

Modernized and Significantly Improved Kepler's Laws

Pavlo Danylchenko* 

Research and Production Enterprise "GeoSystem", Vinnytsia, Ukraine

(Received 29 June 2026; accepted 00 July 2026; published 00 July 2026)

It is substantiated that the Solar System corresponds not to a static Newtonian gravitational field, but to a dynamic gravitational field that is also formed by planetary motion and therefore adapts to changes in the positions of the planets within it. It is because of this that gravitational pseudo-forces, just like centrifugal pseudo-forces of inertia, are orthogonal to the superharmonic quasi-circular orbits of the planets at every point. The correspondence of the superharmonic trajectories of the planets' motion exclusively to the laws of conservation of angular momentum and energies (the Newtonian of inert free rest energy and the Keplerian of ordinary rest energy of matter) is proven. The inconsistency of two-focus elliptical orbits with the law of conservation of angular momentum is substantiated. Equations for superharmonic single-focus quasi-circular (asymmetric oval) planetary orbits have been derived, consistent with both the laws of conservation and astronomical observations. Modernized and substantially improved versions of Kepler's laws have been formulated.

Keywords: Theory of Relativity, Newtonian, Keplerian, Adaptive-Dynamic Gravitational Field, Solar System.

I. INTRODUCTION

The formulations of the laws of classical physics are overly simplified and too straightforward. After all, they, as a rule, do not take into account the presence of negative feedbacks between the physical parameters of matter. But it is precisely these negative feedbacks that ensure the stability of natural formations.

Such simplification was also not avoided in the formulation of the differential equations of the gravitational field of the General Relativity (GR). Therefore, these equations should be taken only as a basis and creatively reinterpreted and supplemented with additional dependencies. After all, the GR equations themselves do not directly take into account the conservation of angular momentum and the corresponding effects of the motion of matter on the gravitational field due to the presence of negative or positive feedbacks. Einstein himself compared the metric tensor of these equations with good-quality marble, and the energy-momentum tensor only with low-quality wood [1]. And indeed, as it turned out much later, the use of ordinary Lorentz transformations of increments of spatial coordinates and time (OLT) in these equations came into conflict with the relativistic and gravitational invariance of thermodynamic parameters and potentials of matter [2-4].

Thus, the equations of the GR did not ensure the absence of the influence of the external gravitational field and motion, both on the

thermodynamic state of matter and on the rate of flow of its proper time. After all, the compensation of the change in the value of the gravitational parameter b (the Schwarzschild solution of the GR gravitational field equations [5]) by the motion of matter by inertia ($b_{cj} = b_i + {}^i v_j^2 c^{-2} = b_{0j} = \text{const}(r, t)$)

ensures the correspondence to reality of the analogous equations of relativistic gravithermodynamics (RGTD, which is only an improved version of the GR [2-4]). Only according to them, the strength of the gravitational field is directly equal at point i to the gravitational acceleration of the common velocity ${}^i v_j = d{}^i \vec{r}_j / dt$ of motion of all substances m , which may have unequal values of their hidden thermodynamic parameters ${}^m b_{0j} = {}^m b_{00} {}^m T_{00}^2 / {}^m T_j^2$ dependent on the absolute temperature T [2-4,6,7]:

$$\begin{aligned} {}^i \hat{g}_j &= -b_i c^2 \frac{d \ln b_i}{2 d \vec{r}_i} = -c^2 \frac{d(b_{cj} - {}^i v_j^2 c^{-2})}{2 d \vec{r}_i} = {}^i v_j \frac{d {}^i v_j}{d \vec{r}_i} = \frac{d {}^i v_j}{dt}, \\ {}^i \hat{g}_j &= -c^2 \frac{d(\ln b_{cj})_v}{2 d \vec{r}_i} = -c^2 \frac{d[\ln(b_i + {}^i v_j^2)]_v}{2 d \vec{r}_i} = \\ &= -c^2 \frac{d(b_{cj} - {}^i v_j^2 c^{-2})}{2 b_{cj} d \vec{r}_i} = \frac{{}^i v_j}{b_{cj}} \frac{d {}^i v_j}{d \vec{r}_i} = \frac{G}{G_{00}} \frac{d {}^i v_j}{dt} \left(\frac{db_{cj}}{d \vec{r}_i} = 0 \right), \end{aligned}$$

where: ${}^i \hat{g}_j$ and ${}^i \hat{g}_j$ are gravitational field strength at point i by the observer's clock and by a hypothetically stationary clock at point i , respectively, $G = G_{00} / b_{cj}$ and G_{00} are true and standard values of the gravitational constant, respectively.

