Unification of Interactions According to the Special Theory of Relativity

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Abstract

This article presents a proposed unified description of physical interactions within the framework of the special theory of relativity. The starting point is the principle of minimum potential energy, interpreted equivalently as the principle of minimum rest mass. According to this approach, every interaction can be described by a general force formula containing the charges of the given interaction, an interaction constant, and a dimensionless function of distance. The commonly used inverse-square dependence is treated here as an experimentally established approximation valid only within a limited range of distances, rather than as a universal law extending from zero to infinity.

After the finite propagation speed of interactions is taken into account, interactions become one-sided: the force exerted by a source charge on another charge is determined by the state of the source at the point from which its interaction ray reaches the affected object. Therefore, the interaction must first be described in the rest frame of the source charge and then transformed to other inertial frames by means of Lorentz transformations.

Using general vectorial Lorentz transformation formulas for U-spaces of arbitrary dimension, together with a relativistic form of Newton's second law of motion, unified vector formulas are derived for the acceleration and force acting on a point object carrying an arbitrary interaction charge. The appendix presents the derivation of the rest-mass derivative used in these formulas and provides numerical checks confirming the consistency of the transformations.

In the article "What is Mass?"[1] it was stated that the fundamental law of physics governing all interactions is the **principle of minimum potential energy**, or, in other words, the **principle of minimum rest mass**. Consequently, there exists one unified formula that describes these interactions. The force with which one charge acts on another results from the gradient of the potential field of the charge in which the second charge is located. The formula for this force was given there in scalar form:

$$\boldsymbol{F} = \boldsymbol{k}\boldsymbol{q}_1\boldsymbol{q}_2\boldsymbol{f}(\boldsymbol{r}) \tag{1}$$

where $\boldsymbol{F}$ is the force with which two bodies interact, these bodies containing, respectively, $\boldsymbol{q_1}$ and $\boldsymbol{q_2}$ amounts of the charge of a given interaction; $\boldsymbol{k}$ is the constant of that interaction, whose task is to adapt the units of measurement used and which, in a certain sense, determines the strength of the given interaction. The function $\boldsymbol{f(r)}$, on the other hand, is a function of the distance $\boldsymbol{r}$, given in metres, between the two bodies. We agreed that this function is dimensionless.

For Newton's law of universal gravitation and Coulomb's law, that is, for the gravitational and electric interactions, it was assumed that the function $\boldsymbol{f(r)}$ is a power function:

$$\boldsymbol{f(r) = \frac{1}{r^2}} \tag{2}$$

This was established experimentally, on the basis of observations of these interactions within the range of distances accessible to the measuring instruments existing at that time. The accuracy of those instruments differed greatly from the precision of modern devices, and the range of distances covered by those instruments was limited. Therefore, function (2) should be treated, for these interactions, as a first approximation.

Unfortunately, it was very quickly and hastily accepted that this function is exact and has this form over the entire range of distances, that is, from zero to infinity. This conviction became so deeply rooted in common awareness that when later observations began to reveal deviations from function (2), especially for the gravitational interaction, it occurred to no one to verify this view. As a result of this attitude, the postulate of the existence of mysterious "dark matter" was created. It therefore seems very rational to adopt the thesis that the original functions $\boldsymbol{f(r)}$ for the individual interactions used by Nature differ from function (2).

> *Note! For the strong interaction, it has been assumed that the attractive force between charges increases with the distance between them, that is, in the opposite manner to that given by function (2). Therefore, the adoption of the principle that the distance function in formula (1) is not, in the general case, equal to function (2) is justified.*

Moreover, for short-range nuclear interactions the functions $\boldsymbol{f(r)}$ are limited, and their course is not known to us in detail; at most we may infer what shape these functions have. We can be certain of only one thing: that the function of the electric interaction and the functions of nuclear interactions pass through zero many times at subatomic distances, creating local minima of the potential, because otherwise the periodic table would not have arisen. All stable chemical compounds and stable atoms of elements are located in such a local minimum of potential energy.

Formula (1), supplemented by (2), has scalar form and is only a certain approximate image of the gravitational and electric interactions, although it was in precisely this form that it was presented by Newton and Coulomb. At that time it was believed that interactions, or forces, propagate instantaneously, and in such a case Newton's third law applies. However, after the announcement of the special theory of relativity, it turned out that interactions propagate with a limited speed and, for this reason, became one-sided.

