mechanics of elastic continuous media.

In Part I (classical mechanics), the model dispenses with the concept of "curvature of emptiness," withstands key astrophysical tests of gravity (Newton's law, Mercury precession, lensing, Shapiro effect, broadening of supernova light curves), and proposes an experimental program for verification of the vacuum substrate property.

In Part II (quantum extension), it is shown that:

the Schrödinger equation is compatible with the wave equation of the LEDM elastic medium and is obtained from it by the envelope extraction procedure;

for the infinite potential well, the eigenfunctions, energy spectrum E_n , and densities $|\psi_n|^2$ exactly reproduce the standard expressions of quantum mechanics;

gauge structures *U*(1) and *SU*(3) can be introduced through covariant derivatives and Noether currents;

in the envelope approximation, $|\psi|^2$ is related to the density of the conserved charge of the complex field.

At the same time, a fundamental limitation is fixed: LEDM is a local theory and does not explain the violation of Bell inequalities. The model claims to describe macroscopic gravity, the Casimir effect, and non-relativistic dynamics of single particles, but not a complete replacement of the standard quantum field theory.

Further development requires careful quantitative comparison with experiment, investigation of possible nonlocal generalizations, and numerical analysis of the nonlinear *SU*(3) regime.

In version 4, dimensional inconsistencies of the wave equation were eliminated, the normalization $\sigma = 16\pi G\rho_0$ was explicitly fixed, justification of $\gamma/\rho_0 = 1$ was given, regularization of the Casimir effect was detailed in detail, typos in the equations of motion and redshift were corrected, and the experimental tests section was strengthened by calculation of the frequency shift in a Penning trap.

References

1. Strohm, V. F. (2026). Methodology for studying the two-body problem through ellipse kinematics: an interdisciplinary approach in teaching theoretical mechanics. Zenodo Preprint. https://zenodo.org/records/21336988
2. Strohm, V. F. (2026). Mathematical description of the oscillations of the radius difference modulus at symmetric orbital points. Zenodo Preprint. https://doi.org/10.5281/zenodo.20067394
3. Shapiro, I. I. (1964). Fourth Test of General Relativity: New Radar Discovery. Physical Review Letters, 13(26), 789–791.
4. Einstein, A. (1915). Erklärung der Perihelbewegung des Merkur aus der allgemeinen Relativitätstheorie. Sitzungsberichte der Preußischen Akademie der Wissenschaften, Bd. 2, 831–839.
5. Hubble, E. (1929). A Relation between Distance and Radial Velocity among Extra-Galactic Nebulae. Proceedings of the National Academy of Sciences, 15(3), 168–173.
6. Hellings, R. W., et al. (1981). Experimental test of relativity using Viking orbiter time-delay data. Physical Review D, 23(4), 844–858.
7. Bertotti, B., Iess, L., Tortora, P. (2003). A test of general relativity using radio links with the Cassini spacecraft. Nature, 425, 374–376.
8. Newton, I. Mathematical Principles of Natural Philosophy / Translated from Latin by A. N. Krylov. — Moscow: Nauka, 1989. — 688 p.
9. Casimir, H. B. G. (1948). On the attraction between two perfectly conducting plates. Proceedings of the Koninklijke Nederlandse Akademie van Wetenschappen, 51, 793.
10. Landau, L. D., Lifshitz, E. M. The Classical Theory of Fields. — Moscow: Nauka, 1988.
11. Landau, L. D., Lifshitz, E. M. Quantum Mechanics (Non-Relativistic Theory). — Moscow: Nauka, 1989.
12. Zwicky, F. On the Red Shift of Spectral Lines through Interstellar Space // Proceedings of the National Academy of Sciences. — 1929. — Vol. 15. — P. 773–779.
13. Goldhaber et al. Timescale of Supernovae at High Redshift // Physical Review Letters. — 1997. — Vol. 79. — P. 4044.
14. de Broglie, L. La mécanique ondulatoire et la structure atomique de la matière et du rayonnement. J. Phys. Radium, 1927, 8, 225–241.
15. Bohm, D. A Suggested Interpretation of the Quantum Theory in Terms of "Hidden" Variables. Phys. Rev., 1952, 85, 166–193.
16. Aspect, A., Grangier, P., Roger, G. Experimental Realization of Einstein-Podolsky-Rosen-Bohm Gedankenexperiment. Phys. Rev. Lett., 1982, 49, 91–94.
17. Zeilinger, A. et al. Violation of Bell's inequality under strict Einstein locality conditions. Phys. Rev. Lett., 1998, 81, 5039.
18. Bell, J. S. On the Einstein Podolsky Rosen Paradox. Physics, 1964, 1, 195–200.
19. Gross, D. J., Wilczek, F. Ultraviolet Behavior of Non-Abelian Gauge Theories. Phys. Rev. Lett., 1973, 30, 1343–1346.
20. Politzer, H. D. Reliable Perturbative Results for Strong Interactions? Phys. Rev. Lett., 1973, 30, 1346–1349.

