MAGIC SQUARES OF THE 4TH AND 8TH ORDERS: MATHEMATICAL PROJECTIONS AND PRACTICAL APPLICATIONS

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Abstract

The study demonstrates that projecting order-4 and order-8 magic squares onto a tesseract preserves their fundamental numerical invariants. Similarly, the 4×4 Dürer's magic square is projected onto the vertices of an octagonal prism with internal spatial connections. The results obtained in this study open up prospects for utilizing multidimensional geometry to develop new data distribution algorithms. The presented mathematical symmetries can find practical applications in high-tech and theoretical disciplines.

1 Introduction

The earliest unique 4×4 magic square was discovered in an 11th-century inscription in the Indian city of Khajuraho. The 4×4 square depicted in Albrecht Dürer's engraving "Melencolia I" (Figure 1) is considered the earliest in European art.

Figure 1 – Magic squares: A. Dürer's engraving "Melencolia I" (left); Dürer's square (top right); Khajuraho square (bottom right)

The initial study demonstrated that the structural invariants of 4×4 pandiagonal squares consist of number pairs summing to one of the two Fibonacci numbers, namely 13 or 21. Various combinations of these symmetric number pairs form the set of 4×4 pandiagonal squares (Figure 2).

1	8	13	12
14	11	2	7
4	5	16	9
15	10	3	6

(a)

1	8	11	14
12	13	2	7
6	3	16	9
15	10	5	4

(b)

1	12	7	14
8	13	2	11
10	3	16	5
15	6	9	4

(c)

7	12	1	14
2	13	8	11
16	3	10	5
9	6	15	4

(d)

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

(e)

Figure 2 – Magic squares: (a, b, c) main variants of 4×4 squares;
(d) Khajuraho square; (e) Dürer's square

Additionally, the second study established formulas for calculating the series sum—the so-called magic constant—of numbers constituting squares and cubes of a specific order. The central symmetry of magic squares from order 2 to 8, as well as magic cubes from order 3 to 5, was analyzed. Based on the underlying logic of constructing magic squares and cubes, two similar magic objects were developed: a "cube-in-a-cube" and "cubes-in-a-cube." The former is built based on the order-4 magic square (Albrecht Dürer, 1514), while the latter is based on the order-8 magic square. These magic figures exhibit "mixed" symmetry.

The third study demonstrates the possibility of formula-based calculation for the patterns of ideal order-4 and order-8 magic squares, as well as order-2 and order-4 magic cubes. Furthermore, it substantiates the hypothesis that the laws of dialectics are applicable to numbers as ideal objects. In particular, the presence of mathematical patterns can characterize the dialectical law of the "unity and struggle of opposites," where the sums of the pattern's numbers act as the opposites. The presence of the golden ratio constant or its derivatives between opposing patterns is presumably due to the unity of the elements comprising such a whole. This unity of the elements constituting the whole is likely determined by the maximum number of structural mathematical connections resulting from the unique properties of golden proportional relationships.

2 The main part

Object of research: Magic numerical matrices (squares and cubes) of the 4th and 8th orders, as well as multidimensional geometric spaces (specifically, the four-dimensional hypercube—the tesseract).

Subject of research: Regularities of spatial-numerical symmetry during the mathematical projection of magic squares into multidimensional structures.

Spatial-numerical symmetry is a harmonious, balanced distribution of quantitative parameters, coordinates, or geometric elements relative to a center, axis, or plane in three-dimensional space. It is essential for ensuring structural stability, proper system operation, and computational accuracy. Below are the key areas where spatial-numerical symmetry plays a critically important role.

Architecture and Structural Engineering:

• Load distribution. The symmetrical arrangement of load-bearing columns and beams evenly distributes the building's weight onto the foundation.

• Seismic resistance. Buildings with correct spatial geometry better withstand ground vibrations and are less prone to torsional twisting during earthquakes.

• Auditorium acoustics. In theaters and philharmonic halls, the numerical proportions of the walls are designed symmetrically for ideal reflection and focusing of sound waves.

