Singularity Projection in Three-Dimensional Space and the Discretization of a Continuous Interval

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Abstract

We study the projection geometry induced by a singular point on the boundary of a three-dimensional fan-shaped cylinder. The cylinder is defined in the first quadrant of the real coordinate plane, with x and y both between zero and one, and a singular point located at x=1 on the boundary. Through an iterative projection process involving the angle bisector of the quadrant, we prove that **the continuous interval [0,1] is precisely discretized to an infinite set of boundary points**.

Keywords: projection geometry, singular point, discretization, fan-shaped cylinder, boundary points
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1. Introduction

The study of projection structures in three-dimensional real space has applications in various areas of geometry and analysis. In this paper, we consider a special class of domains defined by the intersection of a fan-shaped cylinder and a singular boundary point.
The cylinder is defined in the first quadrant of the real coordinate plane by: $0 \leq x < 1, 0 \leq y < 1$
A singular point is located at s=1[1]on the boundary of this domain. We study the iterative projection of this singular point onto the coordinate axes and the angle bisector of the quadrant.
The main result is that the continuous interval [0,1] [1]**, when subjected to this projection geometry, is precisely discretized to an infinite set of boundary points:**

$$B = \left\{ \sqrt{2}^{-n} \,\middle|\, n = 1,2,3, \dots \right\} \cup \{0\}$$

This is not a numerical approximation, nor a truncation. It is an **exact geometric compression**: the interval is not continuous in the sense of structural candidates; rather, all the set of points invariant under the full iterative projection are confined to the discrete set B. The proof is fully elementary and self-contained. It uses only orthogonal projections, similarity of triangles, and infinite descent[2].
The significance of this result extends beyond the specific construction. The discretization of a continuous interval, under a purely geometric projection, challenges a foundational assumption that underlies large parts of analytic number theory. Many classical results in this field rely on the continuity of the domain—typically the interval [0,1] or the critical strip[1]—as a structural given. They treat this continuity as inherent to the domain itself.
What we show is that continuity is not an intrinsic property of the interval, but a property that depends on the geometric framework in which the interval is embedded. When subjected to the projection geometry of a singular point, the interval is precisely reduced to a discrete skeleton. This suggests that the continuity that analytic number theory has long relied upon may be an artifact of the analytic framework, not a structural necessity.

The present paper does not engage in the analytic framework of zeta functions or L-functions. It stays entirely within elementary geometry. Yet the result it establishes has direct bearing on the very notion of domain continuity that underpins that framework. This is not a proof of a conjecture; it is a structural intervention. If correct, it forces a re-examination of the domain assumptions that have been taken for granted in analytic number theory since its inception.

2. Geometric Setup

We work in three-dimensional real space with coordinates (x, y, z). The *x*-axis and y-axis span the real coordinate plane. The z-axis is an auxiliary dimension not involved in the projection process.

Definition 2.1 (Fan-shaped cylinder Fig1). *Let the fan-shaped cylinder be defined by:*
$0 \leq x < 1, 0 \leq y < 1, -\infty < z < \infty.$

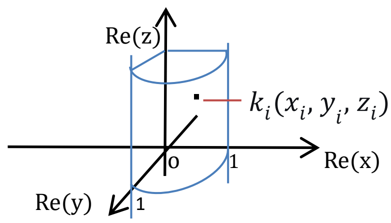

Fig1

The first quadrant of the xy-plane serves as the base of the cylinder. The point s=1 on the x-axis is a singular boundary point.

Definition 2.2 (Angle bisector). *In the first quadrant of the xoy-plane, the angle bisector is the line* $y = x$*, which makes an angle* $\frac{\pi}{4}$ *with the x-axis.*

Definition 2.3 (Quarter-circle). *Draw a quarter-circle of radius 11 centered at the origin, connecting the points (1,0) and (0,1). The intersection of this quarter-circle with the angle bisector is the point:*

$$k_0 = (\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$$

Definition 2.2 and Definition 2.3 See Fig 2

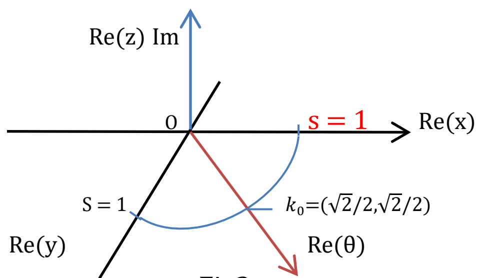

Fig2

3. Iterative Projection and the Discretization Mechanism

3.1 First Projection

Project the point k_0 orthogonally onto the x-axis. Its image is:

on the x-axis. $k_1 = \frac{\sqrt{2}}{2}$ on the x-axis.

Observation. The segment from k_1 to the singular point s=1 on the x-axis is the projection of the arc of the quarter-circle. We call this segment a **zero-free region** (ZFR)[3], as it contains no points that can be reached by further projections from the singular point.

3.2 Projection onto the Angle Bisector

The same geometric obstruction applies to the angle bisector. The segment on the bisector from k_0 to the point at radius 1 (i.e., from $(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2})$ to (1,1) is also a ZFR.

