The Theory of Observational Relativity
Serial Report 6: Towards the Unity of Relativity and Quantum Theories

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Abstract: The theory of Observational Relativity (OR) is a new theory in physics. For ease of understanding, the author reports the significant discoveries and new insights of OR to physicists and readers in the form of serial reports. Now, OR Serial Report 6 reports that, similar to relativistic effects, the quantum effects in the quantum mechanics based on Planck's quantum hypothesis, including wave-like nature and uncertainty, are also sorts of observational effects. But the difference is that relativistic phenomena are caused by the observational locality of observation agents, while quantum phenomena are caused by the observational perturbation of observation agents. The OR axiom system, taking the definition of OR time as the most basic premise or first principle, has not only deduced the theory of OR, including the inertial OR (IOR) and the gravitational OR (GOR), but also deduced the theory of observational quantum (OQ), including the general Planck equation $E=h_\eta f$, general de Broglie relation $p=h_\eta/\lambda$, and ORQ identity $h_\eta\eta\equiv C$ ($C=hc$). Thus, the whole theoretical system based on the OR axiom system is a theory of both relativity and quantum, and may be called the theory of observational relativity-quantum (ORQ). The theory of ORQ is that of the general observation agent OA($\eta|h_\eta$) ($0<\eta<\infty$; $h_\eta>0$) where η and h_η are respectively the speed of the information wave of OA($\eta|h_\eta$) and the energy-frequency ratio of the informons of OA($\eta|h_\eta$). The informons of OA($\eta|h_\eta$) with momentum and energy ($h_\eta>0$) must perturb the observed object P, causing P to exhibit quantum effects. The theory of OQ has discovered that the quantum theory based on Planck's quantum hypothesis $\varepsilon=hf$ is a theory of optical observation adopting the optical agent OA($c|h$), i.e., a special case of the theory of OQ, in which the quantum effects, including wave-like nature and uncertainty, are observational effects caused by the perturbation of the informons (photons) of OA($c|h$), not the intrinsic nature of the physical world or P; the Planck constant h is not a cosmological constant, but rather the energy-frequency ratio of photons, representing the minimum scale or resolution for OA($c|h$) to determine the mass, momentum and energy of P. Now, the whole theoretical system of ORQ has not only generalized and unified Newtonian mechanics and Einstein's theory of relativity, but also generalized and unified Einstein's formula $E=mc^2$ and Planck's equation $E=hf$, marching towards the unification of relativity theory and quantum theory.

Key Words: Einstein formula, Planck equation, Planck constant, quantum effects, observational perturbation.

1 Introduction

The theory of observational relativity (OR) [1-4] is built on the basis of the OR axiom system. The theory of OR has clarified that the only indispensable premise of the OR axiom system is the definition of OR time. Time is the most basic concept in physics, and therefore, the definition of OR time can be regarded as the most basic logical premise or first principle.

Based on the most basic logical premises, the theory of OR has generalized and unified Newtonian mechanics and Einstein's theory of relativity within the same theoretical system under the same axiom system, acquiring the broad perspective and bringing us significant discoveries and new insights.

The author repeatedly emphasizes [5] that the theory of OR is not designed or manufactured, but an accident, an unexpected scientific discovery, being logically inevitable. However, this accident seems to involve not only the relativistic effects but also quantum effects of matter motion and matter interactions. It turns out that the quantum effects in quantum mechanics are also observational effects.

The OR axiom system, starting from the definition of OR time, has not only deduced the theory of OR, including the inertial OR (IOR) and the gravitational OR (GOR), but also deduced the theory of observational quantum (OQ), including the general Planck equation $E=h_\eta f$, de Broglie relation $p=h_\eta/\lambda$, Schrödinger equation, uncertainty principle, and ORQ identity $h_\eta\eta\equiv C$ ($C=hc$).

Thus, the whole theoretical system based on the OR axiom system is a theory of both relativity theory and quantum theory, which has not only generalized and unified Newtonian mechanics and Einstein's theory of relativity, but also generalized and unified Einstein's formula $E=mc^2$ and Planck's equation $E=hf$, marching towards the unification of relativity and quantum theory.

Since OR and OQ are built on the same axiom system and belong to the same theoretical system, it may be referred to as the theory of observational relativity-quantum (ORQ).

As a new theory in physics, for ease of understanding, the author reports the significant discoveries and new insights of ORQ to physicists and readers in the form of serial reports. The OR serial reports 1-5 focus on stating the theory of OR, including the logical deduction and theoretical derivation, generalization and unification of Newtonian mechanics and Einstein's theory of relativity [5], and significant discoveries and new insights of the theory of OR [6-9]. OR serial report 6 and its subsequent reports will focus on stating the theory of OQ, including the logical deduction and theoretical derivation, and significant discoveries and new insights of the theory of OQ.

The theory of ORQ is that of the general observation agent OA($\eta|h_\eta$) ($0<\eta<\infty$; $h_\eta>0$) where η and h_η are respectively the speed of the information wave of OA($\eta|h_\eta$) and the energy-frequency ratio of the informons of OA($\eta|h_\eta$). The informons of OA($\eta|h_\eta$) with momentum or energy ($h_\eta>0$) must perturb the observed object P, causing P to exhibit quantum effects.

Now, OR serial report 6 reports to readers the discovery of the theory of OQ: the quantum theory based on Planck's quantum hypothesis $\varepsilon=hf$ is a theory of the optical observation adopting the optical agent OA($c|h$), that is, a special case of the theory of OQ, in which the quantum effects, including wave-like nature and uncertainty, are sorts of observational effects caused by the observational perturbation of OA($c|h$), not the intrinsic nature of the physical world or P; the Planck constant h is not a cosmological constant, but rather the energy-frequency ratio of photons, representing the minimum scale or resolution for OA($c|h$) to determine the mass, momentum and energy of P. According to the theory of ORQ, Newtonian mechanics is the theory of idealized observation agent OA($\infty|0$), in which the observed object P does not exhibit relativistic effects nor quantum effects.

Specifically, OR serial report 6 will display to physicists and readers the unification of the Einstein equation $E=mc^2$ the represents relativity theory and the Planck equation $E=hf$ that represents quantum theory under the OR axiom system.

Actually, the theory of OQ and the theory of OR were born at the same time. In 2017, the general Planck equation $E=h_\eta f$ and the general de Broglie relation $p=h_\eta/\lambda$ appeared simultaneously in the literature reporting the of OR theory [10]. In 2022, in order to interpret the unification of the Einstein formula $E=mc^2$ and the Planck equation $E=hf$ under the OR axiom system, the author ever elaborated the related content and referred to it as **Dualistic Relativity** [11]. But the dualistic relativity only involved the optical agent OA($c|h$) as a special case of ORQ.

Now, from the broad perspective of the general observation agent OA($\eta|h_\eta$) and based on the general Einstein formula $E=m\eta^2$ and Planck equation $E=h_\eta f$ of ORQ, OR serial report 6 delve deeper into the wave-particle duality of matter motion to reveal the essence of the mass-energy relationship in Einstein's formula $E=mc^2$ and the essence of the Planck constant h in Planck's equation $E=hf$.

2 Physics and Observation Agents

The theory of ORQ has discovered [1-4] that all theoretical systems or spacetime models of mankind's physics, including the Galilean transformation and the Lorentz transformation, including Newtonian mechanics and Einstein's theory of relativity, are linked to specific observation media or specific observation systems, and must be branded by observation without exception.

This is the origin of the names of the theory of **Observational Relativity** (OR) and the theory of **Observational Quantum** (OQ).

However, throughout history, mankind's physics has never clearly specified the indispensable role or status of observation in its theoretical systems or spacetime models.

2.1 The Concept of Observation Agents

In order to clarify the role and status of physical observation in mankind's physics, the theory of ORQ has proposed three important concepts related to physical observation [5]:

(1) Observation Agent;
(2) Information Wave;
(3) Informon

Železnikar [12] ever employed **Informon** to refer to an **Information Entity**, and analogized it with an electron.

Since the theory of OR only involves relativistic effects, the general observation agent is often simply denoted as OA(η) ($0<\eta<\infty$). Einstein's optical agent OA(c) ($\eta=c$) and Newton's idealized agent OA_∞ ($\eta\to\infty$) are just two special cases of OA(η).

In fact, the theoretical system of ORQ not only contains the theory of OR involving relativistic effects, but also contains the theory of OQ involving quantum effects.

Therefore, this article officially denotes the general observation agent of ORQ as

$$\mathrm{OA}(\eta\,|\,h_\eta) \quad (0<\eta<\infty;\ h_\eta>0) \tag{1}$$

where η and h_η are respectively the speed of the information wave of OA($\eta|h_\eta$) and the energy-frequency ratio of the informons of OA($\eta|h_\eta$), being regarded as the eigenvalues of OA($\eta|h_\eta$).

Observation Agent (OA): In mankind's physics, any theoretical system or spacetime model must be linked to a certain observation agent OA($\eta|h_\eta$) or observation system ($O, P, M(\eta|h_\eta)$) where the information of the observed object P must be transmitted to the observer O by a certain observation medium $M(\eta|h_\eta)$ at a certain speed η, so that the observer O can perceive the observed object P.

In theory, all forms of matter motion or matter interactions, not only light or electromagnetic interaction, can serve as mankind's observation agents: any matter wave can be the information wave of OA($\eta|h_\eta$); any matter particle can be the informons of OA($\eta|h_\eta$).

However, one OA($\eta|h_\eta$) of mankind's observation agents must have a finite information-wave speed η and a non-zero energy-frequency ratio: $\eta<\infty$ and $h_\eta>0$, where $\eta<\infty$ suggests that OA($\eta|h_\eta$) has the observational locality and must exhibit relativistic effects and $h_\eta>0$ suggests that OA($\eta|h_\eta$) has the observational perturbation and must exhibit quantum effects. Therefore, mankind's perception and cognition of the physical world not only get benefit from his observation agents, including his sensory organs and the observation instruments he invents, but also are restricted by the observational locality and perturbation of his observation agents.

It must be pointed out that the general observation agent OA($\eta|h_\eta$) ($0<\eta<\infty$; $h_\eta>0$) (Eq. (1)) is not a logical premise of OR and OQ, but a discovery of ORQ, that is, a logical consequence of the OR axiom system.

2.2 Human Cognition and Observation Agents

The natural world has prepared or equipped various observation agents for mankind.

As human sensory organs, the ear is human acoustic agent and the eye is human optical agent. With the sonar he invents, mankind can detect the information from the deep sea; with the radar he invents, mankind can detect material objects that his naked eye cannot see; with the radio telescope he invents, mankind can explore the distant celestial bodies and vast starry sky of the universe.

Restricted by the observational locality and perturbation of his observation agents, mankind can never reach the other shore of objective truth. The objective truth could only be touched by mankind's reason.

The limitations of mankind's observation must be reflected in mankind's physics: the observational locality has led to the birth of Einstein's relativity theory; the observational perturbation has led to the birth of Planck's quantum theory. However, physicists have not truly realized that different observation agents exhibit different observational phenomena, and therefore, observational phenomena do not represent the essence of the natural world.

Even today, the mainstream physics community has not yet realized that the relativistic effects in Einstein's theory of relativity, including the invariance of light speed and spacetime curvature, are the observational effects and apparent phenomena caused by the observational locality ($c<\infty$) of the optical agent OA($c|h$), and that the quantum effects in Planck's quantum theory, including wave-like nature and uncertainty, are observational effects caused by the observational perturbation ($h>0$) of the optical agent OA($c|h$), but not the inherent nature of the natural world.

So, modern physics has formed many erroneous understandings and even absurd theories [5-9], giving rise to many myths and even superstitions that contradict human reason and common sense [13].

2.2.1 Einstein's Observation Agent

The theory of OR has discovered [1-5] that Einstein's theory of relativity, including the special and the general, is that of optical observation adopting the optical agent.

According to the notation of Eq. (1), Einstein's optical agent should be denoted as OA($c|h$) ($\eta=c$; $h_\eta=h$), which has both the observational locality ($c<\infty$) and the observational perturbation ($h>0$), and involves both relativistic effects and quantum effects. However, relative to a macroscopic object P, the momentum of a photon is too small to cause significant perturbation to P. This is the reason why quantum effects are often regarded as exclusive to the microscopic world, and also the reason why Einstein's theory of relativity does not involve quantum effects.