Mechanical Engineering and Aerodynamics:
• Rotor balancing. In turbines, engines, and wheels, mass must be distributed perfectly symmetrically relative to the axis of rotation to eliminate vibrations.
• Aerodynamic shapes. Aircraft wings, rocket fuselages, and automobile bodies require strict mirror-image equality of sides and volumes for airflow stability.
• Mass centering. Symmetrical distribution of heavy components within the craft's space maintains the center of gravity at a strictly specified point.
Information Technology and Graphics:
• 3D modeling. Programming characters and objects in video games relies on the symmetry of polygonal meshes to optimize rendering.
• Spatial audio. Audio distribution algorithms (e.g., Dolby Atmos) utilize precise numerical delays to create a symmetrical soundstage around the listener.
• Computer vision. Robots and autopilots employ symmetrical stereo camera calibration matrices to accurately determine the distance to objects.
Physics, Chemistry, and Crystallography:
• Crystal lattices. The structure of minerals, metals, and semiconductors is determined by a strict spatial symmetry in the arrangement of atoms, which dictates their physical properties.
• Molecular biology. The structure of DNA, protein complexes, and viral capsids often exhibits regular geometric and numerical symmetry to ensure the stability of genetic code storage.
• Quantum mechanics. The spatial symmetry of wave functions simplifies the calculation of electron energy levels in atoms.

Figure 3 presents the 4×4 Dürer's magic square (with numbers from 1 to 16) projected onto the vertices of a tesseract (a four-dimensional hypercube). Here, each of the 16 vertices of the four-dimensional cube is mapped to a single number from the original Dürer's square. The potential applications of this structure (a 16-vertex tesseract) include (Figure 3):
• Hamming codes (7,4) and (8,4). The tesseract vertices are encoded using 4-bit binary numbers (from 0000 to 1111). The symmetrical distribution of weights and numbers is utilized to detect and correct errors during data transmission in RAM and satellite communications.
• Quantum computing. The 16 states of this structure are equivalent to a 4-qubit system ($2^4 = 16$). Numerical symmetry helps describe entangled states and design quantum gates (logic elements).
• Karnaugh map optimization. In digital circuit design, logic functions of four variables are minimized using the hypercube topology, where adjacent vertices differ by exactly 1 bit (Gray code).