Therefore, the gravitational field really determines only the velocity of motion and spatial

*Contact author: pavlo@vingeo.com.

gradients, and not at all the values of unequal hidden parameters ${}^m b_{0j}$ in different substances m , which were previously at rest at point j , and are now moving together at point i . And thus, each substance can have its own spatial distribution of the hidden gravithermodynamic parameter ${}^m b = {}^m v_i^2 c^{-2}$, which is equivalent not only to the absolute temperature T , but also to the limit velocity v_i of individual (separate) motion of a certain substance used in the RGTD (alternative to the false coordinate velocity of light of the GR [2-6]) [7].

Based on the use of the identity of the gravitational mass $m_{gr} = m_{00} / \sqrt{b_c}$ of matter to the inertial mass $m_{inr} = m_{00} \sqrt{b_c}$ of matter only according to its intrinsic clock in the RGTD, a canonical differential equation of the dynamic gravitational field was obtained for a non-solid matter, which does not contain the spatial distribution of the mass density of the matter. Due to the definition of the spatial distribution of the local value of the gravitational radius by this equation, on the contrary, it allows to obtain the required spatial distribution of the mass density of the matter [8,9]:

$$\begin{aligned} S' &= \frac{d[r/a(1-b_c)]}{dr} = \frac{1-r'_g - \Lambda r^2}{(1-b_c)} + \\ &+ \frac{(r-r_g - \Lambda r^3/3)}{(1-b_c)^2} b' = -\frac{b_c S}{r(1-b_c)} + \frac{(1-\Lambda r^2)}{(1-b_c)^2}, \\ S &= \frac{r}{a_c(1-v^2 c^{-2}/b_c)(1-b_c)} = \frac{r-r_g - \Lambda r^3/3}{1-b_c} = \\ &= \frac{r}{a(1-b_c)} = \exp \int \frac{-b_c dr}{(1-b_c)r} \times \int \left[\frac{(1-\Lambda r^2)}{(1-b_c)^2} \exp \int \frac{b_c dr}{(1-b_c)r} \right] dr, \end{aligned}$$

where the parameter S can be abstractly considered as the distance from the pseudo-horizon of the infinitely distant cosmological past and the parameters

$$\begin{aligned} a_c &= (d\hat{r}/dr)^2 = ab_c/b = \\ &= a(b+v^2 c^{-2})/b = a/(1-v^2 c^{-2}/b_0) \quad \text{and} \\ b_c &= v_{lc}^2 c^{-2} = b + v^2 c^{-2} = b(1+v^2 v_i^{-2}) \equiv b_0 = \mathbf{const}(t) \end{aligned}$$

are analogues of the parameters $a = 1/(1-r_g/r - \Lambda r^2/3)$ and b of the Schwarzschild solution of the equations of the static gravitational field of the GR, v is the velocity of the hypothetical circular or real quasi-circular (pseudo-elliptical) orbital motion of the planets of the Solar System, $r_g(r)$ is the radial value of the

gravitational radius of the dynamic gravitational field of the Solar System, $\Lambda = 3H_E^2 c^{-2}$ is the cosmological constant, H_E is the Hubble constant, c is the constant of the velocity of light.

This is what allows us to form the proper spatial distribution of the parameter $b_c(r) = v_{lc}^2 c^{-2} = b + v^2 c^{-2}$ of the adaptive-dynamic gravitational field along the superharmonic orbits of the planets of the Solar System. That is, it allows us to obtain exactly such a distribution of this parameter that would correspond to the conservation of not only the Keplerian of ordinary rest energy of the planet, but also the conservation of the angular momentum of the planet in the process of its orbital motion. The parameter a of the gravitational field can be obtained by solving the canonical differential equation of the dynamic gravitational field based on the formed parameter b_c (similar to its obtaining for a flat galaxy [8,9]).

II. PARAMETERS OF THE MOTION OF PLANETS THAT ARE CONSISTENT WITH THE LAWS OF CONSERVATION OF ENERGY AND ANGULAR MOMENTUM

According to Kepler's laws, which are actually based on Newton's theory of gravity, it is not the Hamiltonians and the Lagrangians that are conserved (due to $v_{lc}^2 = v_i^2 + v^2 = \mathbf{const}(t, r)$) in the process of planetary motion, but the Newtonians of inert free rest energy:

$$\begin{aligned} N &= E_0 v_{lc} / c = m_{00} c v_{lc} = m_{00} c \sqrt{v_i^2 + v^2} \approx \\ &\approx m_{00} c^2 \sqrt{1-r_g/(r_1+r_2)} = \mathbf{const}(t, r) \end{aligned}$$

and the Keplerians of ordinary rest energy:

$$\begin{aligned} K &= W_0 c / v_{lc} = m_{00} c^3 / v_{lc} = m_{00} c^3 / \sqrt{v_i^2 + v^2} \approx \\ &\approx m_{00} c^2 / \sqrt{1-r_g/(r_1+r_2)} = \mathbf{const}(t, r) \end{aligned}$$

of the planetary matter [8-12]. Here r_1 and r_2 are the radii of the planet's orbit at aphelion and perihelion, respectively, and r_g is the orbital gravitational radius of the planet.

