Time Dilation in Atomic Clocks

Florian Michael Schmitt

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Abstract

The Hafele–Keating experiment and GPS technology exhibit a time-dilation effect that has traditionally been explained in terms of special and general relativity. We show, based on a Newtonian concept of a gravitationally entrained ether, that atomic clocks intrinsically exhibit this time-dilation effect when moving relative to gravitational fields. To this end, we develop a mathematical framework that reproduces the values observed in the aforementioned experiments for the apparent time dilation within a Newtonian-physics framework.

Content

1. How Atomic Clocks Work

In simplified terms, the atomic clock compares the frequency (corresponding to energy) required to excite the atoms from the hyperfine state F=3 to F=4 with a frequency generated by a local oscillator, using a feedback control loop. The oscillator operates at a frequency from which a synthesizer generates microwaves close to the atomic transition frequency. These microwaves are coupled into the cavity resonator through a coupling aperture. The desired mode—a spatially defined microwave magnetic field—builds up inside the resonator. The atoms pass through this field and interact with it. After passing through the field, the fraction of excited atoms is detected. The servo electronics then modulates the frequency slightly and symmetrically around the nominal value and compares the response on both sides. If the difference is zero, the frequency is exactly at the atomic resonance. If it is nonzero, the servo applies a correction voltage to the oscillator, causing it to shift its frequency by the required amount. The resulting frequency constitutes the time standard of the atomic clock.

Due to manufacturing tolerances, the resonator cannot be tuned to the transition frequency with atomic precision. This is not critical, however, because it acts as a frequency-dependent amplifier with a certain bandwidth. Consequently, the field amplitude at the applied microwave frequency is maximal when the resonator is perfectly tuned, while otherwise only its amplitude inside the resonator varies. The probability of excitation of the atoms is determined by both the frequency and the amplitude of the signal (Rabi frequency).

In this respect, it should be permissible to hypothesize that a phase shift in the resonator, which consequently leads to a change in this amplitude, should also have an effect on the number of atoms detected (provided that the amplitude is not corrected by additional measures).

In the regime of relativity theory, such a phase shift in the resonator is not possible. In the regime of Newtonian physics, however, it necessarily occurs, as we will demonstrate below.

2. Determining Time Dilation by Restoring Resonance

For simplicity, we assume that a phase shift in the resonator produces a corresponding rate deviation of the atomic clock. We now want to examine, within the framework of Newtonian physics and an ether that is completely entrained by gravity, what it would mean to compensate for such a phase shift by means of a change in wavelength.

As long as we consider the entire atomic clock, and therefore also the resonator, to be at rest relative to the gravitational ether, classical physics continues to give us a resonant wave. The light travel times of the outgoing and returning beams do not change, and for the sum of the two beams we have:

$$t_0 = 2 \cdot \frac{l_0}{c}$$

t_0 is the sum of the light travel times of both beams, l_0 is the length of the resonator.

We illustrate this using a model, for which we have chosen the following values for the quantities:

c = 1,2
v_{Res} = 0,0
λ_0 = 0,7
n = 3
l_0 = 2,1

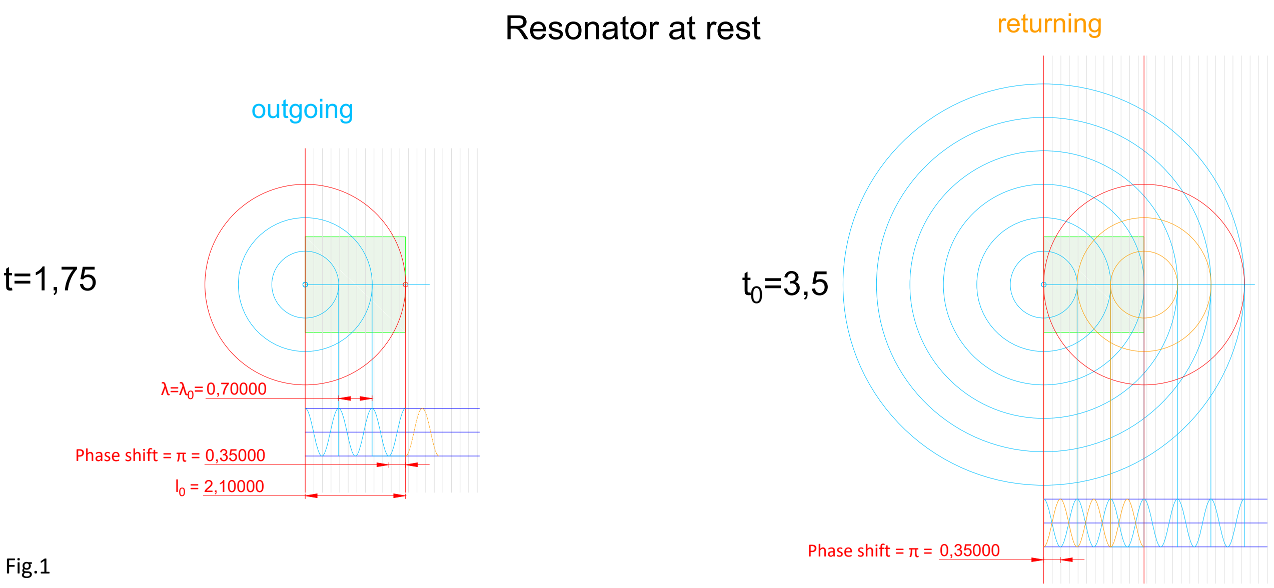

Fig.1

Situation at t = 1.75 (the outgoing beam reaches the stationary right-hand resonator mirror). The phase shift at the mirror is π. The phase of the wave that was emitted with a wave crest likewise reaches the mirror at a wave crest.

Situation at t = 3.5 (the returning beam reaches the stationary left-hand resonator mirror). The phase shift at the mirror is still π. The wave is therefore resonant.

However, if the resonator now moves relative to the gravitational ether, a velocity addition to the velocity of the resonator would result (essentially analogous to the Michelson–Morley experiment [1]). The sum of the light travel times of both beams t_{1+2} is now:

$$t_{1+2} = \frac{l_0}{(c-v)} + \frac{l_0}{(c+v)} = 2 \cdot l_0 \cdot \frac{c}{(c^2-v^2)}$$

And consequently, for the distance traveled:

$$l_{1+2} = 2 \cdot l_0 \cdot c \cdot \frac{c}{(c^2-v^2)} \qquad (1)$$

In classical physics, an additional phase shift would now occur if the distance traveled were no longer an integer multiple of the wavelength. The resonance of the standing wave would break down. For the model, we choose the following values, differing from those in Fig. 1:

v_{Res} = 0,18

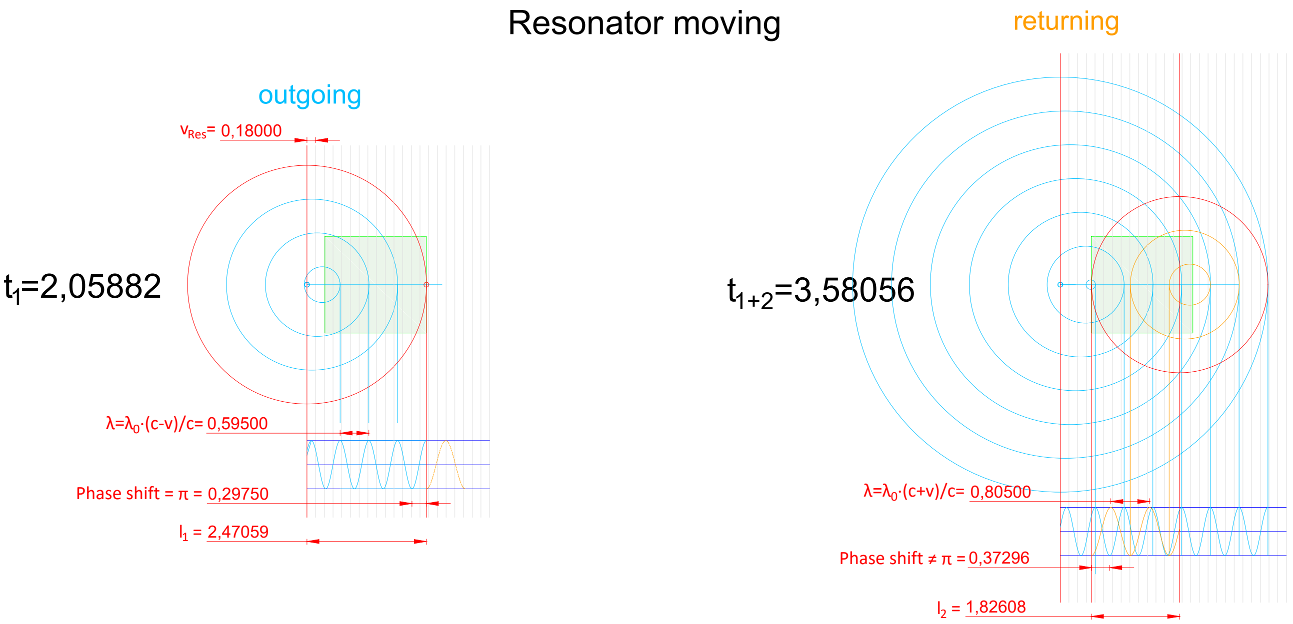

Fig. 2

Situation at t = 2.05882 (the outgoing beam reaches the moving right-hand resonator mirror). The phase shift at the mirror is π. The wavelength changes due to the Doppler effect of the moving source. The Doppler effect of the observer (the right-hand mirror) is determined solely by the time at which the wave arrives. Nevertheless, the phase of the wave emitted with a wave crest continues to arrive at the mirror with a wave crest, since the Doppler effects at the source and receiver cancel each other out.

Situation at t = 3.58056 (the returning beam reaches the moving left-hand resonator mirror). The outgoing and returning waves exhibit a phase shift that differs from π.

We can now adjust the wavelength so that resonance is restored by ensuring that l_{1+2} now constitutes an integer multiple of the wavelength:

$\lambda_0 = \frac{l_0}{n}$ und $l_0 = \lambda_0 \cdot n$

(1) $\Rightarrow l_{1+2} = 2 \cdot \lambda_0 \cdot n \cdot \frac{c^2}{(c^2 - v^2)}$

We can now determine this modified wavelength:

$$\lambda_{new} = \frac{l_{1+2}}{2 \cdot n} = \frac{\lambda_0 \cdot 2 \cdot n \cdot \frac{c^2}{(c^2 - v^2)}}{2 \cdot n} = \lambda_0 \cdot \frac{c^2}{c^2 - v^2} \qquad (2)$$

For the model, we choose the following value, differing from that in Fig. 2:

λ_0 = 0,71611 (calculated according to equation (2))