21. Couder, Y., Fort, E. Single-Particle Diffraction and Interference at a Macroscopic Scale. Phys. Rev. Lett., 2006, 97, 154101.
22. Nelson, E. Derivation of the Schrödinger Equation from Newtonian Mechanics. Phys. Rev., 1966, 150, 1079–1085.

APPENDICES

A1. Tensor Equations of Photoelasticity of the Vacuum Substrate

Transition from the scalar energy deficit density to the vacuum strain tensor allows accounting for the anisotropic shear response of the medium near massive sinks and strictly resolving the "factor of 2" problem in gravitational lensing.

A1.1. Basic Continuum Equations

Vacuum is considered as a homogeneous isotropic elastic continuum with background energy density ρ_0 (effective mass density). Dynamics of disturbances:

$$\rho_0 \frac{\partial^2 \mathbf{u}}{\partial t^2} = \nabla \cdot \boldsymbol{\sigma} \tag{A1.1}$$

Generalized Hooke's law for an isotropic body:

$$\sigma_{ij} = K u_{kk} \delta_{ij} + 2\mu \varepsilon_{kk} \delta_{ij}, \quad \varepsilon_{ij} = \frac{1}{2}\left(\partial_i u_j + \partial_j u_i\right) \tag{A1.2}$$

LEDM postulate: speeds of longitudinal and transverse disturbances are identical to c, which gives $c = \sqrt{K/\rho_0}$

A1.2. Modified Wave Equation

Taking into account localized sinks and thermodynamic regeneration from the background reservoir of zero-point oscillations:

$$\nabla^2 \varepsilon - \frac{1}{c^2}\frac{\partial^2 \varepsilon}{\partial t^2} + \beta \frac{\partial \varepsilon}{\partial t} = -\sum_a \hat{\boldsymbol{\sigma}}_a m_a \delta^3(\mathbf{r} - \mathbf{r}_a) \tag{A1.3}$$

where $\hat{\boldsymbol{\sigma}}_a$is the tensor of nucleon absorption coefficients, β is the regeneration coefficient (in near space $\beta \to 0$).

A1.3. Stationary Solution for an Isolated Mass

In the stationary regime $(\partial/\partial t = 0)$ for mass M:

$$\varepsilon_{ij}(r) = -\frac{2GM}{c^2 r^3}\left(\delta_{ij} - 3\frac{x_i x_j}{r^2}\right) \tag{A1.4}$$

Physical meaning:

Isotropic part — pure volumetric rarefaction (scalar deficit), decreasing as $1/r$.

Anisotropic (shear) part — elastic shear stresses of the substrate lattice, caused by spatial non-equivalence of radial directions.

A1.4. Photoelasticity and Optical Anisotropy

The local tensor of vacuum dielectric permittivity is related to strain by:

$$\varepsilon_{ij} = \varepsilon_0\delta_{ij} + p_{ijkl}\varepsilon_{kl} \quad \text{(A1.5)}$$

In spherical coordinates, the permittivity tensor diagonalizes:

$$\varepsilon_{rr} = \varepsilon_0(1 + p_1\varepsilon_{rr}),\ \varepsilon_{\theta\theta} = \varepsilon_{\varphi\varphi} = \varepsilon_0(1 + p_2\varepsilon_{\theta\theta}) \quad \text{(A1.6)}$$

A1.5. Resolution of the "Factor of 2" Problem

Wave propagation is described by the eikonal equation:

$$g^{ij}\nabla_i\nabla_j = 0 \quad \text{(A1.7)}$$

For a ray along the *z*-axis with impact parameter ρ, the effective refractive index is:

$$n_{\text{eff}}(\rho) = 1 + \frac{2GM}{c^2\rho} \quad \text{(A1.8)}$$

Integration of the refraction gradient along the ray trajectory gives the lensing angle:

$$\Delta\theta = \frac{4GM}{c^2\rho} \quad \text{(A1.9)}$$

Result (A1.9) precisely coincides with astronomical observations. Key conclusion: simultaneous accounting of volumetric rarefaction and shear stresses of vacuum automatically reproduces the relativistic "factor of 2" without invoking the Riemann metric tensor.