Of course, you can also argue that Einstein's observation agent is not the realistic optical agent OA($c|h$), but the OA($c|0$): half realistic, the speed η of the information wave being the speed of light c with the observational locality; half idealized, the energy-frequency ratio h_η of the informons being idealized as zero with no observational perturbation. So, Einstein's observation agent OA($c|0$) only involves relativistic effects but no quantum effects.

The theory of OR has already clarified [1-5] that all the relativistic effects in Einstein's theory of relativity, including the special and the general, including the invariance of light speed and spacetime curvature, are the observational effects and purely apparent phenomena caused by the observational locality ($c<\infty$) of his observation agent OA($c|0$), not the objective physical reality and real physical existence: the speed of light is not really invariant; spacetime is not really curved.

So, Einstein's theory of relativity, no matter the special or the general, only presents us with the optical image of the physical world, and does not represent the completely objective and real physical world.

2.2.2 Planck's Observation Agent

In 1900 [14], in order to theoretically derive the formula of blackbody radiation, Planck proposed the energy-quantum hypothesis $\varepsilon=hf$. In the experiment of blackbody radiation, Planck was the observer O, the object P his observed was light or the photon, and what transmitted the information on P to him was also light or the photon.

So, quantum mechanics based on Planck's quantum hypothesis $\varepsilon=hf$ is also a theory of optical observation adopting the optical agent OA($c|h$) ($\eta=c$; $h_\eta=h$), and must involve both relativistic effects and quantum effects.

However, early old quantum theory and even the quantum mechanics represented by the Schrödinger equation still followed the energy relations of classical mechanics and did not involve relativistic effects, and the observation agent seemed to be OA($0|h$): half idealized, the speed of the information wave being idealized as infinity with no observational locality; half realistic, the informons being photons and the energy-frequency ratio h_η being the Planck constant h with the observational perturbation ($h>0$). So, Planck's observation agent OA($0|h$) only involves quantum effects but no relativistic effects.

The theory of OQ has discovered [1-4.10] that the quantum effects in the quantum mechanics based on Planck's quantum hypothesis $\varepsilon=hf$, including wave-like nature and uncertainty, are also sorts of observational effects, caused by the perturbation of the informons (photons) of the optical agent OA($c|h$).

However, it should be pointed out that, unlike the relativistic effects in relativity theory, the quantum theory presented in the quantum mechanics based on Planck's quantum hypothesis $\varepsilon=hf$ are not apparent phenomena, but the objective and real change in the motion state of the observed object P due to the perturbation of the informons (photons) of the optical agent OA($c|h$).

In this sense, the quantum effects in the quantum mechanics based on Planck's quantum hypothesis $\varepsilon=hf$, including wave-like nature and uncertainty, represent the objective physical reality and real physical existence in the (microscopic) physical world.

2.2.3 Newton's Observation Agent

The theory of ORQ has discovered [1-5] that Newtonian mechanics [15], including Newton's inertial mechanics and Newton's theory of universal gravitation, is a theory of idealized observation.

According to the notation of Eq. (1), Newton's idealized observation agent should be denoted as OA($\infty|0$) that implies two idealized hypotheses: 1) the speed of the information wave η is idealized as infinity ($\eta\to\infty$), and therefore, OA($\infty|0$) has no observational locality; 2) the energy-frequency ratio h_η of the informons is idealized as zero ($h_\eta \to 0$), and therefore, OA($\infty|0$) has no observational perturbation. So, Newtonian mechanics has neither relativistic effects nor quantum effects.

Of cause, there is no idealized observation agent in the objective physical world.

However, the idealized observation agent could exist in human reason. Although both Galileo and Newton did not realize that their observation agent was the idealized agent OA($\infty|0$), Galilean doctrine and Newtonian mechanics have shown us the power of mankind's reason.

The theory of OR has clarified [1-5] that, in terms of macroscopic matter motion, Newtonian mechanics as a theory of idealized observation has no observational locality and apparent relativistic effects, and therefore, represents the objective and real (macroscopic) physical world.

And, according to the theory of OQ, in terms of microscopic matter motion, Newtonian mechanics as a theory of idealized observation has no observational perturbation, and therefore, it cannot reflect the perturbation to the observed object P by realistic observation agents and cannot presents the objective and realistic quantum effects, no matter wave-like nature or uncertainty.

It needs to be emphasized: the observational perturbation caused by realistic observation agents is the objective physical reality and real physical existence; physicists cannot escape such quantum effects in the micro world.

In this sense, in Hawking's words [16], Newtonian mechanics is also a partial theory of mankind's physics.

2.2.4 The ORQ Observation Agent

The theory of OR tells us that, due to the observational locality ($\eta<\infty$) of mankind's observation agent OA($\eta|h_\eta$), the physical world observed by mankind, both the macro or the micro, must exhibit apparent relativistic phenomena. So, mankind's physics gives birth to Einstein's theory of relativity.

The theory of OQ tells us that, due to the observational perturbation ($h_\eta>0$) of mankind's observation agent OA($\eta|h_\eta$), the physical world observed by mankind, both the macro or the micro, must exhibit realistic quantum phenomena. So, mankind's physics gives birth to Planck's theory of quantum.

However, it is not that mankind can only observe the physical world from the perspective of Einstein or Planck. The theory of ORQ, including OR and OQ, has been established on the OR axiom system as the most basic logical premises, and therefore, has a perspective and belongs to a theory of the general observation agent OA($\eta|h_\eta$) ($0<\eta<\infty$; $h_\eta>0$) that might be called **the Eye of God**.

It is exactly due to the broad perspective of OA($\eta|h_\eta$) ($0<\eta<\infty$; $h_\eta>0$) that the theory of ORQ can not only perceive the natural world with mankind's optical agent OA($c|h$) from the perspective of Planck or Einstein, but also cognize the natural world with mankind's reason from the perspective of Galileo and Newton.

So, the theory of ORQ has generalized and unified Newtonian mechanics, Einstein's theory of relativity, and even Planck's quantum physics within the same theoretical system under the same axiom system.

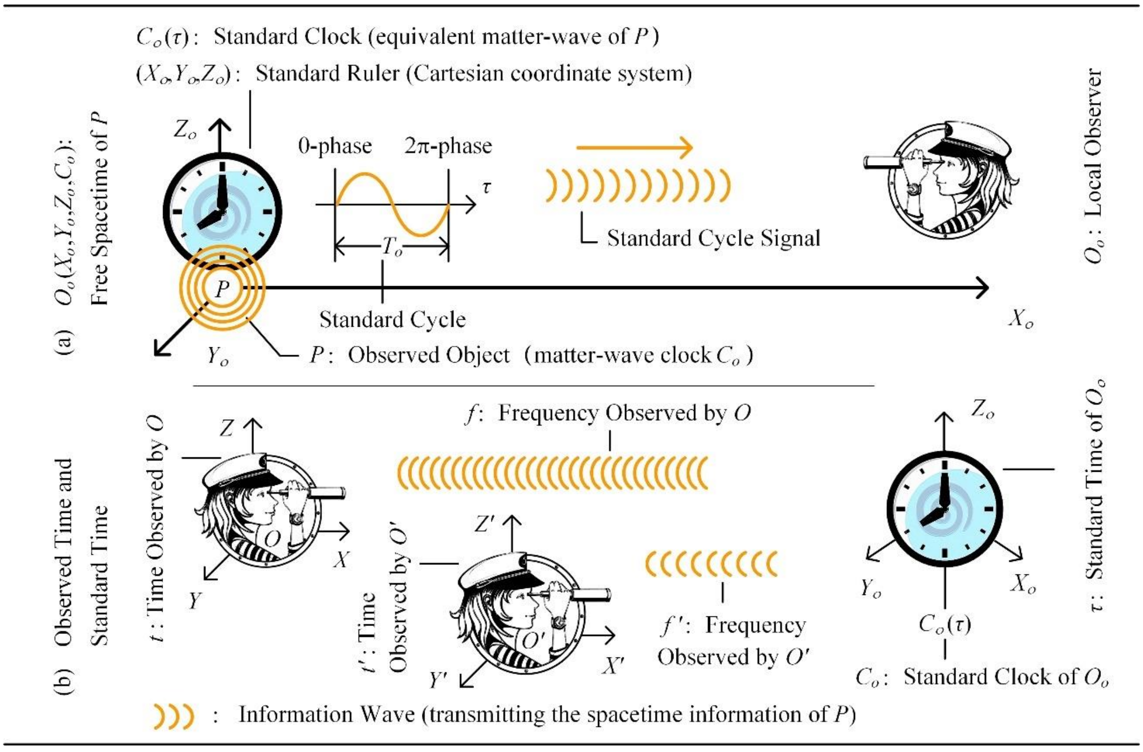

Figure 1 The Standard Clock C_o and the Observed Object P [11]: (a) The standard clock C_o can be defined with the periodic signal originating from the material wave of the observed object P in the inherent observation system $O_o(X_o,Y_o,Z_o)$ of free spacetime; (b) The relativistic nature of time originates from the relativity between the standard time τ of O_o and the observed time t of O.

3 Starting from the Definition of OR Time

The theory of OR [1-4] and OR serial report 1 [5] have clarified that the only indispensable logical premise in the

OR axiom system is the definition of OR time, which is a definition of relativistic time that conforms to the relativistic phenomena observed by mankind.

The OR axiom system, starting from the definition of OR time as the most basic logical premise or first principle, has deduced the invariance of time-frequency ratio, and then has proved the theorem of the invariance of information-wave speeds, deriving the inertial-spacetime transformation in differential form and the general Lorentz transformation in algebraic form. Thereby, following and analogizing Einstein's logic, the whole theoretical system of OR, including IOR and GOR, has been established.

The invariance of time-frequency ratio plays an important role in the theory of ORQ. It was the invariance of time-frequency ratio $dt/f = d\tau/f_o$ that reminded the author of de Broglie's frequency-speed relationship of matter waves, $f = f_o/\sqrt{(1-v^2/c^2)}$, and then, anticipated that Einstein's theory of relativity and de Broglie's theory of matter waves might be unified within the theoretical system of ORQ under the OR axiom system [5,11].

3.1 The Invariance of Time-Frequency Ratio of the Clock

The definition of OR time is the most fundamental logical element of OR axiom system, and the invariance of time-frequency ratio of the clock is the most direct logical inference of the definition of OR time, that is, the theorem of the invariance of time-frequency ratio of ORQ.

The Invariance of Time-Frequency Ratio: Let O be the observer adopting the observation agent $\mathrm{OA}(\eta|h_\eta)$, P the observed object, a periodic signal source with the inherent frequency f_o; according to the definitions of time and standard clock [1,4], define P as a standard clock, then the ratio dt/f of the time-element dt to frequency f observed by O is an invariant, identical to the ratio $d\tau/f_o$ of the inherent time-element $d\tau$ to inherent frequency f_o of P:

$$\frac{dt}{f} = \frac{d\tau}{f_o} \quad \text{or} \quad \Gamma = \frac{dt}{d\tau} = \frac{f}{f_o} \tag{2}$$

As depicted in Fig. 1, Eq. (2) suggests that the rate at which the time of the identical clock flows varies relative to different observation agents or different observers. Of course, this is not because the objective and real time $d\tau$ dilates or contracts, but only because different observers are in different physical environments, or equipped with different observation agents, or moving at different speeds relative to the clock.

The invariance of time-frequency ratio plays an important role in the theory of ORQ including OR and OQ: not only deducing the OR mass-speed relationship $m = m_o/\sqrt{(1-v^2/\eta^2)}$ and the general Einstein formula $E = m\eta^2$ of the general observation agent $\mathrm{OA}(\eta|h_\eta)$, generalizing the Einstein formula $E = mc^2$ of the optical agent $\mathrm{OA}(c|h)$; but also deducing the OQ frequency-speed relationship $f = f_o/\sqrt{(1-v^2/\eta^2)}$ and the general Planck equation $E = h_\eta f$ of the general observation agent $\mathrm{OA}(\eta|h_\eta)$, generalizing the Planck equation $E = hf$ of the optical agent $\mathrm{OA}(c|h)$.

For the detailed proof and related statements of the invariance of time-frequency ratio, please refer to Sec. 2.3.2 of Chapter 2 of the 1st volume IOR in the Monograph of Observational Relativity [1,4].

3.2 The Clock C_o and Observed Object P

In the invariance of OR time-frequency ratio (Eq. (2) and Fig. 1), the clock C_o is the observed object P of the general observation agent $\mathrm{OA}(\eta|h_\eta)$.

In theory, any periodic phenomenon, including matter waves, can be employed to measure time. Any material object can be regarded as a standard clock.