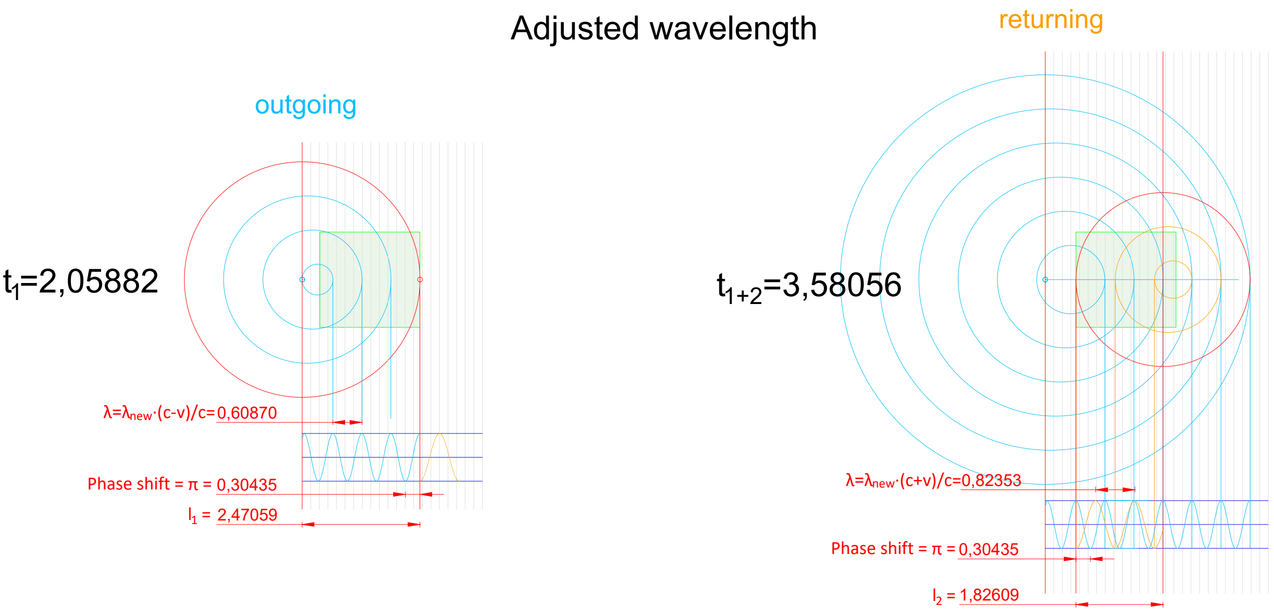

Fig. 3

Situation at t = 2.05882 (the outgoing beam reaches the moving right-hand resonator mirror). The phase shift at the mirror is π. Nevertheless, the phase of the wave emitted with a wave crest continues to arrive at the mirror with a wave crest, since the Doppler effects at the source and receiver cancel each other out.

Situation at t = 3.58056 (the returning beam reaches the moving left-hand resonator mirror). The phase shift at the mirror is π. The wave is resonant again.

The phase correction described above therefore once again maximizes the amplitude of the resonator. The control loop of the atomic clock automatically produces this adjustment by changing the frequency of the oscillator. The ratio of the original wavelength to the adjusted wavelength can therefore be equated with the so-called time dilation.

The ratio of λ_{new} to λ_0 now gives the measure of the time dilation:

$$\frac{\Delta t}{sec} = \frac{\lambda_{new}-\lambda_0}{\lambda_0} = \frac{\lambda_0\cdot\frac{c^2}{c^2-v^2}-\lambda_0}{\lambda_0} = \frac{\lambda_0\cdot\left(\frac{c^2}{c^2-v^2}-1\right)}{\lambda_0} = \frac{c^2}{c^2-v^2}-1 \qquad (3)$$

4. GPS Technology

4.1 Kinematic Component

We first consider the rotational motion of the satellite relative to the stationary gravitational field of the Earth.

c = 299,792,000 m/s (speed of light)

v_{kin} = 3,874 m/s (orbital speed of satellite)

Using Equation (3), we obtain:

$$\Delta t(24h) = \left(\frac{\left(299{,}792{,}000\frac{\mathrm{m}}{\mathrm{s}}\right)^{2}}{\left(299{,}792{,}000\frac{\mathrm{m}}{\mathrm{s}}\right)^{2} - \left(3{,}874\frac{\mathrm{m}}{\mathrm{s}}\right)^{2}} - 1 \right) \cdot 86{,}400s \approx 14.43\mu s$$

4.2 Gravitational Component

Within the framework of a gravitationally dragged aether surrounding the Earth and the satellite, the gravitational influence of the relevant celestial bodies must be taken into account. These influences vary as a function of the distance between the respective masses and the satellite. Their relative magnitudes determine the extent to which the propagation of light is assumed to be affected by gravitational dragging.

The relevant gravitational effects can be illustrated using the Newtonian gravitational relationship.

$$a = G \cdot \frac{m_1 + m_2}{r^2}$$

Gravitational influence of sun (here m_1=100 kg represents the satellite, m_2 the mass of Sun, r the distance between Sun and Earth):

$$a_{sun} = G \cdot \frac{m_1 + m_2}{r^2} = 6.6743 \cdot 10^{-11} \frac{m^3}{kg \cdot s^2} \cdot \frac{100kg + 1.989 \cdot 10^{30}kg}{(1.496 \cdot 10^{11}m)^2} = 5.932 \cdot 10^{-3} \frac{m}{s^2}$$

Gravitational influence of earth (here m_1 represents the satellite, m_2 the mass of Earth, r the radius of the satellite's orbit):

$$a_{orbit} = G \cdot \frac{m_1 + m_2}{r^2} = 6{,}6743 \cdot 10^{-11} \frac{m^3}{kg \cdot s^2} \cdot \frac{100kg + 5{,}972 \cdot 10^{24}kg}{(2{,}6571 \cdot 10^{7}m)^2} = 0{,}56456 \frac{m}{s^2}$$