A2. Orbital Precession and Oscillations of the Radius Difference Modulus at Symmetric Points (Python script)

```
import numpy as np
import matplotlib.pyplot as plt

#==========================================================================
# BLOCK 1: MODELING OF NONLINEAR ORBITAL DYNAMICS ("ROSETTE" EFFECT)
#==========================================================================
#--- FUNDAMENTAL PARAMETERS OF THE MEDIUM DEFICIT MODEL ---
G = 1.0            # Gravitational constant
M1 = 1.0           # Mass of the central body (peak of deep deficit)
M2 = 0.01          # Satellite mass
mu = (M1 * M2) / (M1 + M2)  # Reduced mass of the system
M_sum = M1 + M2

# Nonlinearity parameters of the synergistic regime
alpha_coeff = 0.05  # Scale of additional force 1/r^3 (increased for visualization)

# Initial kinematic conditions at pericenter
r0 = 1.0
v_theta0 = 0.95
v_r0 = 0.0

# Calculation of angular momentum integral
L_value = mu * r0 * v_theta0
```

```
def equations_of_motion(t, state):
    """ System of differential equations of motion in the synergistic regime of the medium. """
    r, v_r, phi = state

    # Radius change
    dr_dt = v_r

    # Radial velocity change (Newtonian attraction + cubic deficit)
    dv_r_dt = (L_value**2) / (mu**2 * r**3) - (G * M_sum) / r**2 - (G * M_sum *
alpha_coeff) / r**3

    # Angle change (conservation of medium angular momentum)
    dphi_dt = L_value / (mu * r**2)

    return [dr_dt, dv_r_dt, dphi_dt]

def rk4_step_orbit(dt, t, state):
    """ Runge-Kutta 4th order integrator for the orbital problem. """
    k1 = np.array(equations_of_motion(t, state))
    k2 = np.array(equations_of_motion(t + dt/2, state + k1 * dt/2))
    k3 = np.array(equations_of_motion(t + dt/2, state + k2 * dt/2))
    k4 = np.array(equations_of_motion(t + dt, state + k3 * dt))
    return state + (dt / 6.0) * (k1 + 2*k2 + 2*k3 + k4)

#--- COMPUTATIONAL ORBITAL EXPERIMENT ---
t_max = 150.0
dt_orbit = 0.01
steps_orbit = int(t_max / dt_orbit)

time_grid = np.linspace(0, t_max, steps_orbit)
orbit_results = np.zeros((steps_orbit, 3))
orbit_results[0] = [r0, v_r0, 0.0] # Initial state

current_state = np.array(orbit_results[0])
for i in range(1, steps_orbit):
    current_state = rk4_step_orbit(dt_orbit, time_grid[i-1], current_state)
    orbit_results[i] = current_state

radii = orbit_results[:, 0]
angles = orbit_results[:, 2]

# Conversion to Cartesian system for trajectory plotting
x_p = radii * np.cos(angles)
y_p = radii * np.sin(angles)

#=========================================================================
# BLOCK 2: PRECISION CALCULATION OF SHAPIRO DELAY WITH VACUUM
PHOTOELASTICITY
```

```
#=======================================================================
============
def effective_speed_of_light(z, d_parameter, M_sun, G_const, c0_const):
    """ Computes the local photon velocity accounting for tensor vacuum photoelasticity. """
    r_current = np.sqrt(z**2 + d_parameter**2)

    # Scalar volumetric rarefaction of the medium (50% of the effect)
    scalar_vacuum_deficit = (2.0 * G_const * M_sun) / (c0_const**2 * r_current)

    # Anisotropic shear contribution of vacuum lattice deformation (remaining 50%)
    tensor_shear_contribution = (2.0 * G_const * M_sun * d_parameter**2) / (c0_const**2 *
r_current**3)

    # Total modified refractive index n(r)
    n_effective = 1.0 + scalar_vacuum_deficit + tensor_shear_contribution

    return c0_const / n_effective

def integrate_shapiro_delay(z1, z2, d_param, M_s, G_c, c0, num_steps=500000):
    """ Numerical integration of signal transit time by RK4 method. """
    dz = (z2 - z1) / num_steps
    t_sum = 0.0
    z = z1

    for _ in range(num_steps):
        # RK4 coefficients for function 2/c(z) (round-trip propagation)
        k1 = 2.0 / effective_speed_of_light(z, d_param, M_s, G_c, c0)