Apparently, the gravitational field is formed by converging spiral waves of space-time modulation of the dielectric and magnetic permeability of the physical vacuum, which (with the de Broglie frequency) support (ensure) the continuous existence of the Solar System [2,3,8-15]. And therefore, it is not the Sun that forms orbital

gravitational fields, but they themselves are formed in the process of forming and maintaining the existence of the Solar System.

The values of the velocities v of orbital motion of independent objects of the Solar System at aphelions and perihelions are determined by the initial conditions of their inclusion inside the Solar System, or even by the conditions of their formation directly inside the Solar System.

Based on the mutual equality of the values of all parameters at aphelion and perihelion of the planet (precisely values of the both Newtonians, values of the both Keplerians, and the values of the both Newtonian angular momenta ($m_{in} v_2 r_2 = m_{in} v_1 r_1$; $v_2 r_2 = v_1 r_1$ due to $m_{in} = \text{const}(r, t)$):

$$b_c = v_{lc}^2 c^{-2} \approx (1 - r_g / r_2) + v_1^2 r_1^2 r_2^{-2} c^{-2} = (1 - r_g / r_2) + v_2^2 c^{-2} \approx (1 - r_g / r_1) + v_1^2 c^{-2},$$

we can find the orbital gravitational radii of the planets of the Solar System:

$$r_g \approx v_1^2 c^{-2} r_1 (1 + r_1 / r_2) = v_2^2 c^{-2} r_2 (1 + r_2 / r_1).$$

But velocities v_1 and v_2 of the orbital motion of the planets at aphelion and perihelion are currently calculated on the basis of a false idea of the ellipticity of the orbits of the planets and, therefore, their calculated values may not be sufficiently reliable. And it is not reliably known how quickly the gravitational radius r_g of the Solar System increases radially due to the presence of the mass of the planets in it. Therefore, in the calculations it is advisable to use more reliable values of the radial distances of the planets at perihelion and at aphelion and to calculate not the orbital rotation period T , but a parameter that additionally takes into account the radial growth of the gravitational radius of the Solar System:

$$\zeta = T \sqrt{r_g} = 2 \sqrt{r_g} \int_0^\pi \frac{d\psi}{\omega} = 2 \kappa \int_0^\pi (r_2 / r_1)^{(\cos \psi)^n} d\psi =$$

$$= \pi (r_1 + r_2)^{3/2} / c [(\cos \psi)^n \equiv \text{sgn}(\cos \psi) |\cos \psi|^n],$$

where: ψ is the angle at which the radius $\rho = r_1 v_1 / v$ of local curvature of the orbit is directed, $\omega = v / \rho = \sqrt{\omega_1 \omega_2} (\omega_2 / \omega_1)^{(\cos \psi)^n / 2} = (\sqrt{r_g} / \kappa) (r_1 / r_2)^{(\cos \psi)^n}$ is angular velocity of the planet's orbital motion; $\kappa = \sqrt{r_1 r_2 (r_1 + r_2)} / c$;

n is the superharmonic index of the equation of planetary orbits, determined according to Kepler's third law from the integral equation:

$$2 \int_0^\pi \left(\frac{r_2}{r_1} \right)^{(\cos \psi)^n} d\psi = \frac{2 \pi a_{so}}{b_{so}} \equiv 4 \int_0^{\pi/2} \cosh[\sqrt{\chi} (\cos \psi)^n] d\psi =$$

$$= \pi \left[\sqrt{\frac{r_2}{r_1}} + \sqrt{\frac{r_1}{r_2}} \right] \equiv 2 \pi \cosh \left(\frac{\sqrt{\chi}}{2} \right),$$

$\chi = [\ln(r_2 / r_1)]^2$, $a_{so} = (r_1 + r_2) / 2$ and $b_{so} = \sqrt{r_1 r_2}$ are the semi-major and semi-minor axes of the superharmonic orbit, respectively.

Expanding the equation into a series and integrating it, we obtain:

$$\frac{2}{\pi} \sum_{k=0}^{\infty} \frac{\chi^k}{(2k)!} \int_0^{\pi/2} (\cos \psi)^{2kn} d\psi =$$

$$= \frac{1}{\sqrt{\pi}} \sum_{k=0}^{\infty} \frac{\chi^k}{(2k)!} \frac{\Gamma(kn + 1/2)}{\Gamma(kn + 1)} = \sum_{k=0}^{\infty} \frac{(\chi/4)^k}{(2k)!},$$

where: $\int_0^{\pi/2} (\cos \psi)^{2kn} d\psi = \frac{\sqrt{\pi}}{2} \frac{\Gamma(kn + 1/2)}{\Gamma(kn + 1)}.$