The corresponding gravitational dragging contribution to the propagation of light along the satellite's orbit, expressed as part of v_{CMB}, is therefore:

$$v_{grav} = v_{CMB} \cdot \frac{a_{sun}}{a_{eart}} = 368{,}000 \frac{m}{s} \cdot \frac{5.932 \cdot 10^{-3} \frac{m}{s^2}}{0.56456 \frac{m}{s^2}} \approx 3{,}828 \frac{m}{s}$$

Compared with the satellite's orbital velocity of 3,874 $\mathrm{m/s}$, this contribution is significant.

The next step is to determine the direction of this additional velocity component throughout one-half of the satellite's orbital revolution.

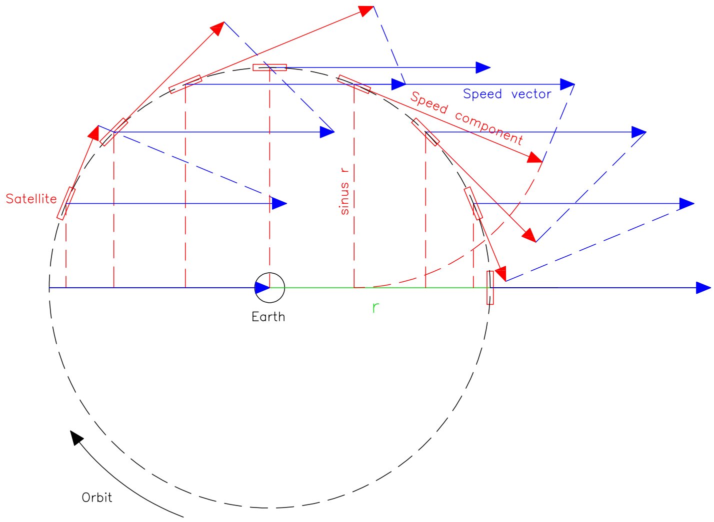

Fig. 4: Velocity vector

As illustrated in Fig. 4, the component of the velocity vector parallel to the resonator axis is given by the sine of the corresponding orbital angle. This component is highlighted in red.

To determine the average applicable velocity component over one-half of the orbital revolution, we consider the following quantities.

For the length of the semicircular arc:

$$f_1(r) = r \cdot \pi$$

For the corresponding component of the velocity vector:

$$f_2(r) = r \cdot \sin(r)$$

The average component over one half of the orbit can then be obtained by integrating both quantities. In other words, the average is determined from the ratio of the area beneath the semicircular arc to the area beneath the corresponding sine curve:

$$\int_0^\pi f_1(r) = \frac{1}{2} r^2 \cdot \pi$$

$$\int_0^\pi f_2(r) = \frac{1}{2} r^2 \cdot \left(-\cos(\pi) + \cos(0)\right) = \frac{1}{2} r^2 \cdot \left(-(-1) + 1\right) = r^2$$

and therefore:

$$\int_0^\pi \frac{f_1(r)}{f_2(r)} = \frac{1/2\, r^2 \cdot \pi}{r^2} = \frac{1}{2} \pi$$

This relationship shows that, over a half-circle interval, the applicable velocity component corresponds to $\pi/2$ times the velocity vector.

Consequently, the total contribution resulting from v_{CMB} is:

$$v_{comp} = \frac{3{,}828\,\frac{m}{s}}{\pi/2} = 2{,}437\,\frac{m}{s}$$

4.3 Total GPS Time Dilation

The total applicable velocity is therefore:

$$v_{total} = v_{kin} + v_{comp} = 3{,}874\,\frac{m}{s} + 2{,}437\,\frac{m}{s} = 6{,}311\,\frac{m}{s}$$

Applying Equation (3) once again gives:

$$\Delta t(24h) = \left(\frac{\left(299{,}792{,}000\,\frac{\mathrm{m}}{\mathrm{s}}\right)^{2}}{\left(299{,}792{,}000\,\frac{\mathrm{m}}{\mathrm{s}}\right)^{2} - \left(6{,}311\,\frac{\mathrm{m}}{\mathrm{s}}\right)^{2}} - 1\right) \cdot 86{,}400s \approx 38.29\mu s$$

The total relativistic clock correction for GPS, including both Special and General Relativity [2], is a net clock gain of approximately 38,5µs, using the corresponding values for the masses, velocities, distances, and other relevant parameters. This result is in good agreement with the experimentally observed value.