In a situation in which interactions propagate with a limited speed, the speed of light, body $\boldsymbol{A}$ acts with a force on body $\boldsymbol{B}$ from the place where $\boldsymbol{A}$ is "seen" by $\boldsymbol{B}$. Body $\boldsymbol{B}$, however, will act on $\boldsymbol{A}$ only in the future, when $\boldsymbol{A}$ finds itself in the cone of interactions of $\boldsymbol{B}$. Therefore, the force with which $\boldsymbol{A}$ acted on $\boldsymbol{B}$ is, in general, not equal to the force with which $\boldsymbol{B}$ will act from that place on $\boldsymbol{A}$, because the distance $\boldsymbol{r}$ between the bodies may be different in these two situations, especially since it is a distance given in the reference frame of the body emitting the ray of interaction. Moreover, the interaction charge contained in $\boldsymbol{A}$ may meanwhile have changed its value, or $\boldsymbol{A}$ may have moved outside the interaction range of body $\boldsymbol{B}$ in the case of short-range interactions.

It is currently believed that interactions are carried by virtual bosons. Virtual bosons are elements of the Standard Model that are completely different from real bosons. Above all, they cannot carry energy. Therefore, in the article "What is Mass?"[1], the abstract, virtual elements carrying information about objects interacting at a distance were called "rays of interaction", and the light cone directed toward the future was called the "cone of interactions". Potential fields are also only information, on the basis of which interaction forces are calculated. We do not directly observe interaction fields; we observe only the effects of the action of these fields.

The unified formula (1), which determines the force of interaction of one charge on another charge within the range of its ray of interaction, applies only in the rest frame of the object emitting the ray of interaction. However, when we consider a larger number of objects in motion, then, to analyse such a situation, we must have a more general formula that will take vector directions into account and will be valid in all reference frames. Interactions must operate in the same way in all inertial reference frames. This is the fundamental paradigm of physics. Naturally, passing from one reference frame to another must be carried out by means of the Lorentz transformation.

In the article "General Vectorial Lorentz Transformation Formulas for Spacetimes of Arbitrary Dimensions"[2], the concept of spacetime was replaced by the concept of U-space, where the speed of light is dimensionless and equal to 1, while the speeds of massive objects lie in the right-open interval $\langle 0\,;1)$ and are also dimensionless. In this situation, the time axis must be dimensioned in units of distance, just like the remaining spatial axes, with $1\mathrm{s} = 299\,792\,458\mathrm{m}$, because in order for speed to be dimensionless, it must be a derivative of distance with respect to distance. In this way, we obtain a homogeneous, four-dimensional, metric space. The axis usually denoted by the letter $\boldsymbol{T}$ was called $\boldsymbol{U}$, the fourth spatial dimension, and hence the name U-space.

> *Note! It must be emphasized that in the special theory of relativity there are no restrictions on the number of dimensions of U-space, or spacetime, and all vector formulas used in the special theory of relativity should be brought to a form valid in U-spaces of arbitrary dimension.*

The task before us is the following: we must find a relativistic formula for the force vector with which one interaction charge acts on another when these charges move with certain velocities and the charge generating the given force has some acceleration. In principle, a more important formula is the formula for the acceleration vector of the object affected by the given interaction, because the worldline of the object is drawn on the basis of this acceleration. These formulas must be valid for U-spaces of arbitrary dimension. On the basis of these formulas, the same worldlines must be determined for every reference frame.

Let us therefore derive the formula for the force vector acting on a point object $\boldsymbol{B}$ containing the charge of an arbitrary interaction. Let us denote the magnitude of this charge by the letter $\boldsymbol{q}$. The potential force field in which $\boldsymbol{B}$ is located is produced by object $\boldsymbol{A}$, which contains charge $\boldsymbol{Q}$ of the same interaction. Let us also denote the remaining parameters of object $\boldsymbol{A}$ by capital letters, and those of object $\boldsymbol{B}$ by lower-case letters.

$$\boldsymbol{A}(\boldsymbol{U}, \vec{\boldsymbol{R}}, \vec{\boldsymbol{V}}, \vec{\boldsymbol{A}}, \boldsymbol{Q}, \boldsymbol{M_r})$$
$$\boldsymbol{B}(\boldsymbol{u}, \vec{\boldsymbol{r}}, \vec{\boldsymbol{v}}, \vec{\boldsymbol{a}}, \boldsymbol{q}, \boldsymbol{m_r})$$

where: $\boldsymbol{u}$ $[\boldsymbol{U}]$ denotes the $\boldsymbol{u}$ coordinate of the object,
vector $\vec{\boldsymbol{r}}$ $[\vec{\boldsymbol{R}}]$ – the position of the object,
vector $\vec{\boldsymbol{v}}$ $[\vec{\boldsymbol{V}}]$ – the velocity of the object,
vector $\vec{\boldsymbol{a}}$ $[\vec{\boldsymbol{A}}]$ – the acceleration of the object,
$\boldsymbol{m_r}$ $[\boldsymbol{M_r}]$ – the rest mass of the object.