De Broglie believed [17-19] that matter has wave-particle duality, both a particle and a wave, the so-called Matter Wave. Thus, any observed object P can be regarded as a natural clock, the so-called **A Rock is A clock** [20], that is, a matter-wave clock: time can be measured with the matter wave of P. Müller believes [21,22] that matter-wave clocks can even be more accurate than atomic clocks.

In fact, according to the definitions of OR time and OR clock [1,4], in theory, regardless of whether the de Broglie wave objectively exists or not, the object P observed by the observation agent $\mathrm{OA}(\eta|h_\eta)$ can be regarded as a standard clock: either P autonomously radiates the information wave of $\mathrm{OA}(\eta|h_\eta)$ to the observer O, or P reflects the information wave of $\mathrm{OA}(\eta|h_\eta)$ to the observer O.

In this way, as shown in Fig. 1, any observation agent $\mathrm{OA}(\eta|h_\eta)$ that carries and transmits information waves naturally or inevitably becomes the standard clock for its related theories or physical models.

3.3 The Spacetime-Transformation Factor: Particle-Like Form and Wave-Like Form

It is because of the invariance of time-frequency ratio derived from the definition of OR time that the transformation factor $\Gamma(\eta,v) = dt/d\tau$ of OR inertial spacetime observed by the general Lorentz transformation has had two different forms [1-4].

The First One: Particle-Like Form

$$\begin{cases} \Gamma(\eta,v) = \dfrac{dt(\eta,v)}{d\tau} = \dfrac{m(\eta,v)}{m_o} = 1\Big/\sqrt{1-v^2/\eta^2} \\ \text{or} \quad m(\eta,v) = \dfrac{m_o}{\sqrt{1-v^2/\eta^2}} \end{cases} \tag{3}$$

where v is the particle speed of P as a matter particle, and m and m_o are respectively the observed mass and inherent mass of P as a matter particle.

The OR spacetime-transformation factor $\Gamma = m/m_o$ in particle-like form (Eq. (3)) is actually the OR mass-speed relationship, which has generalized Einstein's relativistic mass-speed relationship.

The Second One: Wave-Like Form

$$\begin{cases} \Gamma(\eta,v) = \dfrac{dt(\eta,v)}{d\tau} = \dfrac{f(\eta,v)}{f_o} = 1\Big/\sqrt{1-v^2/\eta^2} \\ \text{or} \quad f(\eta,v) = \dfrac{f_o}{\sqrt{1-v^2/\eta^2}} \end{cases} \tag{4}$$

where v is the group speed of P as a matter wave, and f and f_o are respectively the observed frequency and inherent frequency of P as a matter wave.

The OQ spacetime-transformation factor $\Gamma=f/f_o$ in wave-like form (Eq. (4)) is actually the OQ frequency-speed relationship, which has generalized de Broglie's frequency-speed relationship.

Equations (3) and (4) seems to imply that matter motion or observed objects in the physical world exhibits wave-particle duality.

It is the OR factor $\Gamma=m/m_o$ of spacetime transformation in the particle-like form (Eq. (3)) that led to the establishment of the theory of OR, including IOR and GOR. It is the OQ factor $\Gamma=f/f_o$ of spacetime transformation in the wave-like form (Eq. (4)) that led to the establishment of the theory of OQ, including the theory of OQ matter waves.

In this way, the invariance of the time-frequency ratio of a clock links the relativistic effects and quantum effects of matter motion or matter interactions together.

However, the theory of OQ matter waves, the so-called general de Broglie wave, has clarified [1-4] that such wave-particle duality is only a sort of observational effect exhibited by the information wave and informons of the general observation agent OA($\eta|h_\eta$), rooted in the observational locality ($\eta<\infty$) and observational perturbation ($h_\eta<0$) of OA($\eta|h_\eta$).

Subsequent OR serial reports will clarify that both OQ matter waves and de Broglie waves are not inherent matter waves of matter, but rather the information waves of certain observation agents.

3.4 Mass-Frequency Ratio and Energy-Frequency ration

Besides time (t), the mass (m) and energy (E) of matter can be regarded as the other two most fundamental physical quantities in physics.

Restricted by the observational locality ($\eta<\infty$), all physical quantities observed by the observation agent OA($\eta|h_\eta$), including mass and energy, must exhibit relativistic phenomena in observation that are, in fact, observational effects and apparent phenomena. Such observational effects must be related to the dilation of the time $\mathrm{d}t=\mathrm{d}t(\eta,v)$ observed by OA($\eta|h_\eta$), and associated with the invariance of time-frequency ratio.

By combining the invariance of time-frequency ratio (Eq. (2)) and the mass-speed relationship (Eq. (3)), one can derive the invariance of mass-frequency ratio observed by OA($\eta|h_\eta$). By combining the invariance of time-frequency ratio (Eq. (2)) and the general Einstein formula $E=m\eta^2$, one can derive the invariance of energy-frequency ratio observed by OA($\eta|h_\eta$).

3.4.1 The Invariance of Mass-Frequency Ratio

The Invariance of Mass-Frequency Ratio: Let O be the observer adopting the observation agent OA($\eta|h_\eta$), then the ratio m/f of the mass m to frequency f observed by O is an invariant, identical to the ratio m_o/f_o of the inherent mass m_o to inherent frequency f_o of P:

$$\frac{m(\eta,v)}{f(\eta,v)}=\frac{m_o}{f_o}=k_\eta \tag{5}$$

where k_η is the ratio of the mass m_o to frequency f_o of the informons of OA($\eta|h_\eta$).

Equation (5) can be written in the following form and referred to as the mass-quantum equation:

$$m=k_\eta f \tag{6}$$

By analogizing with Planck's energy-quantum hypothesis $\varepsilon=hf$, one can understand that the mass m observed by the general observation agent OA($\eta|h_\eta$) is also quantized.

However, such quantization depends on observation agents and does not represent the objective quantization of material mass, but the quantization of the mass measured by OA($\eta|h_\eta$): the mass of the informons of OA($\eta|h_\eta$) is the minimum scale or resolution for OA($\eta|h_\eta$) to measure material mass. So, the mass measured by OA($\eta|h_\eta$) must be discretized or quantized.

By utilizing the general Einstein formula $E=m\eta^2$ derived in Sec. 4 and the general Planck equation $E=h_\eta f$ derived in Sec. 5, one can derive the mass-frequency ratio k_η of the informons of the general observation agent OA($\eta|h_\eta$) ($0<\eta<\infty$; $h_\eta>0$):

$$k_\eta=\frac{m}{f}=\frac{E}{\eta^2 f}=\frac{h_\eta}{\eta^2} \tag{7}$$

where h_η is the energy-frequency ratio of the informons of OA($\eta|h_\eta$), or called the general Planck constant.

It should be point out that different observation agents must have different informons with different coefficients of mass-frequency ratio.

Naturally, if OA($\eta|h_\eta$) is the optical agent OA($c|h$) ($\eta=c$; $h_\eta=h$), then the mass-frequency ratio of its informons (photons) $k_\eta=k_c=h/c^2$.

3.4.2 The Invariance of Energy-Frequency Ratio

The Invariance of Energy-Frequency Ratio: Let O be the observer adopting the observation agent OA($\eta|h_\eta$), then the ratio E/f of the energy E to frequency f observed by O is an invariant, identical to the ratio E_o/f_o of the inherent energy E_o to inherent frequency f_o of P:

$$\frac{E(\eta,v)}{f(\eta,v)}=\frac{E_o}{f_o}=h_\eta \tag{8}$$

where h_η is the ratio of the energy E_o to frequency f_o of the informons of OA($\eta|h_\eta$), or the general Planck constant.

Equation (8) can be written in the following form and referred to as the general Planck equation:

$$E=h_\eta f \tag{9}$$

By analogizing with the Planck equation $E=hf$, one can understand that the energy E observed by the general observation agent OA($\eta|h_\eta$) is also quantized.

However, such quantization depends on observation agents and does not represent the objective quantization of material energy, but the quantization of the energy measured by OA($\eta|h_\eta$): the energy of the informons of OA($\eta|h_\eta$) is the minimum scale or resolution for OA($\eta|h_\eta$) to measure material energy. So, the energy measured by OA($\eta|h_\eta$) must be discretized or quantized.

It should be point out that different observation agents must have different informons with different coefficients of energy-frequency ratio.

Naturally, if OA($\eta|h_\eta$) is the optical agent OA($c|h$) ($\eta=c$; $h_\eta=h=6.6260693\times10^{-34}$ J·s), then the general Planck constant h_η is exactly the Planck constant h, that is, the energy-frequency ratio of the informons (photons) of the optical agent OA($c|h$).

Now, the invariance of the observational energy-frequency ratio has revealed the essence of Planck's constant h: h is not a so-called cosmological constant, but the coefficient of energy-frequency ratio of photons.

3.5 Towards the Unification of Relativity Theory and Quantum Theory

The OR axiom system, starting from the definition of OR time as a first principle, derives the invariance of time-frequency ratio of the clock. It is exactly the invariance of time-frequency ratio that links the relativistic effects and quantum effects of matter motion and matter interactions together.

The invariance of time-frequency ratio in Sec. 3.1 (Eq. (2)), the spacetime-transformation factor $\Gamma=f/f_o$ in the wave-like form or the frequency-speed relationship in Sec. 3.3 (Eq. (4)), the invariance of mass-frequency ratio in Sec. 3.4.1 (Eqs. (5-7)) and the invariance of energy-frequency ratio in Sec. 3.4.2 (Eqs. (8,9)), are all related to the frequency f observed by the observation agent OA($\eta|h_\eta$), and are all the quantum models of the theory of OQ. In particular, the frequency-speed relationship of OQ (Eq. (4)) lays the foundation for deducing the general Planck equation $E=h_\eta f$ of OQ in Sec. 5.

The spacetime-transformation factor $\Gamma=m/m_o$ in particle-like form in Sec. 3.3 is the relativistic model of the theory of OR, that is, the mass-speed relationship of OR (Eq. (3)), laying the foundation for deducing the general Einstein formula $E=m\eta^2$ of OR in Sec. 4.

In order to clarify how the theory of ORQ generalizes and unifies relativity theory and quantum theory, Sec. 4 of this article will state how the theory of OR derives the general Einstein equation $E=m\eta^2$, generalizes the Einstein equation $E=mc^2$, and reveals the essence of the mass-energy relationship; Sec. 5 of this article will state how the theory of OQ derives the general Planck equation $E=h_\eta f$, generalizes the Planck equation $E=hf$, and reveals the essence of the Planck constant h.

So, Einstein's formula $E=mc^2$ and Planck's equation $E=hf$ become a microcosm of the unification of relativity theory and quantum theory in ORQ.

4 The General Einstein Formula: $E=m\eta^2$

The most famous formula in Einstein's theory of relativity must be the Einstein formula: $E=mc^2$, also known as the Einstein mass-energy relationship. Now, Einstein's formula $E=mc^2$ has already become a hallmark of his theory of relativity.

The theory of OR has discovered that, limited by the observational locality ($c<\infty$) of the optical agent OA($c|h$), Einstein made a wrong interpretation for the physical meaning of $E=mc^2$. Up to today, the mainstream physics community still insists on this erroneous understanding. Now, based on the OR axiom system, the theory of OR has derived the general Einstein formula $E=m\eta^2$, or called the OR mass-energy relationship. Based on the broad perspective of the general observation agent OA($\eta|h_\eta$) ($0<\eta<\infty$; $h_\eta>0$), the theory of OR and the general Einstein formula $E=m\eta^2$ will reinterpret Einstein's formula $E=mc^2$ and reveal the essence of mass-energy relationship.

Naturally, from different logical premises, the Einstein formula $E=mc^2$ and the general Einstein formula $E=m\eta^2$ have different ways of logical deduction.

Einstein's mass-energy relationship $E=mc^2$ and the OR mass-energy relationship $E=m\eta^2$ can both be derived from their respective mass-speed relationships. However, Einstein's mass-speed relationship is based on the hypothesis of the invariance of light speed, where the observation agent is the optical agent OA($c|h$); The OR mass-speed relationship is based on the definition of OR time, or more directly from the invariance of time-frequency ratio, where the observation agent is the general observation agent OA($\eta|h_\eta$).

4.1 The Theoretical Derivation of the General Einstein Formula

There are various theoretical deduction approaches for Einstein's formula or mass-energy relationship $E=mc^2$, mostly starting from the mass-speed relationship in Einstein's theory of special relativity [23,24].