4. Haefele Keating Experiment

The Hafele–Keating experiment [4] is closely related to GPS technology, except that instead of a satellite, we are dealing with two aircraft traveling in opposite directions.

First, we calculate the gravitational component at an altitude of 10,000 m:

$$a_{orbit} = G \cdot \frac{m_1 + m_2}{r^2} = 6.6743 \cdot 10^{-11}\,\frac{m^3}{kg \cdot s^2} \cdot \frac{100kg + 5.972 \cdot 10^{24}kg}{(6.371 \cdot 10^6 m + 1 \cdot 10^4 m)^2} = 9.789\,\frac{m}{s^2}$$

$$v_{grav} = v_{CMB} \cdot \frac{a_{sun}}{a_{eart}} = 368{,}000\,\frac{m}{s} \cdot \frac{5.932 \cdot 10^{-3}\,\frac{m}{s^2}}{9.789\,\frac{m}{s^2}} \approx 222.87\,\frac{m}{s}$$

$$v_{comp} = \frac{222.87\,\frac{m}{s}}{\pi/2} = 141.88\,\frac{m}{s}$$

At the Earth's surface (due to the distance from the center of mass), this component amounts to $141{,}44\,\frac{m}{s}$

The data from the experiment are not scaled to uniform time intervals, so we need to perform some conversion. We have compiled the conditions from the experiment and, alongside them, our results from the equations above as follows:

	data	eastward flight	earth reference east	earth reference west	westward flight
from H+K	c (m/s)	299.792.000,00	299.792.000,00	299.792.000,00	299.792.000,00
	latitude of flight (degree)	34,00	38,90	38,90	31,00
	Earth's radius (m)	6.371.000,00	6.371.000,00	6.371.000,00	6.371.000,00
	earth's circumference at latitude (m)	33.186.517,94	31.153.208,33	31.153.208,33	34.312.555,84
	flight duration (h)	41,20	41,20	48,60	48,60
	equivalent to flight time (s)	148.320,00	148.320,00	174.960,00	174.960,00
	v_{rot} earth rotational speed at latitude (m/s)	385,16	361,56	361,56	398,22
	v_{plane} mean flight speed (m/s)	223,75	0,00	0,00	196,12
according to this paper	v_{grav} gravity component (m/s)	141,88	141,44	141,44	141,88
	v total (m/s)	**467,02**	**220,12**	**220,12**	**60,23**
		$(v_{rot}-v_{grav}+v_{plane})$	$(v_{rot}-v_{grav})$	$(v_{rot}-v_{grav})$	$(v_{rot}-v_{grav}-v_{plane})$
	time dilation per s according to equation (3) (s)	2,43E-12	5,39E-13	5,39E-13	4,04E-14
	total time dilation at flight time (s)	3,60E-07	8,00E-08	9,43E-08	7,07E-09
	total time dilation at flight time (ns)	359,93	79,96	94,33	7,07
	time dilation to earth reference	279,97			-87,25
	published data by H+K	273,00			-59,00

Table 1

These results are in very good agreement with the experimental results, particularly when taking into account the deviation of the raw data, which ranges from −196 to −45 ns.

References and Acknowledgments (in order of appearance):

[1] Michelson, A. A., Morley, E.W, On the Relative Motion of the Earth and the Luminiferous Ether, Vols. 1887, 34 (203): 333–345 1881, American Journal of Science, 1887.

[2] Einstein, Albert, Zur Elektrodynamik bewegter Körper, Annalen der Physik und Chemie. 17, 1905, S. 891–921, 1905.

[3] R. K. J. Hafele, Around the world atomic clocks: predicted relativistic time gains, Science. 177. Jahrgang, Nr. 4044, 1972.

[4] Doppler, Chrisitan, Über das farbige Licht der Doppelsterne und einiger anderer Gestirne des Himmels, Böhmische Gesellschaft der Wissenschaften, 1842.