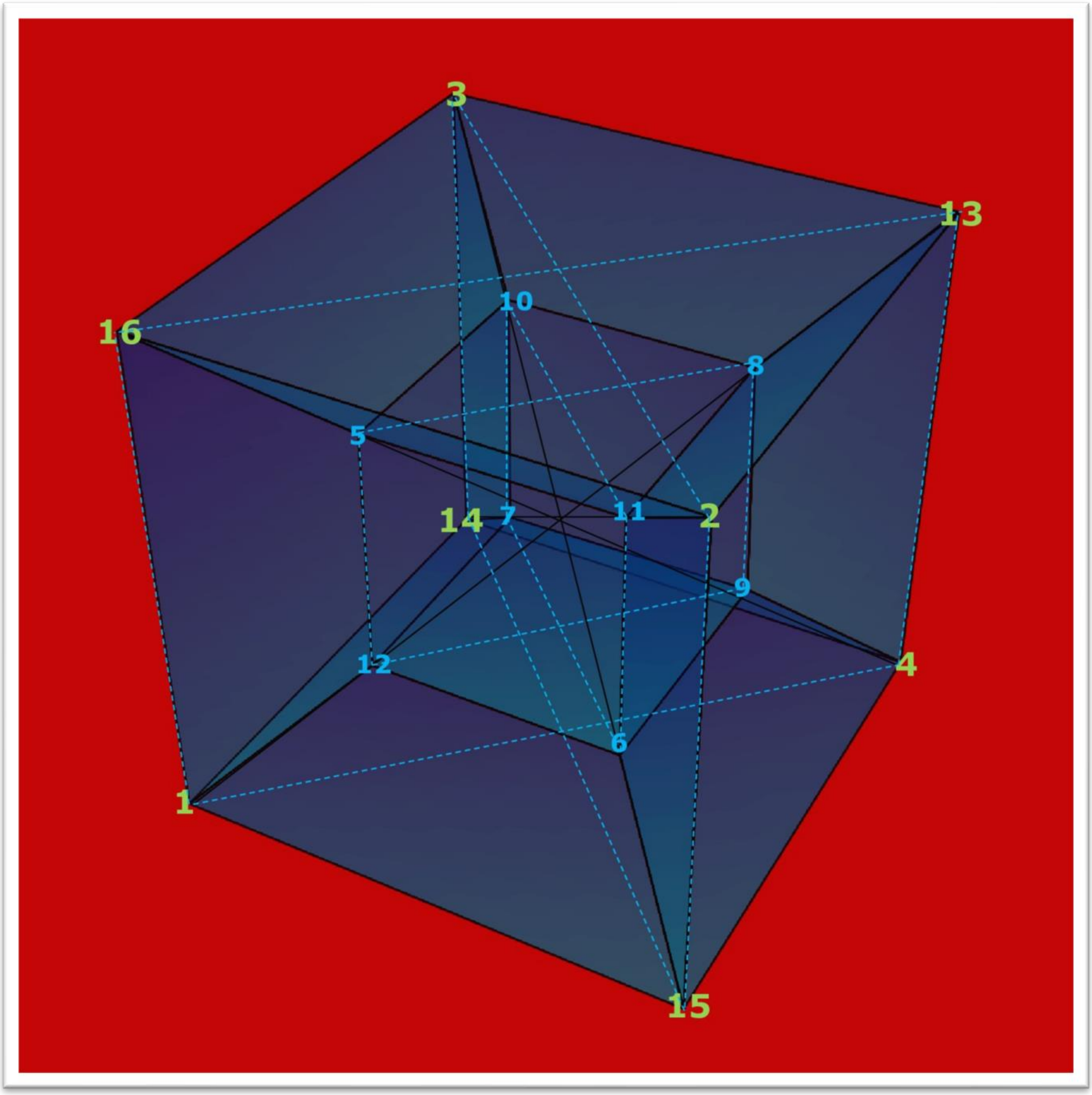

Figure 3 – Dürer's 4×4 magic square projected onto the vertices of a tesseract with internal spatial connections

Figure 4 presents an 8×8 magic square (containing numbers from 1 to 64) projected onto the geometry of a tesseract. This model maintains an ideal balance: the sums of numbers along all edges, planes, sections, and spatial diagonals are equal to a single constant (130).