For an interaction such as the electric interaction, we have two kinds of charges: positive and negative. In the gravitational interaction, however, there is only a positive charge $\boldsymbol{q}$, namely rest mass. (*Note! Somewhat unfortunately, the acceleration vector of object $\boldsymbol{A}$ is denoted by the symbol $\vec{\boldsymbol{A}}$. These two concepts should not be confused, that is, the object should not be confused with its acceleration*).

The acceleration vector $\vec{a}$ of object B is the parameter that we must determine, because on the basis of the acceleration vector and the velocity vector the worldline of object B is drawn. Let us denote the vector $\vec{s}$ as the difference between the position vectors of the objects, where the dimension u is not included:

$$\vec{s} = \vec{r} - \vec{R} \tag{3}$$

Since the point object B is located in the cone of interaction of object A, the two are connected by a ray of interaction in the direction $\hat{s}$, and the following relation holds:

$$|\vec{s}| = u - U \tag{4}$$

The ray of interaction contains all necessary information about object A needed to determine the force vector. Let us list below all formulas that will be used to derive the unified formula for the force vector. From the article "General Vectorial Lorentz Transformation Formulas for Spacetimes of Arbitrary Dimensions"[2], the following formulas will be needed:

$$u' = \frac{u - \vec{r}\cdot\vec{w}}{\sqrt{1-|\vec{w}|^2}} \tag{5}$$

$$\vec{v}' = \frac{\sqrt{1-|\vec{w}|^2}}{1-\vec{v}\cdot\vec{w}}\vec{v} + \frac{\vec{v}\cdot\vec{w} - 1 - \sqrt{1-|\vec{w}|^2}}{(1-\vec{v}\cdot\vec{w})\left(1+\sqrt{1-|\vec{w}|^2}\right)}\vec{w} \tag{6}$$

$$\vec{a}' = \frac{1-|\vec{w}|^2}{(1-\vec{v}\cdot\vec{w})^2}\left[\vec{a} + \frac{\vec{a}\cdot\vec{w}}{1-\vec{v}\cdot\vec{w}}\vec{v} - \frac{\vec{a}\cdot\vec{w}}{(1-\vec{v}\cdot\vec{w})\left(1+\sqrt{1-|\vec{w}|^2}\right)}\vec{w}\right] \tag{7}$$

$$\vec{s}' = \vec{s} - \frac{|\vec{s}|\left(1-\hat{s}\cdot\vec{w}+\sqrt{1-|\vec{w}|^2}\right)}{\sqrt{1-|\vec{w}|^2}\left(1+\sqrt{1-|\vec{w}|^2}\right)}\vec{w} \tag{8}$$

$$\hat{s}' = \frac{\sqrt{1-|\vec{w}|^2}}{1-\hat{s}\cdot\vec{w}}\hat{s} - \frac{1-\hat{s}\cdot\vec{w}+\sqrt{1-|\vec{w}|^2}}{(1-\hat{s}\cdot\vec{w})\left(1+\sqrt{1-|\vec{w}|^2}\right)}\vec{w} \tag{9}$$

From the article "Relativistic Form of Newton's Second Law of Motion"[3], we shall use the formulas:

$$\vec{F} = E_t\left[\vec{a} + \frac{(\vec{a}\cdot\vec{v})}{1-|\vec{v}|^2}\vec{v}\right] + \frac{dm_r}{du}\frac{1}{1-|\vec{v}|^2+\sqrt{1-|\vec{v}|^2}}\vec{v} \tag{10}$$

$$\vec{a} = \frac{1}{E_t}\left[\vec{F} - \left(\frac{dm_r}{du}\frac{\sqrt{1-|\vec{v}|^2}}{1+\sqrt{1-|\vec{v}|^2}} + \vec{F}\cdot\vec{v}\right)\vec{v}\right] \tag{11}$$

In order to derive the unified formulas for acceleration and force, we must first pass to the reference frame in which object $\boldsymbol{A}$ is at rest, that is, the tangent to the worldline of this object is parallel to the $\boldsymbol{U}$ axis, because it is in this reference frame that formula (1) applies. The Lorentz transformation must be carried out with the vector $\vec{w} = \vec{V}$. The vectors obtained as a result of this transformation will be denoted by the subscript 0 (zero). The distance $\boldsymbol{r}$ appearing in formula (1) is simply the length of the vector $|\vec{s}_0|$. We shall denote this length, for short, by the symbol s_0. From formula (8), we have the following relation:

$$s_0 = |\vec{s}_0| = |\vec{s}|\frac{1 - \hat{s}\cdot\vec{V}}{\sqrt{1-|\vec{V}|^2}} \tag{12}$$