Based on the particle-like form of OR spacetime-transformation factor $\Gamma=m/m_o$ in Sec. 3.3, that is, the OR mass-speed relationship (Eq. (3)), by analogizing the logical way of deriving of the Einstein formula $E=mc^2$ [24], one can derive the OR mass-energy relationship, that is, the general Einstein formula $E=m\eta^2$.

4.1.1 The OR Formula of Kinetic Energy

Originally, Einstein's original intention was to establish a relativistic kinetic-energy formula for the observed object P, but unexpectedly derived the so-called mass-energy relationship.

In the theory of OR, the physical quantities of observed objects are measured by the general observation agent OA($\eta|h_\eta$) ($0<\eta<\infty$; $h_\eta>0$). Given the observation agent OA($\eta|h_\eta$) and the inertial object P moving at a speed v with the rest mass m_o and the observed mass $m=m(\eta,v)$, then, according to the mass-speed relationship (Eq. (3)), the observed momentum $p=p(\eta,v)$ and inertial force F of P can be defined as

$$\begin{cases} p = mv = \Gamma(\eta, v)\, m_o v = \dfrac{m_o v}{\sqrt{1 - v^2/\eta^2}} \\ F = \dfrac{dp(\eta, v)}{dt} \end{cases} \tag{10}$$

where η is the information-wave speed of OA($\eta|h_\eta$), v is the particle speed of P, and $p(\eta,v)$ is the relativistic momentum of P measured by OA($\eta|h_\eta$).

By analogizing Einstein's logic of deriving the formula of kinetic energy [24], in the theory of OR, the relativistic kinetic energy of P should be $K=K(\eta,v)$, that is, the kinetic energy measured by OA($\eta|h_\eta$), and can be defined and theoretically derived as follows:

$$\begin{aligned} K &= \int_0^v F dx = \int_0^v \frac{dp(\eta,v)}{dt} dx \\ &= \int_0^v v dp = \int_0^v \Gamma^3 m_o v dv = m_o \eta^2 \Gamma(\eta,v) \Big|_0^v \\ &= (\Gamma(\eta,v) - 1)\, m_o \eta^2 \quad \left(\Gamma = \frac{1}{\sqrt{1 - v^2/\eta^2}}\right) \end{aligned} \tag{11}$$

where $\Gamma(\eta,v)$ is the OR spacetime-transformation factor in the particle-like form (Eq. (3)).

This is the formula of kinetic-energy for the inertial object P observed by OA($\eta|h_\eta$) in the theory of OR.

4.1.2 The OR Formula of Mass Energy

Einstein envisioned that the total energy of the inertial object P should be $E=K+E_o$, where K is the kinetic energy of P, and that, if $v=0$, then $K=0$ and $E=E_o$, therefore E_o should be the rest energy.

By analogizing Einstein's logic, let $E_o=m_o\eta^2$ and $E=K+E_o$, then the OR mass-energy relationship can be derived from the OR kinetic-energy formula (Eq. (11)):

$$\begin{aligned} E &= E(\eta,v) = K + E_o = \Gamma(\eta,v)\, m_o \eta^2 \\ &= \frac{m_o \eta^2}{\sqrt{1 - v^2/\eta^2}} = m\eta^2 \\ &\begin{cases} E_o = E_o(\eta) = m_o \eta^2 \\ K = K(\eta,v) = E - E_o = (\Gamma(\eta,v) - 1)\, m_o \eta^2 \end{cases} \end{aligned} \tag{12}$$

where $E=E(\eta,v)=m\eta^2$ is the total energy or mass-energy of P measured by the observation agent OA($\eta|h_\eta$).

This is the mass-energy formula for the inertial object P observed by OA($\eta|h_\eta$) in the theory of OR. $E=m\eta^2$ is isomorphic to Einstein's formula $E=mc^2$, and is therefore called the general Einstein formula.

4.2 The Implication of the General Einstein Formula $E=m\eta^2$

In Newtonian mechanics or classical mechanics, the mass and energy of matter are independent of each other: mass is just mass, energy is just energy; there is no concepts of **Mass Energy** and **Rest Energy**. Human reason tells us that matter is a contradictory unity of mass and energy: mass and energy are interdependent; energy without mass and mass without energy are unimaginable.

4.2.1 The Einstein Formula and Mass-Energy Relationship

In 1905, based on his hypothesis of the invariance of light speed, Einstein theoretically derived the Lorentz transformation and relativistic mass-speed relationship: $m=m_o/\sqrt{(1-v^2/c^2)}$, establishing his theory of special relativity [23]. Originally, Einstein expected to derive the relativistic kinetic-energy formula of inertial objects based on his mass-speed relationship, but accidentally, deduced the mass-energy relationship $E=mc^2$ that became the hallmark of Einstein's theory of relativity and was known as the Einstein formula. Based on the formula $E=mc^2$, Einstein believed that the total energy of an inertial object should be $E=K+E_o$, not only have the kinetic energy K, but also have the rest energy E_o. In particular, $E=mc^2$ depends on the relativistic mass m of P; $E_o=m_oc^2$ depends on the rest mass m_o, and therefore, E and E_o can be referred to as **Mass Energy**, while $E=mc^2$ as **the Einstein mass-energy relationship**.

Based on his mass-energy relationship $E=mc^2$, Einstein believed that the mass and energy of matter are equivalent. So, mass is also energy and energy is also mass; mass can be transformed into energy and energy can be transformed mass. The mainstream physics community firmly believes in this, which seems to be supported by observations and experiments: during the annihilation of particles and antiparticles, the mass of material particles seems to be transformed into energy. Nowadays, people always talk with relish about Einstein's mass-energy relationship $E=mc^2$.

Now, the general Einstein formula $E=m\eta^2$ provides new insights to physics, and based on the broad perspective of the general observation agent OA($\eta|h_\eta$), provides new interpretation for the Einstein formula $E=mc^2$, revealing the essence of the relationship between material mass and material energy.

4.2.2 The General Einstein Formula and Mass-Energy Relationship

In this article, we do not discuss whether the mass and energy of matter are equivalent or whether they can be transformed into each other, nor do we discuss whether the observation or experiment of the annihilation of particles and antiparticles are optical illusions or the physical reality. The relevant issues are left for subsequent OR serial reports and physicists to study. From the perspective of the general observation agent OA($\eta|h_\eta$), based on the OR mass-energy relationship $E=m\eta^2$ (Eq. (12)), we reexamine Einstein's mass-energy relationship $E=mc^2$, and Einstein's mass-energy E and rest energy E_o.

As a theoretical model with the general observation agent OA($\eta|h_\eta$), the OR mass-energy relationship $E=m\eta^2$ has generalized the Einstein formula $E=mc^2$, extending the Einstein formula $E=mc^2$ from the optical agent OA($c|h$) to the general observation agent OA($\eta|h_\eta$). Naturally, as $\eta\to c$, the general Einstein formula $E=m\eta^2$ reduces to the Einstein formula $E=mc^2$:

$$\begin{aligned} E(c,v) &= \lim_{\eta\to c} E(\eta,v) = \lim_{\eta\to c} m\eta^2 \\ &= \lim_{\eta\to c} \frac{m_o\eta^2}{\sqrt{1-v^2/\eta^2}} = \frac{m_o c^2}{\sqrt{1-v^2/c^2}} = mc^2 \end{aligned} \tag{13}$$

which indicates that the Einstein formula $E=mc^2$ is only a special case of the general Einstein formula $E=m\eta^2$, that is, a product of the optical agent OA($c|h$).

So, Einstein's mass-energy relationship $E=mc^2$ is valid only if OA($\eta|h_\eta$) is the optical agent OA($c|h$).

By comparing the OR kinetic-energy formula $K=K(\eta,v)$ (Eq. (11)) and the OR mass-energy formula (Eq. (12)), it can be seen that the OR mass energy $E=m\eta^2$ and the OR rest energy $E_o=m_o\eta^2$ are the by-products of OR kinetic-energy formula $K=K(\eta,v)$, the upper and lower limits of the OR kinetic-energy integration relation: $K(\eta,v)=E(\eta,v)-E_o(\eta)$, which are not the objective and real energy of matter particles except for informons.

If the observed object P is an informon of the observation agent OA($\eta|h_\eta$), then $v=\eta$. According to the OR mass-speed relationship (Eq. (3)) and Einstein's logic, the rest mass and energy of the informon are zero: $m_o=0$ and $E_o=0$. Then, the general Einstein formula $E=m\eta^2$ is the kinetic energy of the informon of OA($\eta|h_\eta$). Thus, as a special case, the Einstein formula $E=mc^2$ represents the kinetic energy of the informons (photons) of OA($c|h$).

It is worth noting that $E=m\eta^2$ and $E_o=m_o\eta^2$ depend on the information-wave speed η of the general observation agent OA($\eta|h_\eta$). This suggests that, for the same inertial object P, the mass energy E and rest energy E_o of P measured by different observation agents are different. So, the so-called mass energy E and rest energy E_o in the OR mass-energy relationship $E=m\eta^2$ are not the objective physical existence and real material energy. Thus, the mass energy $E=mc^2$ and rest energy $E=mc^2$ in Einstein's mass-energy relationship can only be the optical illusions presented to observers by the optical agent OA($c|h$).

In particular, in the observation of Newton's idealized agent OA($\infty|0$), $\eta\to\infty$, $E\to\infty$, and $E_o\to\infty$, which suggests that both the mass energy $E=m\eta^2$ and rest energy $E_o=m_o\eta^2$ of matter are unobservable. Therefore, in Newtonian mechanics, matter has neither mass energy E nor rest energy E_o. This is in line with human reason and the principles of physical observability [1-5].

So, Newton's view of mass and energy is right.

4.3 The OR Kinetic-Energy Formula: A Microcosm of the Unity of Newton and Einstein

As stated in Sec. 4.2, the so-called mass energy E and rest energy E_o in the general Einstein formula $E=m\eta^2$ are only the upper and lower limits of kinetic-energy integral formula, not the real material energy. The objective and real material energy in the energy relations of inertial OR (IOR) is the kinetic energy of inertial objects: $K(\eta,v)=E(\eta,v)-E_o(\eta)$.

It should be pointed out that the kinetic energy $K=K(\eta,v)$ in the OR kinetic-energy formula (Eq. (11)) is the kinetic energy of inertial objects measured by the general observation agent OA($\eta|h_\eta$), while Newton's classical kinetic energy $K=K(\infty,v)$ (measured by Newton's idealized agent OA($\infty|0$)) and Einstein's relativistic kinetic energy $K=K(c,v)$ (measured by Einstein's optical agent OA($c|h$)) are just two special cases of the OR kinetic-energy formula $K=K(\eta,v)$.

4.3.1 Generalizing Einstein's Kinetic Energy

As $\eta\to c$, the general observation agent OA($\eta|h_\eta$) becomes the optical agent OA($c|h$), and the OR kinetic-energy formula $K=K(\eta,v)$ (Eq. (11)) strictly converges to Einstein's relativistic kinetic-energy formula $K=K(c,v)$:

$$\begin{aligned} K(c,v) &= \lim_{\eta\to c}\left(\left(\sqrt{1-v^2/\eta^2}\right)^{-1}-1\right)m_o\eta^2 \\ &= \left(\left(\sqrt{1-v^2/c^2}\right)^{-1}-1\right)m_oc^2 \end{aligned} \tag{14}$$

Equation (14) indicates that Einstein's kinetic energy $K=K(c,v)$ is indeed that measured by the optical agent OA($c|h$): Einstein's relativistic kinetic-energy formula is only a special case of the OR kinetic-energy formula.

Einstein's observation agent is the optical agent OA($c|h$) that has the observational locality ($\eta=c<\infty$), and therefore, exhibits apparent relativistic phenomena. As shown in Eq. (14), Einstein's kinetic energy $K=K(c,v)$ depends on observation, on the optical agent OA($c|h$), and on the light speed c as the information-wave speed of OA($c|h$)), and is not the completely objective and real kinetic energy of moving objects.

4.3.2 Generalizing Newton's Kinetic Energy

As $\eta\to\infty$, the general observation agent OA($\eta|h_\eta$) becomes the idealized agent OA($\infty|0$), and the OR kinetic-energy formula $K=K(\eta,v)$ (Eq. (11)) strictly converges to Newton's classical kinetic-energy formula $K=K(\infty,v)$:

$$\begin{aligned} K(\infty,v) &= \lim_{\eta\to\infty}\left(\left(\sqrt{1-{v^2}/{\eta^2}}\right)^{-1}-1\right)m_o\eta^2 \\ &= \frac{1}{2}m_ov^2 = \frac{1}{2}m_Iv^2 = \frac{1}{2}m_\infty v^2 \end{aligned} \tag{15}$$

where m_o is the rest mass of the observed object P, while m_I and m_∞ are respectively the inertial mass and classical mass of P. As for why $m_o=m_I=m_\infty$, readers can refer to OR serial report 2 [6]; As for why $h_\eta\to0$ as $\eta\to\infty$, Sec. 7 of this article will discuss.