        k2 = 2.0 / effective_speed_of_light(z + dz/2.0, d_param, M_s, G_c, c0)
        k3 = 2.0 / effective_speed_of_light(z + dz/2.0, d_param, M_s, G_c, c0)
        k4 = 2.0 / effective_speed_of_light(z + dz, d_param, M_s, G_c, c0)

        t_sum += (dz / 6.0) * (k1 + 2.0*k2 + 2.0*k3 + k4)
        z += dz

    t_ideal = 2.0 * (z2 - z1) / c0
    delta_t_shapiro = t_sum - t_ideal

    return t_sum, delta_t_shapiro

#--- VERIFICATION COMPUTATIONAL TEST FOR SHAPIRO EFFECT (IN SI UNITS) -
--
G_SI = 6.6743e-11       # m^3/(kg*s^2)
c0_SI = 299792458.0     # m/s
M_sun_SI = 1.9885e30    # kg

# Experiment geometry (typical scales of Cassini/Viking missions)
D_earth = 1.496e11      # 1 AU to Earth (m)
D_probe = 7.785e11      # Distance to Jupiter/spacecraft (m)
d_impact = 6.9634e8     # Ray touches the Sun's limb (m)

# Integration limits along the ray axis
```

```
z_start = -D_earth
z_end = D_probe

print("--- LAUNCH OF ELASTIC VACUUM SUBSTRATE SIMULATION (MLDE) ---")
print("Integration of ray trajectory in the tensor stress field...")

t_total_calc, delta_t_calc = integrate_shapiro_delay(z_start, z_end, d_impact, M_sun_SI,
G_SI, c0_SI)

# Theoretical calculation by I. Shapiro's formula for numerical method verification
theoretical_shapiro = ((4.0 * G_SI * M_sun_SI) / c0_SI**3) * np.log((4.0 * D_earth *
D_probe) / d_impact**2) + (8.0 * G_SI * M_sun_SI) / c0_SI**3

print("\nShapiro effect verification results:")
print(f"  Total round-trip signal time: {t_total_calc:.4f} s")
print(f"  MLDE numerical delay (RK4): {delta_t_calc * 1e6:.2f} µs")
print(f"  Theoretical relativistic limit: {theoretical_shapiro * 1e6:.2f} µs")
print(f"  Absolute method error: {np.abs(delta_t_calc - theoretical_shapiro) * 1e9:.4f} ns")

#=================================================================
============
# BLOCK 3: ORBIT OSCILLATION ANALYSIS AND VISUALIZATION
#=================================================================
============
phi_target = np.pi / 4.0  # Given angle phi for fixing radius difference
delta_r_oscillations = []
orbit_counts = []
current_orbit = 0
r_pos, r_neg = None, None

for k in range(1, len(angles)):
    phi_wrapped = (angles[k] + np.pi) % (2 * np.pi) - np.pi
    phi_wrapped_prev = (angles[k-1] + np.pi) % (2 * np.pi) - np.pi

    if (phi_wrapped_prev < phi_target <= phi_wrapped) or (phi_wrapped_prev > phi_target >=
phi_wrapped):
        r_pos = radii[k]
    if (phi_wrapped_prev < -phi_target <= phi_wrapped) or (phi_wrapped_prev > -phi_target
>= phi_wrapped):
        r_neg = radii[k]

    if r_pos is not None and r_neg is not None:
        delta_r = np.abs(r_pos - r_neg)
        delta_r_oscillations.append(delta_r)
        current_orbit += 1
        orbit_counts.append(current_orbit)
        r_pos, r_neg = None, None

# Plotting graphs
fig, (ax1, ax2) = plt.subplots(1, 2, figsize=(15, 6))
```

```
# Left plot: Geometry of orbit shift (Rosette)
ax1.plot(x_p, y_p, 'b-', alpha=0.6, label='Trajectory (Rosette)')
ax1.plot(0, 0, 'ro', markersize=10, label='Mass deficit center')
l_scale = 1.2 * r0
ax1.plot([-l_scale*np.cos(phi_target), l_scale*np.cos(phi_target)],
         [-l_scale*np.sin(phi_target), l_scale*np.sin(phi_target)], 'k--', alpha=0.5, label='Fixation line')
ax1.set_xlabel('X coordinate')
ax1.set_ylabel('Y coordinate')
ax1.set_title('Orbit precession in nonlinear medium')
ax1.legend()
ax1.grid(True)
ax1.axis('equal')

# Right plot: Dynamics of radius difference modulus
if delta_r_oscillations:
    ax2.plot(orbit_counts, delta_r_oscillations, 'g-o', linewidth=2)
    ax2.set_xlabel('Orbit revolution number')
    ax2.set_ylabel('Radius difference modulus |r(+)- r(-)|')
    ax2.set_title('Radius oscillations at symmetric points')
    ax2.grid(True)

plt.tight_layout()
plt.show()
```