The force vector, which we shall denote by $\vec{F}_0$, in the reference frame of object $\boldsymbol{A}$ is, according to formula (1):

$$\vec{F}_0 = kQqf(s_0)\hat{s}_0 \tag{13}$$

Now, on the basis of formula (11), the second law of motion, let us determine the vector $\vec{a}_0$ for object $\boldsymbol{B}$. In order to apply (11), we also need a formula for the derivative of the rest mass, or potential energy, of object $\boldsymbol{B}$ in the potential field of object $\boldsymbol{A}$:

$$\frac{dm_r}{du} = -kQqf\left(s_0\right)\left(\frac{1-\left(\vec{v}\cdot\vec{V}\right)}{\sqrt{1-\left|\vec{V}\right|^2}} - L\frac{1-\hat{s}\cdot\vec{v}}{\sqrt{1-\left|\vec{V}\right|^2}\left(1-\hat{s}\cdot\vec{V}\right)}\right) \tag{14}$$

where:

$$L = 1 - \left|\vec{V}\right|^2 + \vec{s}\cdot\vec{A} - \frac{|\vec{s}|\left(\vec{V}\cdot\vec{A}\right)\left(1-\hat{s}\cdot\vec{V}\right)}{1-\left|\vec{V}\right|^2} \tag{15}$$

Note! The derivation of formula (14) is given in the Appendix at the end of the article.

After taking into account the fact that, in the reference frame of object $\boldsymbol{A}$, its velocity is $\vec{V}_0 = \mathbf{0}$, formula (14) simplifies to the form:

$$\frac{dm_r}{du_0} = -kQqf\left(s_0\right)\left[\hat{s}_0\cdot\vec{v}_0 + \vec{s}_0\cdot\vec{A}_0\left(\hat{s}_0\cdot\vec{v}_0 - 1\right)\right] \tag{16}$$

Substituting the expressions from (13) and (16) into formula (11), we obtain the vector $\boldsymbol{\vec{a}_0}$.

To return to the initial reference frame and determine the vector $\boldsymbol{\vec{a}}$, we must use in formula (7) the inverse vector of the previous Lorentz transformation, that is, $\boldsymbol{\vec{w} = -\vec{V}}$.

As a result of these transformations, we obtain a unified formula, applicable to all interactions, for the acceleration vector of point object $\boldsymbol{B}$ in the potential field of object $\boldsymbol{A}$:

$$\boldsymbol{\vec{a} = kQqf\left(s_0\right)\frac{1-\vec{v}\cdot\vec{V}}{E_t\left(1-\hat{s}\cdot\vec{V}\right)}\left[\hat{s}-\vec{v}+L_V\left(\vec{V}-\vec{v}\right)\right]} \tag{17}$$

where:

$$\boldsymbol{L_V = L_1\sqrt{1-|\vec{v}|^2}-\frac{1-\hat{s}\cdot\vec{v}}{1-\vec{v}\cdot\vec{V}}} \tag{18}$$

$$\boldsymbol{L_1} = \frac{\left(1-\hat{s}\cdot\vec{v}\right)\left[\vec{s}\cdot\vec{A}\left(1-\left|\vec{V}\right|^2\right)+\left(1-\left|\vec{V}\right|^2\right)^2-|\vec{s}|\left(\vec{V}\cdot\vec{A}\right)\left(1-\hat{s}\cdot\vec{V}\right)\right]-\left(1-\hat{s}\cdot\vec{V}\right)\left(1-\vec{v}\cdot\vec{V}\right)\left(1-\left|\vec{V}\right|^2\right)}{\left(\sqrt{1-\left|\vec{V}\right|^2}\right)^3\left(1-\vec{v}\cdot\vec{V}+\sqrt{1-\left|\vec{V}\right|^2}\sqrt{1-|\vec{v}|^2}\right)\left(1-\vec{v}\cdot\vec{V}\right)} \tag{19}$$

To obtain the unified formula for the force vector, we must use formula (10), the second law of motion, and substitute into it the expressions from (17) and (14):

$$\boldsymbol{\vec{F} = kQqf\left(s_0\right)\frac{1-\vec{v}\cdot\vec{V}}{\left(1-\hat{s}\cdot\vec{V}\right)}\left(\hat{s}+L_v\vec{v}+L_V\vec{V}\right)} \tag{20}$$

where:

$$\boldsymbol{L_v = L_1\frac{\sqrt{1-\left|\vec{V}\right|^2}-1+\vec{v}\cdot\vec{V}}{1+\sqrt{1-|\vec{v}|^2}}} \tag{21}$$

The coefficients $\boldsymbol{L_V}$ and $\boldsymbol{L_1}$ are the same as previously defined by formulas (18) and (19), respectively.