Perhaps, you foresaw that the OR kinetic-energy formula $K=K(\eta,v)$ could generalize Einstein's relativistic kinetic-energy formula $K=K(c,v)$. However, you may not be able to foresee that the OR kinetic-energy formula $K=K(\eta,v)$ also can generalize Newton's classical kinetic-energy formula $K=K(\infty,v)$.

Equation (15) indicates that Newton's kinetic energy $K=K(\infty,v)$ is indeed that measured by the idealized agent OA($\infty|0$): Newton's classical kinetic-energy formula is also only a special case of the OR kinetic-energy formula.

Newton's observation agent is the idealized agent OA($c|h$) that has no observational locality ($\eta\to\infty$), and therefore, no apparent relativistic phenomena. As shown in Eq. (15), Newton's kinetic energy $K=K(\infty,v)$ does not depend on observation, representing the objective and real kinetic energy of moving objects.

4.3.3 The Unity of Newton and Einstein in OR

Equations (14) and (15) indicate that the OR kinetic-energy formula $K=K(\eta,v)$ (Eq. (11)) has generalized and unified Newton's classical kinetic energy $K(\infty,v)$ and Einstein's relativistic kinetic energy $K(c,v)$.

In fact, the theory of OR has already become a complete theoretical system, generalizing and unifying Newtonian mechanics and Einstein's theory of relativity [1-5]: as $\eta\to c$, all the relations in the theory of IOR strictly converge to the corresponding relations in Einstein's theory of special relativity, while all the relations in the theory of GOR strictly converge to the corresponding relations in Einstein's theory of general relativity; as $\eta\to\infty$, all relations in the theory of IOR strictly converge to the corresponding relations in Newton's inertia mechanics, while all relations in the theory of GOR strictly converge to the corresponding relations in Newton's theory of universal gravitation.

This suggests that the theory of OR is logically self-consistent: logically consistent not only with Einstein's relativity theory but also with Newton's classical mechanics. The unification of Newtonian mechanics and Einstein's theory of relativity in the theory of OR confirms the logical rationality and theoretical validity of the theory of OR.

The unification of Newton's classical kinetic-energy formula and Einstein's relativistic kinetic-energy formula in the theory of OR is just a microcosm of the unity of Newton and Einstein in the theory of OR.

For details on the unification of Newtonian mechanics and Einstein's theory of relativity, please refer to the monograph of OR [1,4] and OR serial report 1 [5].

5 The General Planck Equation: $E=h_\eta f$

Although belonging to two different theoretical systems of quantum theory and relativity theory, Planck's equation $E=hf$ and Einstein's formula $E=mc^2$ are still a pair of dual formulae in modern physics. As for the optical agent OA($c|h$), both Planck's equation $E=hf$ and Einstein's formula $E=mc^2$ represent the kinetic energy of the informons (photons) of OA($c|h$). Therefore, their unification in the theory of ORQ holds special significance.

As stated in Sec. 2.2.2, the theory of OQ has discovered [1-4,10] that, like Einstein's theory of relativity, quantum mechanics based on the Planck energy-quantum hypothesis $\varepsilon=hf$ is also a theory of optical observation, the quantum effects, including wave-like nature and uncertainty, are also observational effects. Differently, Einstein's relativistic effects are apparent phenomena rather than the objective physical reality and real physical existence, while Planck's quantum effects are not apparent phenomena, but rather the objective and real change of the motion state of observed objects due to the observational perturbation of the informons (photons) of the optical agent OA($c|h$).

Now, the theory of OQ will theoretically derive the general Planck equation $E=h_\eta f$. Based on the broad perspective of the general observation agent OA($\eta|h_\eta$) ($0<\eta<\infty$; $h_\eta>0$), the theory of OQ and the general Planck equation $E=h_\eta f$ will reinterpret the Planck equation $E=hf$ and reveal the essence of the quantum effects exhibited in the modern quantum physics.

It should be pointed out that the Planck equation $E=hf$ is only a hypothesis in the law of blackbody radiation. The general Planck equation $E=h_\eta f$ in the theory of OQ, however, is a logical consequence of the OR axiom system.

5.1 Historical Review: The Concept of Quantum

Thank to the great Mr. Max Planck!

In 1900, Planck proposed the hypothesis of energy quanta, $\varepsilon=hf$, that laid the first cornerstone for quantum mechanics [14], marking the birth of quantum physics and revealing quantum effects and quantum phenomena in the physical world [25,26].

Actually, Planck himself seemed reluctant to quantize the energy of matter, as it goes against the traditional concept of material energy. However, in order to derive the blackbody-radiation formula that conformed to the blackbody-radiation experiment, Planck had to propose the energy-quantum hypothesis of $\varepsilon=hf$, discretizing the originally continuous energy of matter in physics.

Now, the theory of OQ has discovered that, like Einstein's theory of relativity, the quantum theory based on Planck's quantum hypothesis $\varepsilon=hf$ is also a theory of optical observation adopting the optical agent OA($c|h$).

In fact, both the Planck equation $E=hf$ and Planck's blackbody radiation law are the theoretical models of the optical agent $E=hf$, in which Planck is the observer O, the object P observed by Planck is light, and the observation medium M that transmits the information on P to Planck is also light. Photons, as the informons of optical agent OA($c|h$), represents the minimum scale or resolution of the optical measurement of OA($c|h$). Naturally, the energy quantum $\varepsilon=hf$ is the minimum scale or resolution of energy measured by OA($c|h$). Therefore, the discretization and quantization of material energy measured by OA($c|h$) is natural and inevitable.

So, Mr. Planck did not need to have agonized over his hypothesis of energy quanta.

The mainstream physics community has long lacked a correct understanding of Einstein's relativistic effects; similarly, the mainstream physics community has long lacked a correct understanding of Planck's quantum effects.

Mankind's cognition and understanding of the natural world must be a process from the macroscopic world to the microscopic world, from relativistic phenomena to quantum phenomena.

However, before mankind could properly understand the relativistic phenomena of matter motion that mainly present in the macroscopic world, he had to face the quantum phenomena of matter motion that mainly present in the microscopic world. Of course, the problem is not whether we should enter the microscopic world to explore the quantum world; the problem is that we have brought our erroneous understanding of the relativistic phenomena of matter motion in the macroscopic world into our exploration of the quantum phenomena of matter motion in the microscopic world!

Originally, the Planck equation $E=hf$ was a hypothesis about photon energy proposed by Planck so that he could theoretically derive the formula of blackbody radiation [14]. However, in the 1920s, de Broglie imagined [27] that matter motion had wave-particle duality. Based on this idea, he extended Planck's equation $E=hf$ from photon to all matter particles, derived the de Broglie relation $p=h/\lambda$, and established the matter-wave theory, known as de Broglie wave [28,29], laying the second cornerstone of quantum mechanics. Afterwards, inspired by de Broglie's theory of matter waves, Schrödinger established the quantum wave equation, laying the third cornerstone of quantum mechanics, that is, the famous Schrödinger equation [30].

Perhaps, it is de Broglie's extension of the Planck equation $E=hf$ that makes the Planck constant h a cosmological constant.

Neither Planck, nor de Broglie, nor Schrödinger, nor even the mainstream physics community today, have realized that Planck's energy quanta are photons, more precisely, the informons of the optical agent OA($c|h$); and that Planck's quantum effects are observational effects caused by the perturbation of informons (photons) of the optical agent OA($c|h$) exerting on observed objects.

The issue of whether de Broglie wave truly exist, as well as the issues of the Schrödinger equation [30] and Heisenberg uncertainty [31], are left for subsequent OR serial reports. This article focuses on the general Planck equation $E=h_\eta f$, and from the perspective of the general observation agent OA($\eta|h_\eta$) ($0<\eta<\infty$; $h_\eta>0$), reexamines the Planck equation $E=hf$, revealing the essence of the Planck constant h.

5.2 The Theoretical Derivation of the General Planck Equation

The wave-like form of OR spacetime-transformation factor $\Gamma=f/f_o$ stated in Sec. 3.3 naturally forms the OQ frequency-speed relationship (Eq. (4)), laying the foundation for deriving the general Planck equation in the theory of OQ.

In a simple way, by combining the OQ frequency-speed relationship (Eq. (4)) with the general Einstein formula $E=m\eta^2$ (Eq. (12)), one can obtain the general Planck equation implied in the invariance of energy-frequency ratio (Eqs. (8,9)):

$$\begin{aligned} E &= m\eta^2 = \Gamma(\eta,v)m_o\eta^2 = \frac{f(\eta,v)}{f_o}m_o\eta^2 \\ &= \frac{m_o}{f_o}\eta^2 f(\eta,v) = h_\eta f(\eta,v) \\ &\qquad \left(h_\eta = \frac{m_o}{f_o}\eta^2 = k_\eta\eta^2\right) \end{aligned} \tag{16}$$

Equation (16) has special significance as it naturally and logically links the general Einstein equation $E=m\eta^2$ of OR and the general Planck equation $E=h_\eta f$ of OQ together, becoming a pair of dual formulae that have been unified in the theoretical system of ORQ.

In fact, the theory of OQ can derive the general Planck equation $E=h_\eta f$ in a more fundamental logical way without relying no the general Einstein formula $E=m\eta^2$ in the theory of OR.

5.2.1 The OQ Formula of Kinetic Energy

Given the observation agent OA($\eta|h_\eta$), by combining the particle-like form $\Gamma=m/m_o$ (Eq. (3)) and wave-like form $\Gamma=f/f_o$ (Eq. (4)) of the OR spacetime-transformation factor $\Gamma=\mathrm{d}t/\mathrm{d}\tau$, one can obtain:

$$\mathrm{d}\Gamma = \frac{\Gamma^3}{\eta^2}v\mathrm{d}v \quad \text{and} \quad \mathrm{d}\Gamma = \frac{1}{f_o}\mathrm{d}f$$
$$\begin{cases} \Gamma = \Gamma(\eta,v) = {}^{\mathrm{d}t(\eta,v)}\!/_{\mathrm{d}\tau} \\ \quad = \dfrac{m(\eta,v)}{m_o} = \dfrac{f(\eta,v)}{f_o} = \dfrac{1}{\sqrt{1-{}^{v^2}\!/_{\eta^2}}} \end{cases} \tag{17}$$

Thus, $\Gamma^3 v\mathrm{d}v = \left(\eta^2/f_o\right)\mathrm{d}f$.

Based on the same logic as deriving the general Einstein formula $E=m\eta^2$ (Eq. (12)), consider an inertial object P moving at a speed v with the total energy $E=K+E_o$, where K and E_o is respectively the relativistic kinetic energy and rest energy of P. Substituting Eq. (17) into the definition of kinetic energy K (Eq. (11)) and the definition of force F (Eq. (10)), then in the theory of OQ, the kinetic energy of P measured by OA($\eta|h_\eta$) is

$$\begin{aligned} K &= \int_0^v F\mathrm{d}x = \int_0^v \frac{\mathrm{d}p}{\mathrm{d}t}\mathrm{d}x = \int_0^v v\mathrm{d}p \\ &= \int_0^v \Gamma^3 m_o v\mathrm{d}v = \int_0^v m_o\frac{\eta^2}{f_o}\mathrm{d}f = h_\eta f\Big|_0^v \\ &= h_\eta(f(\eta,v)-f_o) \quad \left(h_\eta = \frac{m_o}{f_o}\eta^2 = k_\eta\eta^2\right) \end{aligned} \tag{18}$$

where k_η and h_η are respectively the mass-frequency ratio (Eq. (5)) and energy-frequency ratio (Eq. (8)) of the informons of OA($\eta|h_\eta$).

5.2.2 The OQ Formula of Frequency Energy

Let $E_o=h_\eta f_o$ is the rest energy of the moving object P in inertial spacetime, then the frequency-energy formula of OQ can be obtained from Eq. (18):

$$E(\eta,v) = h_\eta f(\eta,v) \quad \left(h_\eta = \frac{E_o(\eta)}{f_o}\right) \tag{19}$$

Obviously, the frequency-energy formula $E=h_\eta f$ (Eq. (19)) of OQ is isomorphic to the Planck equation $E=hf$, and can be referred to as the general Planck equation, and the energy-frequency ratio h_η can be referred to as the general Planck constant.