By equating the terms with the same powers k of the parameter χ in the two sums of expansion terms, we numerically determine the coefficients C_k of these powers k of the parameter χ in the expansion of the superharmonic exponent n :

$$n = 1 + C_1 \chi + C_2 \chi^2 + C_3 \chi^3 + C_4 \chi^4 + C_5 \chi^5 + \dots + C_1 \chi^k,$$

$$\text{where: } C_1 = 4.8368476128470179,$$

$$C_2 = 0.3962571729803806,$$

$$C_3 = 0.01363661822922468,$$

$$C_4 = 0.000296002992543841,$$

$$C_5 = 0.00000364017172797243,$$

$$C_6 = 0.00000004207706175127,$$

$$C_7 = 0.0000000000958023035.$$

According to Kepler's third law, planets (the parameters of which are obtained in the JPL SBDB and calculated according to the JPL Solar System Dynamics) have different values of the superharmonic index n .

Tables I and II show precisely those known approximate values of the orbital parameters and the calculated velocities of motion of various planets, which allowed us to obtain the calculated values of the orbital gravitational radii of the planets of the Solar System with the smallest deviations from their most probable values.

TABLE I. Parameters of the planet.

Planet	r_1 mln. km	r_2 mln. km	v_1 km/s	v_2 km/s
Mercury	69.817	46.001	38.86	58.98
Venus	108.94	107.48	34.78	35.26
Earth	152.10	147.10	29.29	30.29
Mars	249.23	206.67	21.98	26.51
Jupiter	816.00	740.68	12.45	13.72
Saturn	1507.2	1351.0	9.125	10.18
Uranus	3006,3	2735,0	6.488	7.132
Neptune	4537,0	4460,8	5.389	5.482
Pluto	7375,9	4436,8	3.682	6.121

TABLE II. Orbital periods of planets and orbital gravitational radii

Planet	T days	ζ 10^{-6} s $\text{km}^{1/2}$	r_g km
Mercury	87,97	13,0617	2,95325
Venus	224,70	33,3639	2,95335
Earth	365,26	54,2336	2,95326
Mars	686,67	102,008	2,95625
Jupiter	4331,87	643,626	2,95722
Saturn	10783,1	1601,63	2,95411
Uranus	30688.3	4558,80	2,95613
Neptune	60185.9	8942,63	2,95739
Pluto	90472.4	13454,2	2,96244

Table II shows that the calculated values of the orbital gravitational radii of Solar System planets are almost identical. But these values, although not very significantly, still increase with distance from the center of gravity due to the presence of peripheral matter in the Solar System.

And this takes place despite the neglect (in the calculations) of the presence of both a slight evolutionary weakening (Λ -reduction) of centrifugal pseudo-forces of inertia, and the influence of planets on each other. And this confirms not only the correspondence of the Newtonians and the Keplerians to these planets, but also the absence of relativistic time dilation in them.

All planets in the Solar System have $1 - b_c \gg 1 - b_{\max} = 3(\Lambda r_g^2 / 12)^{1/3} = 1.387 \cdot 10^{-15}$ ($r_{\max} = (3r_g / 2\Lambda)^{1/3}$; $\Lambda = 3H_E^2 c^{-2} = 1.35457 \cdot 10^{-52} \text{ m}^{-2}$).

This means that they did not enter the Solar System from outside, but formed directly on its outskirts. Moreover, although the planets influence

each other, they do not experience any violation of the laws of conservation of energy and angular momentum, nor any non-gravitational influence on the thermodynamic characteristics of their matter. Therefore, their motion (from the very beginning of the formation of Solar System) is formed by de Broglie high-frequency spiral waves (converging towards the center of gravity) of the space-time modulation of the dielectric and magnetic permeability of the physical vacuum.

Gravitational pseudo-forces are clearly compensated in the adaptive-dynamic gravitational field of the Solar System by centrifugal pseudo-forces of inertia [8,9]:

$$\mathbf{F}_{gr} = m_{0gr} \hat{g} = -\frac{m_{0in} c^2}{2} \frac{d \ln b}{d \rho} = -\frac{m_{0in} c^2}{2b} \times \frac{d(b_c - v^2 c^{-2})}{d \rho} = -\frac{m_{0in} c^2}{2b} \frac{d(b_c - v_1^2 c^{-2} \rho_1^2 \rho^{-2})}{d \rho} =$$

$$= -\frac{m_{0in} v_1^2 \rho_1^2}{b \rho^3} = -\frac{m_{0in} v^2}{b \rho} = -\mathbf{F}_{in},$$

$$\text{where: } \hat{g} = -\frac{L^2}{m_{in}^2 \rho^3} = -\frac{v_1^2 \rho_1^2}{\rho^3} = \frac{c^2 r_g}{2r_0 \sqrt{r_1 r_2}} \left(\frac{2r_0}{r} - 1 \right)^{3/2};$$

$$b = 1 - \frac{r_g}{r} = 1 - \frac{r_g}{r_1 + r_2} - \frac{r_1 r_2 r_g}{(r_1 + r_2) \rho^2};$$