Note! For the gravitational interaction, the symbols $\boldsymbol{Q}$ and $\boldsymbol{q}$ denote, respectively, the rest masses $\boldsymbol{M_r}$ and $\boldsymbol{m_r}$, that is, gravitational charges. For the gravitational interaction, the coefficient $\boldsymbol{k_G}$ has a negative value because like gravitational charges, rest masses, attract one another. The negative value of $\boldsymbol{k_G}$ follows from formula (13). When the masses in (17) and (20) *are given in joules $[J]$, then $k_G = -\frac{1}{c^4}G = -8,26245\times10^{-45}\left[\frac{1}{Nm^2}\right]$. For the electric interaction, on the other hand, k_E has a positive value because like electric charges repel one another. The coefficient k_E has the dimension $\left[\frac{N}{C^2}\right]$. It must be remembered that the functions $f(s_0)$ are dimensionless, with their argument given in metres.*

When object B is located in potential fields of different interactions produced by many different objects lying in its observation cone, the force and acceleration vectors calculated according to formulas (17) and (20) add in the usual, linear way. These formulas, when applied, for example, to the electric interaction, also include all forces associated with the so-called magnetic field. It should be added that formulas (17) and (20) are valid in U-spaces of arbitrary dimension.

Appendix

Derivation of the Formula for the Rest-Mass Derivative

Formula (14) for the derivative of the rest mass of an object containing charge q of an arbitrary interaction in the potential field of charge Q of the same interaction is as follows:

$$\frac{dm_r}{du} = -kQqf\left(s_0\right)\left[\frac{1-\vec{v}\cdot\vec{V}}{\sqrt{1-|\vec{V}|^2}} - L\frac{1-\hat{s}\cdot\vec{v}}{\sqrt{1-|\vec{V}|^2}\left(1-\hat{s}\cdot\vec{V}\right)}\right] \tag{A.1}$$

where:

$$L = 1-\left|\vec{V}\right|^2 + \vec{s}\cdot\vec{A} - \frac{|\vec{s}|\left(\vec{V}\cdot\vec{A}\right)\left(1-\hat{s}\cdot\vec{V}\right)}{1-\left|\vec{V}\right|^2} \tag{A.2}$$

The vectors $\vec{V}$ and $\vec{A}$ are the velocity and acceleration of the object possessing charge Q of some interaction, which acts on the second object, of rest mass m_r, possessing charge q of the same interaction. Charge q is located in the cone of interaction of charge Q.

$\vec{v}$ is the velocity of charge q.

$\vec{s}$ is the vector $\vec{r}-\vec{R}$, where $\vec{r}$ is the position of charge q, and $\vec{R}$ is the position of charge Q.

$\boldsymbol{k}$ is the constant of the given interaction.

$\boldsymbol{f(s_0)}$ is the function of the given interaction.

$\boldsymbol{s_0} = |\vec{\boldsymbol{s}}_0|$, where $\vec{\boldsymbol{s}}_0$ is the vector $\vec{\boldsymbol{s}}$ after transformation to the reference frame of charge $\boldsymbol{Q}$. All other vectors and quantities in the reference frame of charge $\boldsymbol{Q}$ will also be denoted by the subscript zero (0). To derive formula (A.1), we shall start from the equation:

$$\frac{dm_r}{du} = \frac{dm_r}{du_0}\frac{du_0}{du} \tag{A.3}$$

From formula (5) and from the assumption that we pass to the reference frame of charge $\boldsymbol{Q}$, where the Lorentz transformation vector is the vector $\vec{\boldsymbol{w}} = \vec{\boldsymbol{V}}$:

$$\frac{du_0}{du} = \frac{d\frac{u-\vec{r}\cdot\vec{V}}{\sqrt{1-|\vec{V}|^2}}}{du} = \frac{1-\frac{d\vec{r}}{du}\cdot\vec{V}}{\sqrt{1-|\vec{V}|^2}}$$

$$\frac{du_0}{du} = \frac{1-\vec{v}\cdot\vec{V}}{\sqrt{1-|\vec{V}|^2}} \tag{A.4}$$

On the other hand:

$$\frac{dm_r}{du_0} = -F_0\frac{ds_0}{du_0} = -kQqf(s_0)\frac{ds_0}{du_0} \tag{A.5}$$

Therefore, we must determine the derivative $\frac{ds_0}{du_0}$. First, we shall calculate this derivative resulting from the expansion or contraction of U-space in the direction $\hat{\boldsymbol{s}}_0$ due to the acceleration of charge $\boldsymbol{Q}$. We assume that the velocity of charge $\boldsymbol{Q}$ has increased from zero to $d\vec{\boldsymbol{V}}_0$, while at the same time $\boldsymbol{s_0}$ has increased by $\boldsymbol{ds_0}$. Thus, from formula (12), we have:

$$s_0 + ds_0 = |\vec{s}_0|\frac{1-\hat{s}_0\cdot d\vec{V}_0}{\sqrt{1-|d\vec{V}_0|^2}} = \frac{s_0-\vec{s}_0\cdot d\vec{V}_0}{\sqrt{1-|d\vec{V}_0|^2}} \tag{A.6}$$