Thus, following Einstein's logic, $E(\eta,v)=K(\eta,v)+E_o$ can be understood as the total energy of the observed object P in inertial spacetime measured by the observation agent OA($\eta|h_\eta$), that is, the sum of the kinetic energy $K(\eta,v)$ (Eq. (18)) and rest energy $E_o=h_\eta f_o$ of P measured by OA($\eta|h_\eta$).

We sometimes refer to $E=hf$ and $E=h_\eta f$ as frequency-energy formulae, as we sometime refer to $E=mc^2$ and $E=m\eta^2$ as mass-energy formulae.

5.3 The Implication of the General Planck Equation $E=h_\eta f$

Planck's equation $E=hf$ has been in existence for over a hundred years. However, in the physics community, the understanding of the Planck equation $E=hf$ and Planck constant h as well as quantum phenomena is still vague and even erroneous. The mainstream physics community has not yet realized that quantum physics based on Planck's energy-quantum hypothesis $\varepsilon=hf$, including quantum mechanics and quantum-field theory, like Einstein's theory of relativity, is only a theory of optical observation, adopting the optical agent OA($c|h$).

Now, the theory of OQ has derived the general Planck equation $E=h_\eta f$, generalizing the Planck equation $E=hf$ and surpassing de Broglie's imagination, not only extending the Planck equation $E=hf$ from photons to any matter particle, but also from the optical agent OA($c|h$) to the general observation agent OA($\eta|h_\eta$) ($0<\eta<\infty$; $h_\eta>0$).

Based on the broad perspective of the general observation agent OA($\eta|h_\eta$), the general Planck equation $E=h_\eta f$ has had significant discoveries and brought new insights to physics, giving new interpretations to the Planck equation $E=hf$ and Planck constant h, and even the quantum physics that Planck laid the foundation for.

5.3.1 On Planck's Equation $E=hf$

Equations (18) and (19) indicate that the general Planck equation $E=h_\eta f$ depends on observation, on observation agents, on the speed η of the information wave of OA($\eta|h_\eta$) and the energy-frequency ratio h_η of the informons of OA($\eta|h_\eta$).

In theory, any form of matter motion or matter interaction can serve as the observation agent OA($\eta|h_\eta$) of OQ; any matter wave can serve as the information wave of OA($\eta|h_\eta$) transmitting observed information to observers; any material particle can serve as the informons of OA($\eta|h_\eta$) exerting perturbation on observed objects. So, for the same observed object P, different observation agents with different information waves and informons must exert different perturbations on P, resulting in different quantum effects or phenomena.

As a theoretical model of the general observation agent OA($\eta|h_\eta$), the general Planck equation $E=h_\eta f$ has generalized the Planck equation $E=hf$ as a theoretical model of the optical agent OA OA($c|h$). As $\eta\to c$, the general Planck equation reduces to the Planck equation:

$$\begin{aligned} E(c,v) &= \lim_{\eta\to c} E(\eta,v) = \lim_{\eta\to c} h_\eta f(\eta,v) \\ &= h_c f(c,v) = hf \quad \left(h_c = \frac{E_o(c)}{f_o} = h\right) \end{aligned} \tag{20}$$

where $h_c=h$ is the energy-frequency ratio of the informons (photons) of OA($c|h$), i.e., the Planck constant h.

So, the Planck equation $E=hf$ is only a special case of the general Planck equation $E=h_\eta f$, that is, the product of the optical agent OA($c|h$).

It should be pointed out that, in Eq. (20), the Planck equation $E=hf$ as a special case of the general Planck equation $E=h_\eta f$, is no longer a hypothesis, but a logical consequence of theory the of OQ.

As stated in Sec. 5.1, Planck's equation $E=hf$ is the product of blackbody-radiation experiment adopting the optical agent OA($c|h$), in which Planck is the observer O, the observed object P is light, and the medium M that transmits information on P is also light. Actually, Planck's equation $E=hf$ is the energy-quantum hypothesis $\varepsilon=hf$ that Planck had to set to derive his blackbody-radiation formula, which represents the kinetic energy of a photon in a light or electromagnetic wave with frequency f, and is also the minimum scale or resolution for the optical agent OA($c|h$) to measure the mass, momentum, and energy of matter. Planck's energy quanta are photons.

In this regard, it is natural and logical for Planck to take his energy-quantum hypothesis $\varepsilon=hf$ as the logical premise for deducing the blackbody-radiation formula.

For the informons of the optical agent OA($c|h$), the speed $v=\eta$; according to the OQ frequency-speed relationship (Eq. (4)) and Einstein's logic, the inherent frequency f_o and rest energy E_o of the informons are zero: $f_o=0$ and $E_o=hf_o=0$. So, the general Planck equation $E=h_\eta f$ is if fact the kinetic energy of the informons of OA($\eta|h_\eta$).

Accordingly, as a special case of the general Planck equation $E=h_\eta f$, the Planck equation $E=hf$ is just the kinetic energy of the informons (photons) of the optical agent OA($c|h$).

5.3.2 On Planck's Constant h

In the dictionary of modern physics, the Planck constant h is regarded as a cosmological constant.

In de Broglie's theory of matter waves, and even throughout the whole quantum physics, Planck's constant h seems to be universally applicable to all matter particles, all matter waves, and even all quantum situations.

However, in theory of OQ, the general Planck equation $E=h_\eta f$ (Eq. (19)) has brought a new physical quantity: the general Planck constant h_η.

As a theoretical model of the general observation agent OA($\eta|h_\eta$), the energy quanta in the general Planck equation $E=h_\eta f$ are the informons of OA($\eta|h_\eta$), and the general Planck constant h_η is the energy-frequency ratio of the informons of OA($\eta|h_\eta$). In theory, the informons of OA($\eta|h_\eta$) or the energy quanta in the general Planck equation $E=h_\eta f$ can be any material particle, not necessarily just photons, and the energy-frequency ratio h_η can take any value, not necessarily just the Planck constant h.

As a system parameter or eigenvalue of the general observation agent OA($\eta|h_\eta$), the general Planck constant h_η has generalized the Planck constant h as a system parameter or eigenvalue of the optical agent OA($c|h$).

As $\eta\to c$, the general Planck constant h_η reduces to the Planck constant h:

$$h_c = \lim_{\eta\to c} h_\eta = \lim_{\eta\to c} \frac{E_o(\eta)}{f_o} = \frac{E_o(c)}{f_o} = \frac{m_o c^2}{f_o} = h \tag{21}$$

The general Planck equation $E=h_\eta f$ (Eq. (19)) in the theory of OQ and Eq. (21) tell us that the Planck constant h is only the energy-frequency ratio of photons or electromagnetic particles, and is valid only if the observation agent OA($\eta|h_\eta$) is the optical agent OA($c|h$).

Accordingly, Planck's constant h is only a constant for electromagnetic interaction, not the so-called cosmological constant.

5.3.3 On Quantum Physics

Up to this day, the Planck equation $E=hf$ remains a hallmark of quantum physics; the Planck constant h still plays the role of a cosmological constant in physics and is universally applied in quantum physics.

So, modern quantum physics, including quantum mechanics and quantum-field theory, is actually that based on Planck's quantum hypothesis of $\varepsilon=hf$.

It should be pointed out that, similar to that for macroscopic matter motion, most of human observations and experiments for microscopic matter motion also employ the optical agent OA($c|h$)), including the experiment of blackbody radiation. This is the main reason why the Planck equation $E=hf$ and Planck constant h seem to be universally applicable in quantum physics.

According to the general Planck equation $E=h_\eta f$, the Planck equation $E=hf$ is a theoretical model of optical observation adopting the optical agent OA($c|h$), which presents us only with the optical image of microscopic quantum world. If mankind employs non-optical agents OA($\eta|h_\eta$) ($\eta\neq c$; $h_\eta\neq h$) to detect microscopic matter motion, such as the electronic agent OA($v_e|h_e$), then he will see a quantum world different from that presented by the optical agent OA($c|h$).

The double-slit experiment with electrons is such an example [32]. The mystery of the double-slit experiment with electrons is listed as one of the fifteen big puzzles in physics in the monograph of OR [1,4]: BP-08.

The phenomena of electron self-coherence and electron decoherence exhibited in the double-slit experiment with electrons are quite weird and remain a mystery in quantum physics [33,34]. The double-slit experiment with electrons will leave interference fringes on the interference screen, regardless of whether the emitted electrons are a continuous electron flow or individual electrons. However, electrons seem to have the art of making themselves invisible: if you try to spy on the whereabouts of an electron, set up a camera, and see if it is crossing the left or right slit, it will hide and the interference fringes on the interference screen will disappear; if you stop spying and turn off the camera, the screen will reproduce electronic interference fringes.

The mystery of the double-slit experiment with electrons has long plagued physicists. Physicists have not yet fully understood the self-coherence and decoherence phenomena of electrons in the double-slit experiment.

Now, the theory of OQ and the general Planck equation $E=h_\eta f$ have solved the puzzle of the double-slit experiment with electrons. The reason seems quite simple: in order to spy on the whereabouts of electrons, experimental physicists have to switch observation agents during the double-slit experiment with electrons.

In the double-slit experiment with electrons, the object observed by the physicist is the electron. Before turning on the camera, the observation agent is the electronic agent OA($v_e|h_e$), and it is the electron itself that transmits information of its own; after turning on the camera, the observation agent is switched to the optical agent OA($c|h$), then it is the photon that transmits information on the electron. OA($v_e|h_e$) and OA($c|h$) have different information waves with different speeds ($v_e\neq c$) and different informons with different energy-frequency ratios ($h_e\neq h$). Therefore, regarding the motion of the electron, OA($v_e|h_e$) and OA($c|h$) not only exhibit different relativistic phenomena, but also different quantum phenomena.

Thus, the phenomena of electron self-coherence and electron decoherence presented by the electronic agent OA($v_e|h_e$) are much different from that presented by the optical agent OA($c|h$), and cannot be described or interpreted by the quantum mechanics based on the Planck energy-quantum hypothesis $\varepsilon=hf$.

This is the answer of the theory of OQ to the mystery of the double-slit experiment with electrons.

In the theory of OQ, the general Planck equation $E=h_\eta f$ has not only generalized the Planck equation $E=hf$, but also indicated that the theory of OQ will generalize and reshape quantum physics.

Now, according to the general Planck equation $E=h_\eta f$, the mainstream physics community needs to reexamine the quantum physics that is based on the Planck energy-quantum hypothesis $\varepsilon=hf$, and reinterpret the quantum phenomena of matter motion and matter interactions from the perspective of the general observation agent OA($\eta|h_\eta$) ($0<\eta<\infty$; $h_\eta>0$).

6 ORQ: The Unification of Relativity Theory and Quantum Theory

The same physical world should have the same logic and follow the same laws.

In modern physics, however, Newtonian mechanics and Einstein's theory of relativity belong to different theoretical systems, and also, Einstein's theory of relativity and Planck's quantum theory belong to different theoretical systems.

Hawking said in his book **A Brief History of Time** [16]: "If we discover a complete theory, it would be the ultimate triumph of human reason – for then we should know the mind of God." Hawking believed that modern physics has been divided into many partial theories and the ultimate goal of physics is to unify them.

As stated in Sec. 3, starting from the definition of OR time as first principle, the OR axiom system has derived the invariance of time-frequency ratio (Eq. (2)). It is based on the invariance of time-frequency ratio that the theory of OR has established the OR spacetime transformation in the differential form, and then, acquired its algebraic form, the so-called general Lorentz transformation, in which the factor $\Gamma(\eta,v)=\mathrm{d}t/\mathrm{d}\tau$ of spacetime transformation has two forms: one is the particle-like form $\Gamma=m/m_o$ (Eq. (3)); the other is the wave-like form $\Gamma=f/f_o$ (Eq. (4)).

It is based on the particle-like form $\Gamma=m/m_o$ of OR spacetime-transformation factor that the theory of ORQ has established the theory of observational relativity (OR). It is based on the wave-like form $\Gamma=f/f_o$ of OR spacetime-transformation factor that the theory of ORQ has established the theory of observational quantum (QR)

In this way, the originally separate relativity theory and quantum theory has been generalized and unified within the same theoretical system under the same axiom system, becoming the theory of observational relativity-quantum (ORQ).