$L = m_{in} v \rho = m_{in} \omega \rho^2 = \text{const}(r, t)$ is angular momentum; $\rho^2 = r_1 r_2 r / (r_1 + r_2 - r)$ is the square of the radius of curvature of the planet's orbit ($\rho_1 \equiv r_1$, $\rho_2 \equiv r_2$); $r_0 = (r_1 + r_2) / 2$ corresponds to the maximum value of the planet's angular pseudo-momentum $m_{in} v r$, which is equal to the angular momentum of the planet in the case of its circular rotation $(v^2 r^2 - v_0^2 r_0^2 = -c^2 r_g (r - r_0)^2 / 2r_0 \leq 0;$

$$v_0^2 r_0^2 = c^2 r_g (r_1 + r_2) / 4 > c^2 r_g r_1 r_2 / (r_1 + r_2) = v_1^2 r_1^2).$$

And thus, not only in perihelion and aphelion, but even in the hypothetical circular motion of astronomical bodies, the centrifugal pseudo-forces of inertia compensate for the gravitational pseudo-forces, which are inversely proportional not to the square, but to the cube of the distance to the center of gravity. And this, of course, is a paradox of the gravitational field. After all, it acts differently on bodies that fall freely and on bodies that move in orbits in this field.

At the same time, since:

$$b_c = \frac{v_{lc}^2}{c^2} = b + \frac{v^2}{c^2} = 1 - \frac{r_g}{r} + \frac{v^2}{c^2} =$$

$$= 1 - \frac{r_1 r_2 r_g}{2r_0 \rho^2} - \frac{r_g}{2r_0} + \frac{v^2}{c^2} = 1 - \frac{r_g}{2r_0},$$

according to the adaptive-dynamic gravitational field the squares of the true velocities of the planets:

$$\begin{aligned} v^2 &= \frac{c^2 b \rho}{2} \frac{d \ln b}{d \rho} = \frac{c^2 \rho}{2} \frac{d(1 - r_1 r_2 r_g \rho^{-2} / 2r_0)}{d \rho} = \\ &= \frac{c^2 r_1 r_2 r_g}{2r_0 \rho^2} = c^2 r_g \left(\frac{1}{r} - \frac{1}{r_1 + r_2} \right) \end{aligned}$$

significantly differ from their values in a static gravitational field:

$$\begin{aligned} v_s^2 &= (c^2 r \sqrt{ab} / 2) d \ln b / d r = \\ &= (c^2 / \sqrt{b})(r_g / 2r - \Lambda r^2 / 3) \approx c^2 r_g / 2r. \end{aligned}$$

The centrifugal pseudo-forces of inertia determined by these velocities are directed away from the local centers of curvature of the trajectory of the planet's motion and, therefore, are orthogonal to this trajectory. Therefore, the gravitational pseudo-forces, which are compensated by the centrifugal pseudo-forces of inertia, are also orthogonal to the trajectory of the planet's motion.

And it is because of this that the lines of pseudo-forces of the adaptive-dynamic gravitational field (in contrast to the static gravitational field) are curved. And this curvature of theirs is related to the rotation of the Sun about its axis together with its spiral-wave structure in the surrounding "empty" space [13-15]. That is why all the planets revolve around the Sun in exactly the same direction as the Sun rotates about its axis. Therefore, the planets revolve not in circular, but in superharmonic quasi-circular orbits precisely because they are dragged by the external spiral-wave structure of the Sun.

Thus, the superharmonic trajectory of the planet, like an ellipse, has two major semiaxes $a \equiv r_0 = (r_1 + r_2) / 2$ and two minor semiaxes $b = \sqrt{r_1 r_2}$. But unlike an ellipse, it is asymmetric along the major axis and has only one focus, which contains the Sun and is the center of curvature of trajectory not only in perihelion, but also in aphelion. In addition, the superharmonic orbit has a multitude of instant local foci (centers of gravity) located on the major axis in the centers of local curvatures of the orbit. It is to these local centers of gravity of the adaptive-dynamic gravitational field that gravitational pseudo-forces are directed, the strength of which is inversely proportional to the

cube of the distance ρ to the corresponding instantaneous center of rotation of the planet and, consequently, of its angular momentum $L = m_{in} v \rho$.

And therefore, after the planet passes perihelion, its instantaneous centers of gravity begin to gradually migrate from the center of the Solar System to the center of the planet's ecliptic ($r_{ce} = (r_1 - r_2) / 2$

when $r_0 = (r_1 + r_2) / 2$ and $\rho_0 = \sqrt{r_1 r_2}$). And then they gradually return to the center of the Solar System and reach it again at aphelion. And such their continuous migration occurs cyclically.