As $d\vec{\boldsymbol{V}}_0$ tends to zero, the expression $\sqrt{1-|d\vec{V}_0|^2} = 1$, and therefore:

$$ds_0 = -\vec{s}_0\cdot d\vec{V}_0$$

$$\frac{ds_0}{du_0} = -\vec{s}_0\cdot\frac{d\vec{V}_0}{du_0} = -\vec{s}_0\cdot\vec{A}_0 \tag{A.7}$$

We therefore see that for a non-inertial object, U-space in the direction of the ray of interaction $\hat{\boldsymbol{s}}_0$ expands or contracts linearly with the velocity given by formula (A.7). The farther a given point of space lies, the faster U-space expands or contracts at that point. This velocity is greatest in the direction parallel to the vector $\vec{A}_0$ and decreases to zero in the direction perpendicular to $\vec{A}_0$. Owing to the linearity of space, the velocities from formula (A.7) are not subject to any limit, because in the special theory of relativity there is also no restriction on acceleration. In a non-inertial reference frame, U-space may expand or contract at velocities many times greater than the speed of light.

Next, we calculate the increment $\frac{ds_0}{du_0}$ resulting from the velocity $\vec{v}_0$ of charge q. In this case, we assume that $\vec{A}_0 = 0$.

$$s_0 = |\vec{s}_0| = |\vec{r}_0 - \vec{R}_0|$$

$$\frac{ds_0}{du_0} = \hat{s}_0 \cdot \frac{d\vec{r}_0}{du_0} - \hat{s}_0 \cdot \frac{d\vec{R}_0}{du_0} = \hat{s}_0 \cdot \vec{v}_0 - \hat{s}_0 \cdot \vec{V}_0$$

Since $\vec{V}_0 = 0$, we have:

$$\frac{ds_0}{du_0} = \hat{s}_0 \cdot \vec{v}_0 \tag{A.8}$$

The increments $\frac{ds_0}{du_0}$ from formulas (A.7) and (A.8) do not add linearly, because although the increment, or the velocity of expansion or contraction of space, from formula (A.7) belongs to a linear space, the increment, or velocity, from formula (A.8) comes from a nonlinear space. From the sum of the increments $\frac{ds_0}{du_0}$ from the above formulas, a nonlinear term in the form of the product of these increments must be subtracted. Thus, we obtain the resultant formula:

$$\frac{ds_0}{du_0} = \hat{s}_0 \cdot \vec{v}_0 - \vec{s}_0 \cdot \vec{A}_0 + (\hat{s}_0 \cdot \vec{v}_0)(\vec{s}_0 \cdot \vec{A}_0)$$

$$\frac{ds_0}{du_0} = \hat{s}_0 \cdot \vec{v}_0 + \vec{s}_0 \cdot \vec{A}_0 (\hat{s}_0 \cdot \vec{v}_0 - 1) \tag{A.9}$$

To check formula (A.9), we shall perform certain calculations. Let us assume that in a three-dimensional U-space, with a Cartesian coordinate system imposed on it, with axes U, X, Y, a physical point revolves around the U axis on a circle of radius $|\vec{R}| = 600$. In essence, this point moves along the U axis on a spiral. To simplify the discussion, we shall not specify in what units the distance is measured. Let us assume that at a certain moment this point occupies the position $\vec{R} = (0\,;\, 600)$, and its velocity is $\vec{V} = (0{,}3\,;\, 0)$. Then the centripetal acceleration of this point is $\vec{A} = -\frac{|\vec{V}|^2}{|\vec{R}|}\,\hat{R}$. Let us further assume that in the cone of interactions of this object there is a second physical point, which orbits the $\boldsymbol{U}$ axis with the same angular velocity, but on a circle with a radius, for example, one and a half times larger, equal to $|\vec{\boldsymbol{r}}| = \mathbf{900}$. Consequently, its velocity is $|\vec{\boldsymbol{v}}| = \mathbf{0,45}$. Let us denote the position of this second object by the vector $\vec{\boldsymbol{r}}$. Let us also determine the vector $\vec{\boldsymbol{s}} = \vec{\boldsymbol{r}} - \vec{\boldsymbol{R}}$.