Project the endpoint of this ZFR orthogonally onto the x-axis. Because the bisector makes an angle $\pi/4$ with the x-axis, the projection reduces the length by a factor $\cos\frac{\pi}{4}=\frac{\sqrt{2}}{2}$ Thus:

$k_3=\frac{\sqrt{2}}{2}\times\frac{\sqrt{2}}{2}=\frac{1}{2}$. **Fig**3:

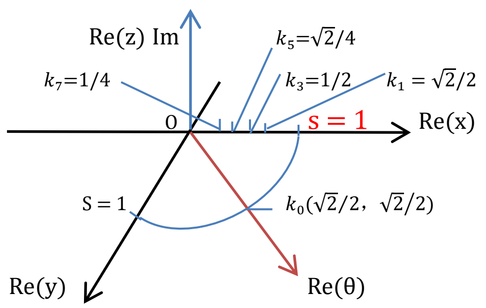

Fig3

3.3 Inductive Step

The segment $(\frac{1}{2},\frac{\sqrt{2}}{2})$ on the x-axis becomes a new ZFR. Projecting its right endpoint back onto the bisector and then onto the x-axis again gives:

$$k_5=\frac{1}{2}\times\frac{\sqrt{2}}{2}=\frac{\sqrt{2}}{4}$$

Repeating the process:

$$k_7=\frac{\sqrt{2}}{4}\times\frac{\sqrt{2}}{2}=\frac{1}{4},k_9=\frac{1}{4}\times\frac{\sqrt{2}}{2}=\frac{\sqrt{2}}{8}\ldots$$

4. Main Result: Exact Discretization of the Interval

Theorem. The iterative projection process generates an infinite set of boundary points in the interval [0,1]:

$$\mathrm{B}=\left\{\sqrt{2}^{-n}\ \middle|\ \mathrm{n}=1,2,3,\ldots\right\}\cup\{0\}$$

Consequently, any meaningful position in the interval must belong to this set.

Proof.

The first projection gives $k_1=\sqrt{2}/2$ *. Subsequent projections alternate between the x-axis and the angle bisector. Each projection from the bisector to the x-axis reduces the length by a factor .* $\sqrt{2}/2$

Thus the sequence of boundary points is:

$$b_1 = \frac{\sqrt{2}}{2}, b_2 = \frac{1}{2}, b_3 = \frac{\sqrt{2}}{4}, b_4 = \frac{1}{4}, b_5 = \frac{\sqrt{2}}{8}, b_6 = \frac{1}{8} \dots$$

In closed form, for n=1,2,3,... $b_n = \sqrt{2}^{-n}$
The sequence converges to 0, which is the limit point of the set.

Hence $B = \left\{ \sqrt{2}^{-n} \mid n = 1,2,3, \dots \right\} \cup \{0\}$. *Since every point in the interval that is not in B lies inside a zero-free region and is excluded by the projection, the interval [0,1] is* ***exactly discretized*** *to B.* □
The point0 is included as the limit point of the sequence; it is not generated by a finite number of iterations.

5. Discussion: Continuous Interval as a Discrete Skeleton

The geometric construction presented here reveals a structural phenomenon: **a continuous interval, when subjected to the projection geometry induced by a singular point, is precisely compressed to a discrete skeleton.**
This is not an approximation or a truncation. The discretization is exact and is entirely determined by the geometry of the fan-shaped cylinder, the singular boundary point, and the iterative projection through the angle bisector.
The set B is not just a set of points; it is the complete set of structural positions in the interval. Any point that is not in B lies inside a zero-free region and cannot carry any structure inherited from the singular point. Therefore, the projection geometry forces a **discrete structure** on an interval that is normally considered continuous.

6. Conclusion

The results presented here suggest that **the continuity of the interval is not a structural given, but a framework-dependent property.** When the framework is changed—when the interval is embedded in a projection geometry induced by a singular point—it dissolves into a discrete skeleton. This is a purely geometric fact, but its implications reach beyond geometry, into the foundations of analytic number theory.
We have shown that a continuous interval, under the projection geometry of a fan-shaped cylinder and its singular boundary point, is **precisely reduced to a discrete skeleton**:

$$\mathrm{B} = \left\{ \sqrt{2}^{-n} \mid \mathrm{n} = 1,2,3, \dots \right\} \cup \{0\}$$

“The significance of this result extends beyond the specific construction. The discretization of a continuous interval under a purely geometric projection raises a foundational question: is continuity an intrinsic property of a domain, or does it depend on the geometric framework in which the domain is embedded? Our construction shows that, under the projection geometry of a singular point, a continuous interval is precisely reduced to a discrete skeleton. This suggests that continuity, often taken as a structural given in analysis, may in some contexts be an artifact of the analytic framework rather than a necessity.”

1. The authors declare that there are no conflicts of interest regarding the publication of this paper.

2. All references cited in this paper are from publicly available online sources.

References

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[3] Bohrium SciencePedia. Nontrivial Zeros: *The Key to the Music of the Primes [EB/OL]. Beijing: Bohrium, n.d.***Proof completed**