The superharmonic trajectory of the planet's motion in the adaptive-dynamic gravitational field is determined in polar coordinates by the radii ρ of its curvature and the angles ψ of inclination of these radii (and therefore the inclination of the lines of force of the external gravitational field that is adapted by the movement) by the following equation:

$$\rho^2 = r_1 r_2 (r_2 / r_1)^{(\cos \psi)^n},$$

where: $[(\cos \psi)^n \equiv \text{sgn}(\cos \psi) |\cos \psi|^n]$.

Accordingly, we have the parameters of the equation of the asymmetric superharmonic orbit of the planet in polar coordinates r and φ :

$$\begin{aligned} r &= \frac{r_1 + r_2}{1 + (r_1 / r_2)^{(\cos \psi)^n}}, \\ \sin \varphi &= \frac{1 + (r_1 / r_2)^{(\cos \psi)^n}}{r_1 + r_2} \sqrt{r_1 r_2 \left(\frac{r_2}{r_1} \right)^{(\cos \psi)^n}} \sin \psi = \\ &= \pm \sqrt{\frac{r_1 r_2}{r(r_1 + r_2 - r)}} \left[1 - \eta \sqrt{\left(\frac{\ln(r_1 + r_2 - r) - \ln r}{\ln r_1 - \ln r_2} \right)^2} \right], \end{aligned}$$

where: $(\cos \psi)^n \equiv \text{sgn}(\cos \psi) |\cos \psi|^n$,

$$|\cos \psi| = \eta \sqrt{\left| \frac{\ln(r_1 + r_2 - r) - \ln r}{\ln r_1 - \ln r_2} \right|},$$

$$\sin \psi = \text{sgn}(\sin \varphi) \sqrt{1 - \eta \sqrt{\left(\frac{\ln(r_1 + r_2 - r) - \ln r}{\ln r_1 - \ln r_2} \right)^2}};$$

$\cos \psi < 0$ when $x < x_b$ and $\cos \psi > 0$ when $x > x_b$ (the cosines of the angles ψ and φ have different signs only when $x_b < x < 0$).

III. LAWS OF ORBITAL MOTION OF ASTRONOMICAL BODIES

FIRST LAW

Astronomical bodies move in quasi-stationary superharmonic orbits that are asymmetric with respect to the minor axis and have only one focus, at which the much more massive astronomical body is located. The length of the semi-major axis of the orbit is the arithmetic mean of the radial distances to the bodies at perihelion (pericenter) and aphelion (apocenter), whereas the length of the semi-minor axis is the geometric mean of these distances. All centers of local curvatures of the orbit lie on the major axis between the focus and the point of intersection of the axes. At the same time, the centers of curvature of only the perihelion and aphelion are located precisely at the focus, whereas the centers of curvature of only the points of the orbit lying on the minor axis are located at the point of intersection of the axes.

SECOND LAW (LAW OF AREAS)

The radius of curvature of the trajectory is directed along the normal to the trajectory and sweeps out nearly equal areas in equal intervals of time, which is equivalent to the conservation of the planet's angular momentum. This is due to the orthogonality of the gravitational pseudo-forces to the planetary trajectories and, consequently, to the curvature of the lines of force of the gravitational field of the Sun, which rotates about its axis in the same direction as the planets orbit around it.

THIRD LAW

The product of the orbital value of the gravitational radius, the square of the constant of velocity of light, and the square of the orbital period of an astronomical body is equal to the product of the number π and the cube of the major axis of the superharmonic orbit of the astronomical body. This is precisely what makes it possible to determine the exact orbital value of the gravitational radius, which increases with distance from the center of gravity both because the true value of the gravitational coefficient ("constant") $G = G_0/b$ is inversely proportional to the parameter b of the Schwarzschild solution of the gravitational field equations and because of the increase in peripheral mass of the Solar System enclosed within the orbit [2,3,8,9]. Therefore, because the orbital value of the gravitational radius differs among astronomical bodies, the square of their orbital period is not proportional to the cube

of the semi-major axis of their orbit by the same factor.

FOURTH LAW

The sum of the square of the velocity of the astronomical body and the product of the square of the constant velocity of light and the parameter b of the Schwarzschild solution of the gravitational field equations remains invariant throughout its orbital motion. And this is due to the conservation of both the Newtonian of the inert free energy and the Keplerian of the ordinary energy of the astronomical body in the process of its inertial motion in the gravitational field [2,3,8-10].

FIFTH LAW

The lines of force of the orbital gravitational field, as well as the centrifugal pseudo-forces of inertia that compensate for the gravitational pseudo-forces, are orthogonal to the orbital trajectory of the astronomical body, and the strengths of the orbital gravitational field are inversely proportional to the cubes of the radii of local curvatures of its trajectory of motion. And this is due to the conservation of the angular momentum of the astronomical body, which is equal to the product of its linear velocity of motion and the radius of curvature of its trajectory of motion.