The vector $\vec{\boldsymbol{s}}$ in the considered reference frame does not change its length while the points move around the $\boldsymbol{U}$ axis on spirals. Therefore, this vector also does not change its length in the reference frame of the point moving with velocity $\vec{\boldsymbol{V}}$. It follows that after performing the Lorentz transformation with the vector $\vec{\boldsymbol{w}} = \vec{\boldsymbol{V}}$, the derivative $\frac{\boldsymbol{ds_0}}{\boldsymbol{du_0}}$ from formula (A.9) should be equal to zero.

To be certain, we examined two cases. In the first variant, we assumed that the angle between the vectors $\vec{\boldsymbol{R}}$ and $\vec{\boldsymbol{r}}$ is $\boldsymbol{\pi/3 = 60°}$, as shown in Table A.1.

Table A.1

No.		value
1	$\vec{V}$	(0.3 , 0)
2	$\vec{A}$	(0 , - 0.00015)
3	$\vec{R}$	(0 , 600)
4	$\vec{v}$	(0.225 , 0.389711431703)
5	$\vec{r}$	(- 779.42286340599 , 450)
6	$\vec{s}$	(- 779.42286340599 , - 150)
7	$\vec{A}_0$	(0 , - 0.00016483516483516)
8	$\vec{v}_0$	(- 0.080428954423592 , 0.39867132647915)
9	$\vec{s}_0$	(- 1066.6722574042 , - 150)
10	$\vec{s}_0 \cdot \vec{A}_0$	0.024725274725275
11	$\hat{s}_0 \cdot \vec{v}_0$	0.024128686327078
12	(10)x(11)	0.000596588398197
13	ds_0/du_0	(11)+(12)-(10) = 0

Source: own elaboration

Table A.2

No.		value
1	$\vec{V}$	(0.3 , 0)
2	$\vec{A}$	(0 , - 0.00015)
3	$\vec{R}$	(0 , 600)
4	$\vec{v}$	(- 0.18726607644621 , 0.40918384207156)
5	$\vec{r}$	(- 818.36768414311 , - 374.53215289243)
6	$\vec{s}$	(- 818.36768414311 , - 974.53215289243)
7	$\vec{A}_0$	(0 , - 0.00016483516483516)
8	$\vec{v}_0$	(- 0.46134764730942 , 0.36957391067572)
9	$\vec{s}_0$	(- 1258.0877008676 , - 974.53215289243)
10	$\vec{s}_0 \cdot \vec{A}_0$	0.16063716805919
11	$\hat{s}_0 \cdot \vec{v}_0$	0.13840429419282
12	(10)x(11)	0.0222328738663
13	ds_0/du_0	(11)+(12)-(10) = 0

Source: own elaboration

In the second variant, the angle between the vectors is $\mathbf{2 \approx 114,56^\circ}$, as shown in Table A.2. In both of these tables, in item 13, the derivative $\frac{ds_0}{du_0}$ is equal to zero, and thus we have shown that formula (A.9) is correct.

When we substitute the expressions from (A.4), (A.5), and (A.9) into formula (A.3), we obtain:

$$\frac{dm_r}{du} = -kQqf(s_0)\left[\hat{s}_0 \cdot \vec{v}_0 + \vec{s}_0 \cdot \vec{A}_0(\hat{s}_0 \cdot \vec{v}_0 - 1)\right]\frac{1 - \vec{v} \cdot \vec{V}}{\sqrt{1 - |\vec{V}|^2}} \tag{A.10}$$

Now we must express the scalar products $\hat{s}_0 \cdot \vec{v}_0$ and $\vec{s}_0 \cdot \vec{A}_0$ in terms of the parameters of the objects from the original reference frame. For this purpose, we must use the Lorentz transformation formulas (6)–(9), where the transformation vector is $\vec{w} = \vec{V}$. We therefore have:

$$\vec{v}_0 = \frac{\sqrt{1-|\vec{V}|^2}}{1-\vec{v}\cdot\vec{V}}\vec{v} + \frac{\vec{v}\cdot\vec{V} - 1 - \sqrt{1-|\vec{V}|^2}}{\left(1-\vec{v}\cdot\vec{V}\right)\left(1+\sqrt{1-|\vec{V}|^2}\right)}\vec{V} \tag{A.11}$$

$$\vec{A}_0 = \frac{1-|\vec{V}|^2}{\left(1-|\vec{V}|^2\right)^2}\left[\vec{A} + \frac{\vec{A}\cdot\vec{V}}{1-|\vec{V}|^2}\vec{V} - \frac{\vec{A}\cdot\vec{V}}{\left(1-|\vec{V}|^2\right)\left(1+\sqrt{1-|\vec{V}|^2}\right)}\vec{V}\right]$$