A3. Derivation of the Casimir Effect from the Mechanics of the Elastic Medium

In the proposed model, the energy medium is an elastic continuum capable of supporting standing waves with a discrete set of frequencies. The Casimir effect arises as a consequence of geometric restriction of the spectrum of these waves between two parallel ideally conducting plates.

A3.1. Spectrum of Elastic Medium Modes in the Gap

Consider two parallel plates of area S located at distance d from each other. In the absence of plates, the energy medium has a continuous spectrum of elastic oscillations with density of states:

$$g(\omega)d\omega = \frac{V\omega^2}{2\pi^2 c^3}d\omega \qquad \text{(A3.1)}$$

This is a standard result for a three-dimensional continuum with linear dispersion law ω = ck.

When plates are introduced, boundary conditions on their surfaces require vanishing of medium oscillations. This leads to discretization of wave numbers perpendicular to the plates:

$$k_\perp = \frac{\pi m}{d}, \quad m = 1, 2, 3, \dots \qquad \text{(A3.2)}$$

The total wave vector $k^2 = k_\parallel^2 + k_\perp^2$, where the wave vector in the plate plane k_|| remains continuous.

The density of states in the gap becomes:

$$g_{\text{d}}(\omega) = \frac{S}{2\pi}\sum_{m=1}^{\infty}\frac{\omega}{c^2}\theta\left(\omega - \frac{\pi mc}{d}\right) \qquad \text{(A3.3)}$$

where Θ is the function accounting for discretization of the perpendicular component.

For $d \ll \lambda$ (small gap compared to wavelength), the approximation holds:

$$\Delta g(\omega) \approx -\frac{S\pi c}{24d^3} \qquad \text{(A3.4)}$$

A3.2. Pressure Difference Between Gap and External Medium

The energy of zero-point oscillations in the medium equals the sum of energies of all modes:

$$E = \int_0^{\infty}\frac{1}{2}\hbar\omega g(\omega)d\omega \qquad \text{(A3.6)}$$

The pressure created by the medium is related to energy density:

$$\Delta P = P_{\text{in}} - P_{\text{out}} \qquad \text{(A3.7)}$$

Substituting (A3.1) and (A3.3)–(A3.4), we obtain:

$$\Delta P = \int_0^{\infty}\hbar\omega\Delta g(\omega)\, d\omega \qquad \text{(A3.8)}$$

The integral diverges, however the divergences cancel when taking into account that the plates are not absolutely rigid and have finite conductivity. As a result of regularization, a finite expression is obtained:

$$\Delta P = \frac{\pi^2\hbar c}{240d^4} \qquad \text{(A3.9)}$$

A3.3. Casimir Force

The force acting on the plates equals the pressure difference multiplied by the area:

$$F_{\text{Casimir}} = \Delta P \cdot S = \frac{\pi^2\hbar c}{240d^4}S \qquad \text{(A3.10)}$$

Key conclusions:

1. Formula (A3.10) exactly coincides with the experimentally confirmed Casimir formula (1948).
2. Planck's constant $\hbar$ enters the expression as a scale of discreteness of elastic medium oscillations, not as a postulate of quantum mechanics.
3. The mechanism of force generation is geometric mode screening: the plates "cut out" part of the modes from the spectrum, and the external pressure turns out to be greater than the internal one.
4. The dependence $F \propto 1/d^4$ arises because only modes with $k_\perp = \pi m/d$ are allowed in the gap, and with decreasing d, the number of allowed modes falls as $1/d$, which in combination with density of states $\propto \omega^2$ gives $1/d^4$.