6.1 OR: The Theory of Observational Relativity

The theory of ORQ, based on the definition of OR time and the invariance of time-frequency ratio, has proved the theorem of the invariance of information-wave speeds and derived the general Lorentz transformation, and thereby established the theory of inertial OR (IOR). The theory of IOR, as that of the general observation agent OA($\eta|h_\eta$), has generalized and unified Newton's inertial mechanics of the idealized agent OA($\infty|0$) and Einstein's special relativity of the optical agent OA($c|h$).

The basic theorems and relations of IOR include

(1) The theorem of the invariance of time-frequency

ratio: $\mathrm{d}t_\eta/f_\eta=\mathrm{d}\tau/f_o$;

(2) The theorem of the invariance of information-wave speeds: $\forall v\in(-\eta,\eta)\ \eta\oplus v=\eta$;

(3) The general Lorentz transformation: $O'(x',y',z',t')\Leftrightarrow O(x,y,z,t)$;

(4) The general speed-addition law: $u=(u'+v)/(1+u'v/\eta^2)$;

(5) The general mass-speed relationship: $m=m_o/\sqrt{(1-v^2/\eta^2)}$;

(6) The general momentum-speed relationship: $p=mv=m_ov/\sqrt{(1-v^2/\eta^2)}$

(7) The general kinetic-energy-speed relationship: $K=(1/\sqrt{(1-v^2/\eta^2)}-1)m_o\eta^2$;

(8) The general Einstein formula: $E=m\eta^2$.

For the details of the theory of IOR, please refer to the 1st volume IOR of the Monograph of OR [1,4].

After IOR, by taking the invariance of information-wave speeds as a new logical starting point, utilizing the principles of GOR equivalence and GOR covariance as well as the principle of general correspondence, following and analogizing the logic of Einstein's theory of general relativity, the theory of ORQ has established the theory of gravitational OR (GOR). The theory of GOR, as that of the general observation agent OA($\eta|h_\eta$), has generalized and unified Newton's theory of universal gravitation of the idealized agent OA($\infty|0$) and Einstein's theory of general relativity of the optical agent OA($c|h$).

The most fundamental formulae of GOR are

(1) GOR Field Equation:

$$R_{\mu\nu}(\eta)-\frac{1}{2}g_{\mu\nu}R=-\kappa_{\mathrm{GOR}}T_{\mu\nu}\quad\left(\kappa_{\mathrm{GOR}}=\frac{8\pi G}{\eta^4}\right)\tag{22}$$

which has generalized and unified Newton's law of universal gravitation in the Poisson equation form and Einstein's field equation in general relativity.

(2) GOR Motion Equation:

$$\frac{\mathrm{d}^2x^\mu}{\mathrm{d}\tau^2}+\Gamma^{\ \mu}_{\alpha\beta}(\eta)\frac{\mathrm{d}x^\alpha}{\mathrm{d}\tau}\frac{\mathrm{d}x^\beta}{\mathrm{d}\tau}=0\quad(\mu=0,1,2,3)\tag{23}$$

which has generalized and unified Newton's law of universal gravitation in the gravitational-acceleration form and Einstein's motion equation in general relativity.

For the details of the theory of GOR, please refer to the 2nd volume GOR of the Monograph of OR [1,4].

In ORQ, the theory of OR, including IOR and GOR, has already formed a complete theoretical system, and has generalized and unified Newton's classical mechanics and Einstein's relativity theory within the same theoretical system under the same axiom system.

6.2 OQ: The Theory of Observational Quantum

The theory of ORQ, based on the definition of OR time and the wave-like form of OR spacetime-transformation factor $\Gamma=f/f_o$ from the invariance of time-frequency ratio, has deduced the theory of OQ of the general observation agent OA($\eta|h_\eta$). The theory of OQ has extended Planck's equation $E=hf$ and de Broglie's theory of matter waves from the optical agent OA($c|h$) to the general observation agent OA($\eta|h_\eta$), marching towards the direction of generalizing and even reshaping quantum physics.

The basic theorems and relations of OQ include

(1) The theorem of the invariance of time-frequency ratio: $\mathrm{d}t_\eta/f_\eta=\mathrm{d}\tau/f_o$;

(2) The theorem of the invariance of mass-frequency ratio: $m_\eta/f_\eta=m_o/f_o$;

(3) The theorem of the invariance of energy-frequency ratio: $E_\eta/f_\eta=E_o/f_o$;

(4) The general frequency-speed relationship: $f=f_o/\sqrt{(1-v^2/\eta^2)}$;

(5) The general Planck equation: $E=h_\eta f$;

(6) The general de Broglie relation: $p=h_\eta/\lambda$;

(7) The speed relations of the general de Broglie wave: $v=v_g=\eta^2/v_pv_g$;

(8) The general Schrödinger equation: $\mathrm{i}\hbar_\eta\partial\Psi/\partial t=-(\hbar_\eta^2/2m)\nabla^2\Psi+V\Psi$;

(9) The principle of general uncertainty: $\sigma_x\sigma_p\geq\hbar_\eta/2$; and

(10) The ORQ identity $h_\eta\eta\equiv C\ (C=hc)$.

Now, the theory of OQ has already formed a basic theoretical framework.

Just as the Planck equation $E=hf$ represents the birth of quantum physics as its first cornerstone, the general Planck equation $E=h_\eta f$ represents the birth of the theory of OQ as its first cornerstone. And moreover, the general Planck equation $E=h_\eta f$ has generalized the Planck equation $E=hf$; the OQ matter-wave theory, or called the general de Broglie wave, has generalized de Broglie waves; the general Schrödinger equation has generalized the Schrödinger equation; the principle of general uncertainty has generalized Heisenberg's uncertainty principle.

For the details of the theory of OQ, please refer to Chapter 6 in the Monograph of OR [1,4].

6.2 $E=m\eta^2=h_\eta f$: The Unity of Planck and Einstein

The theory of ORQ, including OR and OQ, is not only on relativity theory, but also of quantum theory: the theory of OR represents the unity of Newton and Einstein; the theory of OQ represents the unity of Planck and Einstein.

Actually, in the theory of ORQ, it implies the unification of relativity theory and quantum theory that the general Einstein formula $E=m\eta^2$ is derived by the theory of OR and the general Planck equation $E=h_\eta f$ is derived by the theory of OQ; it implies the unity of Planck and Einstein that Einstein's formula $E=mc^2$ is generalized by the general Einstein formula $E=m\eta^2$ and Planck's equation $E=hf$ is generalized by the general Planck equation $E=h_\eta f$.

As stated in Sec. 5, although derived from different logical premises and belonging to different theoretical systems, Einstein's equation $E=mc^2$ and Planck's equation $E=hf$ are a pair of dual formulae: both represent the kinetic energy of the informons (photons) of the optical agent OA($c|h$): $E=mc^2=hf$.

Now, based on the same axiom system, the theory of OR has extended Einstein's formula $E=mc^2$ from the optical agent OA($c|h$) to the general observation agent

OA($\eta|h_\eta$), and the theory of OQ has extended Planck's equation $E=hf$ from the optical agent OA($c|h$) to the general observation agent OA($\eta|h_\eta$). Thus, the general Einstein formula $E=m\eta^2$ and the general Planck equation $E=h_\eta f$ have become a pair of dual formulae in the same theoretical system: both represent the kinetic energy of the informons of the general observation agent OA($\eta|h_\eta$): $E=m\eta^2=h_\eta f$.

Thereby, the theory of ORQ has revealed the logical rationality of Einstein's formula $E=mc^2$ and Planck's equation $E=hf$ as a pair of dual formulae in physics.

In fact, it is exactly because both represent the kinetic energy of the informons (photons) of the optical agent OA($c|h$) that Einstein's formula $E=mc^2$ becomes the iconic formula of Einstein's theory of relativity, and Planck's equation $E=hf$ became the iconic formula of quantum physics based on the Planck energy-quantum hypothesis $\varepsilon=hf$. Following the same logic, as the kinetic energy of the informons of the general observation agent OA($\eta|h_\eta$), the general Einstein equation $E=m\eta^2$ can be regarded as the hallmark of the theory of OR, and the general Planck equation $E=h_\eta f$ can be regarded as the hallmark of the theory of OQ.

Therefore, in order to clarify the unification of relativity theory and quantum theory in the theory of ORQ, Sec. 4 specifically states how the theory of OR deduces the general Einstein formula $E=m\eta^2$ generalizing Einstein's formula $E=mc^2$, and Sec. 5 specifically states how the theory of OQ deduces the general Planck equation $E=h_\eta f$ generalizing Planck's equation $E=hf$. In this way, this article tries clarifies that, based on the same axiom system, the theory of ORQ not only deduces the theory of OR, but also deduces the theory of OQ.

Now, the theory of ORQ has not only unified Newton and Einstein in the theory of OR, but also unified Planck and Einstein in the theory of OQ, marching towards the unification of relativity theory and quantum theory.

7 The ORQ Identity: $h_\eta\eta\equiv C$

For the general Planck equation $E=h_\eta f$ and general Planck constant h_η, the theory of ORQ has established a significant identical equation: $h_\eta\eta\equiv hc$, called the ORQ Identity, where h_η is the general Planck constant and η is the speed of information wave of the general observation agent OA($\eta|h_\eta$) ($0<\eta<\infty$; $h_\eta>0$); h is the Planck constant and c is the speed of light in vacuum.

The ORQ Identity $h_\eta\eta\equiv hc$ is a heuristic physical model, which first appeared in a preprint of viXra [10] in 2017 and China Science Paper Online [35] in 2018.

7.1 From Bohr's Correspondence Principle to the ORQ Identity

It should be pointed out that the ORQ Identity $h_\eta\eta\equiv hc$ is the mathematical formalization of the Bohr correspondence principle.

Bohr believed that there must be some inherent logical corresponding relationship between quantum mechanics and classical mechanics, and under certain conditions, both could be transformed into each other. Based on this fundamental idea, Bohr established the principle of correspondence that can be simply expressed as follows.

Bohr's correspondence principle: If the Planck constant $h\to 0$, then a quantum physical model must converge to its corresponding classical physical model.

It was based on his correspondence principle that Bohr established his hydrogen atom model [37-39].

According to the theory of OQ, the constant $h_\eta=h(\eta)$ in the general Planck equation $E=h_\eta f$ is the energy-frequency ratio of the informons of the general observation agent OA($\eta|h_\eta$), which depends on OA($\eta|h_\eta$): different observation agents have different informons with different energy-frequency ratios or different Planck constants. The Planck constant $h=h(c)$ is just a special case of the general Planck constant $h_\eta=h(\eta)$, referring to the energy-frequency ratio of the informons (photons) of the optical agent OA($c|h$), that is, an electromagnetic-interaction constant, not the so-called cosmological constant.

So, what value should be taken for the general Planck constant $h_\eta=h(\eta)$ in the general Planck equation $E=h_\eta f$.

According to the theory of ORQ and Bohr's correspondence principle, it can be seen that, in Newton's classical case, the information-wave speed η of the idealized agent OA($\infty|0$) is idealized as infinity: $\eta\to\infty$, while the energy-frequency ratio h_η of the informons is idealized as zero: $h_\eta\to 0$; and that, in Planck's quantum case, the information-wave speed η of the optical agent OA($c|h$) is the speed of light: $\eta=c$, and the energy-frequency ratio h_η of the informons (photons) is the Planck constant:: $h_\eta=h$.

In summary, it should hold true

$$h_\eta \to 0 \text{ if } \eta \to \infty \quad \text{and} \quad h_\eta = h \text{ if } \eta = c \tag{24}$$

Following the principle of simplicity, the simplest mathematical relation for "x→0 if y→∞" should be the inverse-proportional function: xy=k (k is a constant).

Thus, Eq. (24) can be expressed as an identity:

$$h_\eta\eta \equiv C \quad (C = hc) \quad \begin{cases} h_\eta \to 0 & \text{if} \quad \eta \to \infty \\ h_\eta \to h & \text{if} \quad \eta \to c \end{cases} \tag{25}$$

where, according to "$h_\eta=h$ if $\eta=c$" in Eq. (24), $C\equiv hc$.

This is the ORQ identity $h_\eta\eta\equiv hc$, which goes beyond Bohr's correspondence principle and extends it from the optical agent OA($c|h$) to the general observation agent OA($\eta|h_\eta$).

7.2 The ORQ Identity and Electromagnetic Fine-Structure Constant

Physics has a dimensionless constant called the fine-structure constant:

$$\alpha = \frac{v_1}{c} = \frac{e^2}{2\varepsilon_0 hc} = \frac{e^2}{4\pi\varepsilon_0 \hbar c} = \frac{1}{137.03599976} \tag{26}$$

where e is the electron charge, v_1 is the Bohr first-orbital speed of the electron, ε_0 is the vacuum dielectric constant, and $\hbar=h/2\pi$ is the reduced Planck constant.