$$\vec{A}_0 = \frac{1}{1-|\vec{V}|^2}\left[\vec{A} + \frac{\vec{A}\cdot\vec{V}}{\sqrt{1-|\vec{V}|^2}\left(1+\sqrt{1-|\vec{V}|^2}\right)}\vec{V}\right] \tag{A.12}$$

$$\vec{s}_0 = \vec{s} - \frac{|\vec{s}|\left(1 - \hat{s}\cdot\vec{V} + \sqrt{1-|\vec{V}|^2}\right)}{\sqrt{1-|\vec{V}|^2}\left(1 + \sqrt{1-|\vec{V}|^2}\right)}\vec{V} \tag{A.13}$$

$$\hat{s}_0 = \frac{\sqrt{1-|\vec{V}|^2}}{1 - \hat{s}\cdot\vec{V}}\hat{s} - \frac{1 - \hat{s}\cdot\vec{V} + \sqrt{1-|\vec{V}|^2}}{\left(1 - \hat{s}\cdot\vec{V}\right)\left(1 + \sqrt{1-|\vec{V}|^2}\right)}\vec{w} \tag{A.14}$$

After multiplying vector (A.14) scalarly by (A.11), and vector (A.13) by (A.12), we obtain:

$$\hat{s}_0\cdot\vec{v}_0 = 1 - \frac{(1 - |\vec{V}|^2)(1 - \hat{s}\cdot\vec{v})}{(1 - \hat{s}\cdot\vec{V})(1 - \vec{v}\cdot\vec{V})} \tag{A.15}$$

$$\vec{s}_0\cdot\vec{A}_0 = \frac{1}{1-|\vec{V}|^2}\left[\vec{s}\cdot\vec{A} - \frac{|\vec{s}|(\vec{V}\cdot\vec{A})(1-\hat{s}\cdot\vec{V})}{1-|\vec{V}|^2}\right] \tag{A.16}$$

In turn, by substituting (A.15) and (A.16) into (A.10), we obtain formula (A.1) for the rest-mass derivative.

To check the correctness of the transformations performed, random vectors $\vec{V}$, $\vec{A}$, $\vec{v}$, and $\vec{s}$ were generated for sample numerical data. Then, using formulas (A.11), (A.12), (A.13), and (A.14), $\frac{dm_r}{du}$ was calculated from formula (A.10), after which the same derivative was calculated from formula (A.1), and the results were compared.

Two cases were considered, shown in Tables A.3 and A.4. For the calculations, $\boldsymbol{kQq} = \mathbf{1000}$ and $\boldsymbol{f(s_0)} = \frac{1}{s_0^2}$ were assumed. In items 9 and 10 of the above tables, the results of the calculations performed in these two ways are given. As can be seen, they do not differ from one another, and therefore the transformation of formula (A.10) into formula (A.1) was carried out correctly.

Table A.3

No.	vector	formula	X	Y	Z
1	$\vec{V}$		0.0793	0.7093	- 0.1973
2	$\vec{A}$		- 0.6782	- 2.639	1.827
3	$\vec{v}$		- 0.0319	- 0.0722	0.3268
4	$\vec{s}$		3267	6265	2187
5	$\vec{A}_0$	(A.12)	- 1.8586783779979	- 9.0373318896385	4.9335539474616
6	$\vec{v}_0$	(A.11)	- 0.095102765720708	- 0.7225530689849	0.38532880787307
7	$\vec{s}_0$	(A.13)	2695.6911083954	1154.9193339828	3608.4280493518
8	$\hat{s}_0$	(A.14)	0.57973403928632	0.24837639908166	0.77602680886165
9	dm_r/du	(A.10)	0.16951393427447		
10	dm_r/du	(A.1)	0.16951393427447		

Source: own elaboration

Table A.4

No.	vector	formula	X	Y	Z
1	$\vec{V}$		0.0493	- 0.1826	0.7132
2	$\vec{A}$		0.6505	0.7068	- 0.0054
3	$\vec{v}$		0.1521	0.2173	- 0.7116
4	$\vec{s}$		- 4030	- 2763	7526
5	$\vec{A}_0$	(A.12)	1.418220894468	1.5872069919904	- 0.15149565834832
6	$\vec{v}_0$	(A.11)	0.024340236069639	0.25206643608844	- 0.9244075460274
7	$\vec{s}_0$	(A.13)	- 4437.9996759148	- 1251.8287865711	1623.6598608023
8	$\hat{s}_0$	(A.14)	- 0.907811532183	- 0.2560668525812	0.33212646543855
9	dm_r/du	(A.10)	- 1.1342517413542		
10	dm_r/du	(A.1)	- 1.1342517413542		

Source: own elaboration

Note! Tables A.1 to A.4 were filled in automatically by means of a computer program written in PHP.

References

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