SIXTH LAW

The periodic displacements of perihelia of planets are approximately proportional to the ratio of the square of the semi-minor axis to the semi-major axis of their trajectories. Deviations from this ratio can be attributed to the mutual influence of the planets.

IV. CONCLUSION

1. The gravitational field is only a manifestation of the spatial inhomogeneity of the thermodynamic state of matter, and therefore, is not an independent form and a special type of matter. The emergence of the spatial inhomogeneity of the thermodynamic state of matter is not due to gravity, but due to the electromagnetic interaction of micro-objects of matter due to the non-conservation of momentum by quanta of electromagnetic radiation in the process of the origin of this inhomogeneity. Therefore, a hypothetical ideal gas is fundamentally unable to form a spatial inhomogeneity of its thermodynamic state, and,

thus, is unable to have a gravitational field [2,3]. And therefore, the search for a special gravitational interaction of micro-objects of matter and attempts to combine it with other types of interactions is as naive as the search for "dark energy" and "non-baryonic dark matter" [9].

2. The gravitational field initiates only the desire of matter to achieve a more stable spatially inhomogeneous thermodynamic state of its own. After all, it only stimulates the motion of matter, without changing its thermodynamic state in the process of the motion. And therefore, the gravitational field determines only the velocities of motion and spatial gradients, and not at all the values of the hidden thermodynamic parameters that are different in different substances [2,3,6]. And this may be associated with the spiral-wave nature of both matter and the Universe as a whole [13-15].

3. But the parameters of the motion of any matter itself, and therefore the planets of the Solar System, are not determined by the gravitational field at all. They are determined by the laws of conservation of the thermodynamic state of matter and the stability of the main parameters of its motion. Namely, they are determined by the laws of conservation of the Newtonian of inert free energy and the Keplerian of ordinary rest energy of matter and the Newtonian angular momentum $L = m_{in} v \rho = \text{const}(r, t)$ of matter, respectively. After all, the gravitational field strength $\hat{g} = -L^2 m_{in}^{-2} \rho^{-3} = -g_{00} (2r_0 / r - 1)^{3/2}$ is determined differently everywhere. It can be inversely proportional not only to the square of the radial distance or to the distance itself (as in flat galaxies [8,9]), but also to the cube of a radial distance ρ , which determines the angular momentum conserved in the process of motion. And it is even fractional (3/2), with respect to the radial distance r from the center of gravity. In addition, the fact that gravitational forces exhibit not only an inverse-square but also an inverse-cube dependence on radial distance constitutes a paradox of the gravitational field, which indicates the primacy (priority) of conservation laws over a field-based interpretation of the phenomenon of gravity. At the edge of the Solar System, this is what could have led to the anomalous motion of the "Pioneer spacecrafts" [16-18].

4. The gravitational field of the Solar System is not stable. After all, it is forced to adapt to changes in the spatial arrangement of the planets, thereby

guaranteeing the flawless operation of the conservation laws. And therefore, as the planets approach the Sun, its strength increases. And thus, it is the motion of the planets that forms the adaptive-dynamic gravitational field they need, which is actually a consequence of not only the spatially inhomogeneous thermodynamic state of the entire gravithermodynamically bound matter of the Solar System, but also the motion of the planets (and not at all the cause of both the spatial inhomogeneity of the thermodynamic state of the matter of the Solar System and the motions of its planets) [2,3].

5. An elliptical orbit is symmetrical, and therefore, can have only the same radii of curvature of the planet's trajectory at aphelion and perihelion ($\rho_1 = \rho_2 = b^2 / a = b^2 / r_0$). And this does not ensure the conservation of the planet's angular momentum due to the same radii of curvature of the trajectory and the unequal velocities of its motion at aphelion and perihelion ($L_1 = m_{in} v_1 \rho_1 \neq L_2 = m_{in} v_2 \rho_2$). Planets actually move in superharmonic quasi-circular orbits ($\rho_1 \equiv r_1 \neq \rho_2$, $\rho_2 \equiv r_2 \neq b^2 / a = r_1 r_2 / r_0$), whose radius of curvature regularly decreases as they approach the Sun. And this is what ensures the conservation of the planet's angular momentum ($L = m_{in} v \rho = \text{const}(r, t)$). Therefore, even calling the orbits of planets asymmetric quasi-elliptical would not be entirely correct. And therefore, hypothetical circular orbits are degeneracies of superharmonic, rather than elliptical, orbits.

6. More resistant to external influences is not a circular, but a superharmonic quasi-circular orbit, which cyclically expands (until reaching aphelion), and then narrows while maintaining the location of the orbital focus. Even the Moon rotates in a superharmonic quasi-circular (pseudo-elliptical) orbit due to the influence on it not only of the Earth's gravitational field, but also of the Sun's gravitational field. And this is due to the significantly smaller angular momentum of bodies moving in a superharmonic quasi-circular, rather than a circular, orbit.