As is well known, the electromagnetic fine-structure constant α is an extremely important constant in physics, defined as the ratio of the Bohr first-orbital speed v_1 to the speed of light c, which is in a certain sense a measurement of the coupling strength between charges in electromagnetic interaction, representing the strength of electromagnetic interaction. According to Sec. 7.1, the ORQ identity $h_\eta\eta\equiv hc$ is a heuristic model based on the principles of simplicity and Bohr correspondence. It perhaps implies a certain logical or physical sense that the constant $C=hc$ of ORQ identity appears in the electromagnetic fine-structure constant α (Eq. (26)). Based on Eq. (26), the logical validity of the ORQ identity $h_\eta\eta\equiv hc$ can be confirmed by analogy between electromagnetic coupling strength and gravitational coupling strength.

First, consider the case where the electron m_e of a hydrogen atom orbits around its nucleus m_H on an orbit with a radius of r, and the linear-speed under the action of electromagnetic force F_E is v_E. Second, suppose that the gravitational force F_G replaces the electromagnetic force F_E, and the linear-speed of the electron m_e on the orbit with the radius of r is v_G. Then the corresponding electromagnetic force F_E and gravitational force F_G are

$$\begin{cases} F_E = K\dfrac{e^2}{r^2} = \dfrac{e^2}{4\pi\varepsilon_0 r^2} = m_e\dfrac{v_E^2}{r} \\ F_G = \dfrac{Gm_H m_e}{r^2} = m_e\dfrac{v_G^2}{r} \end{cases} \tag{27}$$

where K is the electrostatic constant and G is the gravitational constant.

Let the gravitational-radiation speed be κ. According to Laplace's calculation [40], κ is much greater than the speed of light c: $\kappa>7\times10^6c$. Following the logic of Bohr deducing hydrogen atom model, based on the general Planck equation $E=h_\eta f$, the orbital speed v_G of electron m_e under the gravitational force F_G of nuclear m_H can be calculated by

$$v_G = Gm_H m_e/\hbar_\kappa \quad \left(\hbar_\kappa = {}^{h_\kappa}\!/_{2\pi}\right) \tag{28}$$

where h_κ is the general Planck constant of the gravitational observation agent OA($\kappa|h_\kappa$), and $\hbar_\kappa=h_\kappa/2\pi$ is the reduced value of h_κ.

Bohr defined the strength of electromagnetic force as $\alpha_E=v_E/c$; by analogizing Bohr's logic, the strength of gravitational force can be defined as $\alpha_G=v_G/\kappa$. Then, on the orbit with the radius of r, it holds that

$$\frac{\alpha_G}{\alpha_E} = \frac{F_G}{F_E} = \frac{4\pi\varepsilon_0 Gm_H m_e}{e^2} \tag{29}$$

By substituting $\alpha_E=\alpha$ into Eq. (29), the gravitational-radiation speed κ is

$$\begin{aligned}\kappa &= \frac{v_G}{\alpha_G} = \frac{v_G F_E}{\alpha_E F_G} \\ &= \frac{Gm_H m_e}{\hbar_\kappa}\frac{4\pi\varepsilon_0\hbar c}{e^2}\frac{e^2}{4\pi\varepsilon_0 Gm_H m_e} = \frac{\hbar c}{\hbar_\kappa} = \frac{hc}{h_\kappa}\end{aligned} \tag{30}$$

This suggests that, for the gravitational observation agent OA($\kappa|h_\kappa$), it holds that

$$h_\kappa\kappa = hc \quad \left(\eta = \kappa \mid h_\eta = h_\kappa\right) \tag{31}$$

Equations (30) and (31) do not mean that the ORQ identity $h_\eta\eta\equiv hc$ has been theoretically rigorously proven. However, by making use of the electromagnetic fine-structure constant α and the gravitational observation agent OA($\kappa|h_\kappa$), Eq. (31) indeed confirms the logical or theoretical validity of the ORQ identity $h_\eta\eta\equiv hc$.

One thing is certain: As the mathematical formalization of Bohr's correspondence principle of $\eta\to c$ if $h_\eta\to h$ and $\eta\to\infty$ if $h_\eta\to0$, the ORQ identity $h_\eta\eta\equiv hc$ and Bohr's correspondence principle are logically consistent, and their rationality and validity must be equivalent.

7.3 The Implication of the ORQ Identity $h_\eta\eta\equiv hc$

The ORQ identity $h_\eta\eta\equiv hc$ is an extension of the general Planck equation $E=h_\eta f$ in the theory of OQ. It is the formalization of Bohr's correspondence principle, bringing new and significant insights to physics.

(1) On the Essence of Quantum Phenomena

In the physical world, there must be inherent quantum effects in matter motion and matter interaction, including wave-like nature and uncertainty.

However, according to the ORQ identity $h_\eta\eta\equiv hc$, different observation agents must have different information waves with different speeds and different informons with different energy-frequency ratios, and therefore, must exert different degrees of perturbation on the same observed object P, making it exhibit different degrees of quantum effects or quantum phenomena. Accordingly, the quantum effects exhibited in the theory of OQ, including wave-like nature and uncertainty, depend on the observation agent OA($\eta|h_\eta$) of OQ, being the observational effects caused by the observational perturbation of OA($\eta|h_\eta$), rather than the inherent quantum effects and uncertainties of matter motion or interactions in the physical world.

The quantum mechanics based on Planck's quantum hypothesis $\varepsilon=hf$ is just a special case of the theory of OQ, in which quantum effects, including wave-like nature and uncertainty, depend on the optical agent OA($c|h$) ($\eta=c$; $h_\eta=h$), being sorts of observational effects caused by the observational perturbation of OA($c|h$), rather than the inherent quantum effects and uncertainty of matter motion or interactions in the physical world.

According to the ORQ identity $h_\eta\eta\equiv hc$: the energy-frequency ratio h_η of the informons (photons) of Planck's optical agent OA($c|h$) is exactly the Planck constant h; $h_\eta\to h$ is exactly the case of Planck's quantum physics in Bohr's correspondence principle, where photons have momentum ($p=h/\lambda>0$) and must perturb the observed object P. Therefore, Planck's optical agent OA($c|h$) have the observational perturbation ($h>0$) and makes observed objects exhibit quantum effects.

In fact, Newton's classical mechanics is also a special case of the theory of OQ.

According to the ORQ identity $h_\eta\eta\equiv hc$: the energy-frequency ratio h_η of the informons of Newton's idealized agent OA($\infty|0$) is idealized as zero; $h_\eta\to0$ is exactly the case of Newton's classical physics in Bohr's correspondence principle, where the idealized informons has no momentum ($p=h_\eta/\lambda=0$). Therefore, Newton's idealized agent OA($c|h$) has no the observational perturbation ($h_\eta=0$).

And therefore, Newtonian mechanics has not involved quantum effects or quantum phenomena.

(2) The Unification of Relativity Theory and Quantum Theory

It is worth noting that the $h_\eta\eta$ in the ORQ identity $h_\eta\eta\equiv hc$ is the product of the eigenvalues η and h_η of the general observational agent OA(($\eta|h_\eta$), in which the information-wave speed η represents the theory of observational relativity (OR), and the general Planck constant h_η represents the theory of observational quantum (OQ); and that the hc in the ORQ identity $h_\eta\eta\equiv hc$ is the product of the eigenvalues c and h of the optical agent OA(($c|h$), in which the light speed c represents Einstein relativity theory, and the Planck constant h represents Planck's quantum theory. So, the ORQ identity $h_\eta\eta\equiv hc$ has demonstrated the unification of relativity theory and quantum theory in the theory of ORQ, that is, the unity of Planck and Einstein.

Conclusion

The theory of ORQ, starting from the definition of OR time as first principle, not only has deduced the theory of observational relativity (OR), but also has deduced the theory of observational quantum (OQ). As a theory of the general observation agent OA($\eta|h_\eta$) ($0<\eta<\infty$; $h_\eta>0$), the theory of ORQ, including OR and OQ, has had a broad perspective bring significant discoveries and new insights to physics.

The OR serial reports attempt to interpret the theory of ORQ for physicists and readers.

In ORQ, the theory of OR has formed a complete theoretical system, generalizing and unifying Newtonian mechanics and Einstein's theory of relativity.

The OR serial reports 1-5 [5-9] focused on the significant discoveries of the theory of OR: the speed of light is not really invariant and spacetime is not really curved [5]; the rest mass of photons is not really zero [6]; Einstein's prediction on gravitational waves was a mistake [7]; Einstein's prediction on the precession of Mercury's perihelion is not the objective and real precession of Mercury's perihelion [8]; The universe may not have really originated from a Big Bang [9].

Starting from this article, the focus of the OR serial report is going to turn to the theory of OQ.

In ORQ, the theory of OQ has formed a basic theoretical framework, including the general Planck equation $E=h_\eta f$, de Broglie relation $p=h_\eta/\lambda$, matter-wave theory, Schrödinger equation $i\hbar_\eta\partial\Psi/\partial t=-(\hbar_\eta^2/2m)\nabla^2\Psi+V\Psi$, uncertainty principle $\sigma_x\sigma_p\geq\hbar_\eta/2$, and ORQ identity $h_\eta\eta\equiv C$ ($C=hc$). The subsequent OR serial reports will successively report significant discoveries and new insights of the theory of OQ.

The theory of OQ implies that quantum theory and relativity theory will be unified within the same theoretical system under the same axiom of ORQ.

In order to clarify the unification of relativity theory and quantum theory within the theory of ORQ, Sec. 4 of this article specifically stated how the theory of OR deduces the general Einstein formula $E=m\eta^2$, generalizes the Einstein formula $E=mc^2$, revealing the essence of the relationship between mass and energy; Sec. 5 of this article specifically stated how the theory of OQ deduces the general Planck equation $E=h_\eta f$, generalizes the Planck equation $E=hf$, revealing the essence of the Planck constant h.

As stated in Sec. 4, the Einstein formula $E=mc^2$ is just a special case of the general Einstein formula $E=m\eta^2$, and the mass-energy E and rest-energy E_o are only the optical illusions and apparent phenomena presented to observers by the optical agent OA($c|h$), rather than the objective and real material energy.

As stated in Sec. 5, the Planck equation $E=hf$ is just a special case of the general Planck equation $E=h_\eta f$, and the Planck constant h is just a special case of the general Planck constant h_η, that is, the energy-frequency ratio of the informons (photons) of the optical agent OA($c|h$), not a cosmological constant. In the theory of OQ, the Planck equation $E=hf$ is no longer a hypothesis, but a logical consequence of the theory of OQ. The theory of OQ tells us that the quantum effects presented in the quantum physics based on Planck's energy-quantum hypothesis of $\varepsilon=hf$, including quantum mechanics and quantum-field theory, are observational effects caused by the perturbation of the optical agent OA($c|h$), rather than the inherent quantum effects and uncertainties in the physical world.

In fact, although deduced from different logical premises and belonging to different theoretical systems, Einstein's formula $E=mc^2$ and Planck's equation $E=hf$ are a pair of dual formulas, both of which represent the kinetic energy of the informons (photons) of the optical agent OA($c|h$): $E=mc^2=hf$. This suggests that there must be some intrinsic logical connection between Einstein's formula $E=mc^2$ and Planck's equation $E=hf$, and even that between relativity theory and quantum theory.

Now, based on the same axiom system, the theory of OR has extended Einstein's formula $E=mc^2$ from the optical agent OA($c|h$) to the general observation agent OA($\eta|h_\eta$), and the theory of OQ has extended Planck's equation $E=hf$ from the optical agent OA($c|h$) to the general observational agent OA($\eta|h_\eta$). Thus, the general Einstein formula $E=m\eta^2$ and the general Planck equation $E=h_\eta f$ become a pair of dual formulas under the same axiom system, both of which represent the kinetic energy of the informons of the general observation agent OA($\eta|h_\eta$): $E=m\eta^2=h_\eta f$. In this way, the theory of ORQ has revealed the intrinsic logical connection between Einstein's formula $E=mc^2$ and Planck's equation $E=hf$.

As a new theory in mankind's physics, the theory of ORQ not only has unified Newton and Einstein in the theory of OR, but also has unified Planck and Einstein in the theory of OQ. As a microcosm, the unification of Einstein's formula $E=mc^2$ and Planck's equation $E=hf$ within the theory of ORQ indicates that the theory of ORQ is marching towards the unification of relativity theory and quantum theory.

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