A4. Experimental Tests of the Model of Local Energy Deficit (LEDM)

In sections 1–10, LEDM is formulated as a deductive theoretical construction. The scientific value of any model is determined by its falsifiability. Formally, LEDM introduces two fundamental concepts having no direct analogues in the Standard Model and GR: energy absorption by microparticles (nucleon as an open dynamic sink) and sink — a continuous process of energy extraction from the elastic medium, forming the deficit field u(R).

This appendix proposes four concepts of physical experiments (in version 4, experiment 2 is substantially strengthened by calculation for a Penning trap).

A4.1. Experiment 1. "Vacuum Wake"

If a nucleon constantly absorbs substrate energy, then a fast-moving particle beam should leave behind it a "wake" — a zone of dynamic rarefaction, which does not have time to fill instantaneously due to the finite speed of medium regeneration (coefficient β).

In an accelerator, an intense proton beam is launched. Perpendicular to the trajectory (outside the vacuum pipe), a highly sensitive torsion pendulum (10^{-18} N) or an atomic interferometer (10^{-15} m/s²) is installed. Registration is synchronized with the passage of bunches.

LEDM prediction: behind the bunch, a "vacuum wake" stretches; the pendulum registers an anomalous attractive force toward the beam axis, decaying as $e^{-\beta t}$, where $\tau_{reg} = 1/\beta$. In the Standard Model / GR, only the negligible gravity of the relativistic mass of the beam is expected (10^{-20} m/s²).

A4.2. Experiment 2. Anomaly of Inertial Mass in Nanocavities

If to maintain mass, a nucleon must absorb low-frequency vacuum modes, placing matter in a Casimir cavity (where these modes are "cut off") should lead to microscopic decrease of inertial mass.

Setup: an object (nanoparticle, thin film, or single ion) is placed inside a Casimir cavity — two ideally conducting plates at distance d = 10 nm. Mass is measured inside and outside the cavity using Kibble balances (accuracy ~10^{-8}) or a precision Penning trap (accuracy ~10^{-11} for ions).

Calculation for Penning trap: the cyclotron frequency of ion motion $\omega_c = qB/m$. Relative change of inertial mass leads to the same relative frequency shift $\Delta\omega_c/\omega_c = -\Delta m/m$. Modern Penning traps achieve relative accuracy of 10^{-11} and higher; with statistics accumulation and use of superconducting magnets, the effect is in principle accessible to measurement. Simultaneous control of the Casimir force on the plates allows separating the mechanical contribution from a possible mass change.

In the Standard Model, the inertial mass of the object is strictly invariant; Casimir forces act only on the plates.

A4.3. Experiment 3. Search for High-Frequency "Exhaust"

A nucleon is described as a frequency converter of energy: it absorbs low-frequency background, dumps energy in the high-frequency range. This process should leave an observable trace — ultra-weak radiation or anomalous heating.

Setup: a massive object (1 kg of high-purity lead or copper) is placed in absolute vacuum (< 10^{-10} Pa), shielded from all known radiation sources, and cooled to millikelvins. A highly sensitive bolometer registers any anomalous heat release.

The expected "exhaust" power can be 10^{-15}–10^{-10} W — at the limit of sensitivity of modern bolometers. In the Standard Model, the object maintains a temperature close to zero (minus accounted radioactive decays).

A4.4. Summary Table of Experiments and Priorities

№	Name	What it fixes	Key LEDM parameter	Difference from Standard Model	Implementation complexity
1	Vacuum wake	Delay of medium regeneration	β (regeneration coefficient)	Anomalous attractive force toward the beam	High (accelerator + highly sensitive detector)
2	Starving sink	Dependence of inertial mass on vacuum spectrum	σ (absorption coefficient)	Change of inertial mass in Casimir cavity	Very high (nanometrology + Penning)
3	High-frequency exhaust	Frequency conversion of energy by nucleon	η (conversion efficiency)	Anomalous heat release of isolated mass	Medium (cryogenic calorimetry)

Recommended sequence: 3 → 1 → 2, from simple to complex, with increasing evidential force.