7. The curved field lines of the adaptive-dynamic gravitational field of the Solar System are orthogonal precisely to the superharmonic quasi-circular orbits of its planets.

8. Due to the absence of the influence of both the external gravitational field and the motion on the thermodynamic state of matter, along with the conservation (not related to the absorption of solar

radiation) of the thermodynamic state of matter, the rate of flow of the planets' proper time is also conserved. And this is due to the conservation of the Newtonian and the Keplerian respectively (instead of the Hamiltonian and the Lagrangian), in the process of their inertial motion in the gravitational field [8-12].

ACKNOWLEDGEMENTS

The author thanks the developers of artificial intelligence algorithms and programs for

significantly facilitating the solution of very complex integral equations.

CONFLICTS OF INTEREST

The author declares no conflicts of interest regarding the publication of this paper.

FUNDING STATEMENT

The research was carried out without financial support.

-
- [1] Albert Einstein, Physik und Realität, *Journal of The Franklin Institute: Devoted to Science and the Mechanic Arts*, vol. 221, no. 3, pp. 313-347 (1936).
 - [2] Pavlo Danylchenko, Foundations and consequences of Relativistic Gravithermodynamics, Vinnytsia, Ukraine: *Nova knyga*, 5 (2020).
 - [3] Pavlo Danylchenko, Foundations of Relativistic Gravithermodynamics. Vinnytsia, Ukraine: *TVORY* (2022).
 - [4] Pavlo Danylchenko, Relativistic Transformations of Coordinate Increments and Metric Segments of Bodies Moving in a Gravitational Field by Inertia, in Proceed. VI Int. Conference APFS'2025. Lutsk, Ukraine: *Volyn University Press "Vezha"*, 26 (2025).
 - [5] Christian Möller, The Theory of Relativity, Oxford: *Clarendon Press* (1972).
 - [6] Pavlo Danylchenko, Thermodynamics with Nonspecific Hidden Variables. *Eng OA*, 4(4), 01-18 (2026).
 - [7] Richard Tolman, Relativity thermodynamics and cosmology. Oxford: *Clarendon press* (1969).
 - [8] Pavlo Danylchenko, Solutions of the Standard Differential Equation of the Dynamic Gravitational Field of a Flat Galaxy, *Crystal Journal of Physics*, 1, Iss. 1, 1-16 (2025).
 - [9] Pavlo Danylchenko, Ignoring the Compensation of Gravitational Time Dilation by Inertial Motion of Matter and Other Theoretical Misconceptions and Imaginary Entities in Physics, Astronomy, and Cosmology. *Int Nat Sci Int Rese*, 1(1), 01-46 (2026).
 - [10] Pavlo Danylchenko, Alternatives to the Hamiltonian and Lagrangian, *Curr Res Traffic Transport Eng*, 4(1), 01-05 (2026).
 - [11] Pavlo Danylchenko, On the justification for using a dynamic gravitational field in an expanding Universe, in Proceed. XIII Int. Conference RNAOPM2026. Lutsk, Ukraine: *Volyn University Press "Vezha"*, 128 (2026).
 - [12] Pavlo Danylchenko, Ordinary synchronization-compensation transformations of coordinate and time increments, in Proceed. XIII Int. Conference RNAOPM2026. Lutsk, Ukraine: *Volyn University Press "Vezha"*, 131 (2026).
 - [13] Pavlo Danylchenko, The spiral-wave nature of elementary particles. Proceedings of International scientific conference "D.D. Ivanenko – outstanding physicist-theorist, pedagogue", ed. Rudenko O.P., Poltava, Ukraine: *PDPU*, 44 (2004).
 - [14] Pavlo Danylchenko, Spiralwave model of the Universe, in Proceedings of all-Ukrainian seminar on theoretical and mathematical physics: in honour of 85th anniversary of Anatoly Swidzynski. Lutsk, Ukraine: *Volyn University Press "Vezha"*, 21 (2014).
 - [15] Pavlo Danylchenko, About possibilities of physical unrealizability of cosmological and gravitational singularities in General relativity and Relativistic Gravithermodynamics. *Curr Res Traffic Transport Eng*, 4(1), 01-23 (2026).
 - [16] John D. Anderson, Philip A. Laing, Eunice L. Lau, Anthony S. Liu, Michael Martin Nieto, Slava G. Turyshch, Study of the anomalous acceleration of Pioneer 10 and 11. *Physical Review D*. Vol. 65, no. 8, 4322–4329 (2002).
 - [17] Carl Johan Masreliez, A cosmological explanation to the Pioneer anomaly, *Ap&SS*, 299, no. 1, 83-108 (2005).
 - [18] Robert A. Jacobson, The orbits of the Neptunian satellites and the orientation of the Neptune. *The Astronomical Journal*, **137**, N. 5, 4322 (2009).