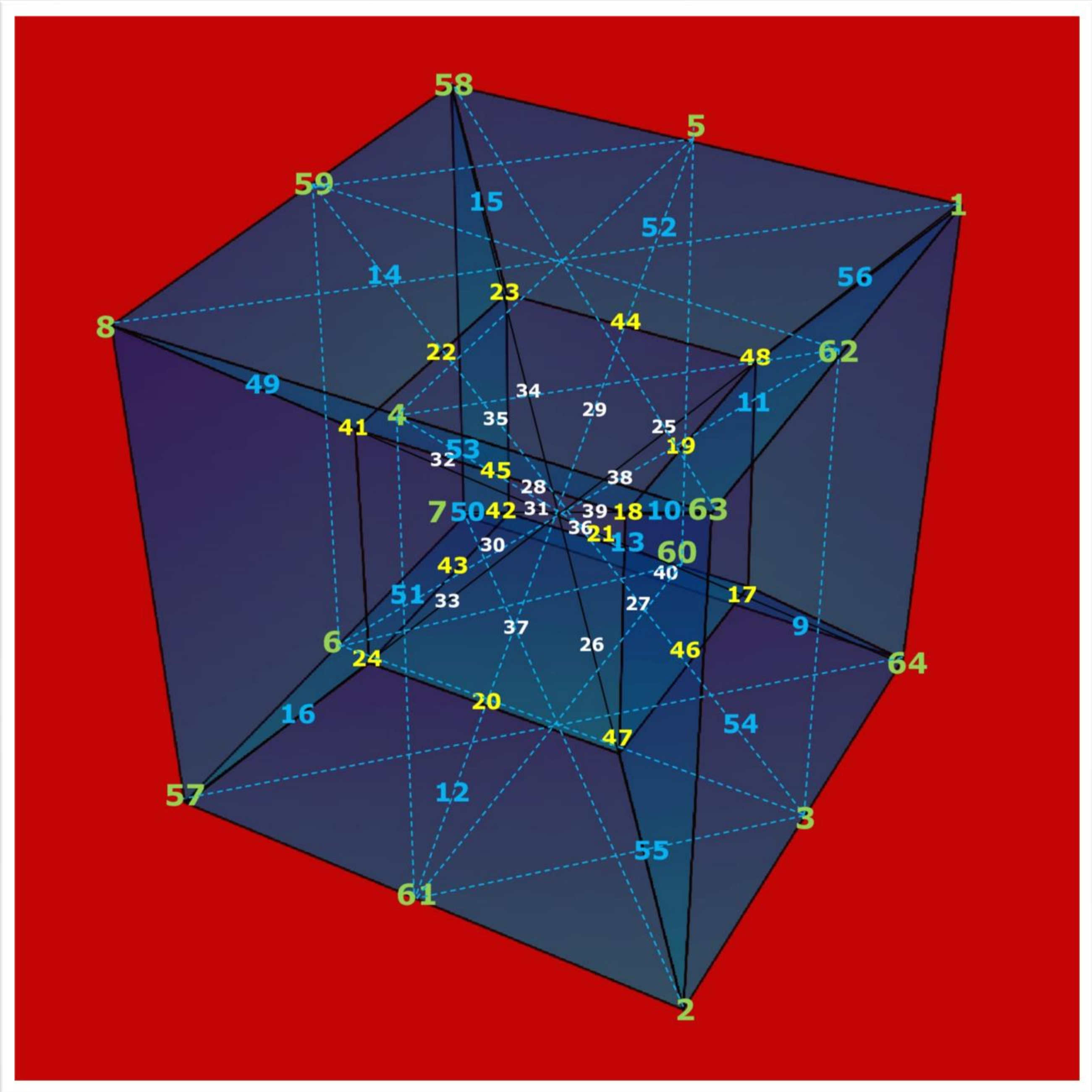

Figure 4 – An 8×8 magic square projected onto the geometry of a tesseract with internal spatial connections

This unique spatial and numerical symmetry (Figure 4) can find applications in high-tech and theoretical disciplines:

Cryptography and Data Encryption:

• S-box design. Symmetrical hypercube matrices are utilized to create substitution tables (S-boxes) in block ciphers (e.g., in AES-type algorithms).

• Maximum entropy. The uniform distribution of numbers complicates the mathematical analysis of ciphertext and protects data against differential cryptanalysis.

• Pseudorandom number generation. Algorithms for traversing the vertices of such a hypercube serve as the foundation for cryptographically secure noise generators.

Steganography and Digital Content Protection:

• Information hiding. The numerical matrix of the hypercube determines the pixel coordinates where hidden watermarks or encrypted messages are embedded.

• Detection resistance. Due to the numerical balance, changes in the file are distributed uniformly, rendering them invisible to statistical and spectral analysis systems.
Network Technologies and Supercomputers:
• Interprocessor interconnect architecture. The "hypercube" topology is used to connect thousands of processors in supercomputers.
• Traffic routing. Symmetrical address distribution across the tesseract vertices minimizes latencies (the number of "hops") during data packet transmission between nodes.
Coding Theory and Signaling:
• Error-correcting codes. Spatial numerical structures aid in developing noise-immune data encoding algorithms.
• Multidimensional modulation. Symmetry helps distribute signals in wireless networks (e.g., in 5G/6G systems) to minimize mutual interference.
Design of Experiments (Combinatorics):
• Multifactorial analysis. Hypercubic matrices allow scientists to test dozens of independent parameters simultaneously with a minimal number of physical trials.

Figure 5 presents Dürer's 4×4 magic square projected onto the vertices of an octagonal prism with internal spatial connections, the vertices of which are numbered with numbers from 1 to 16.

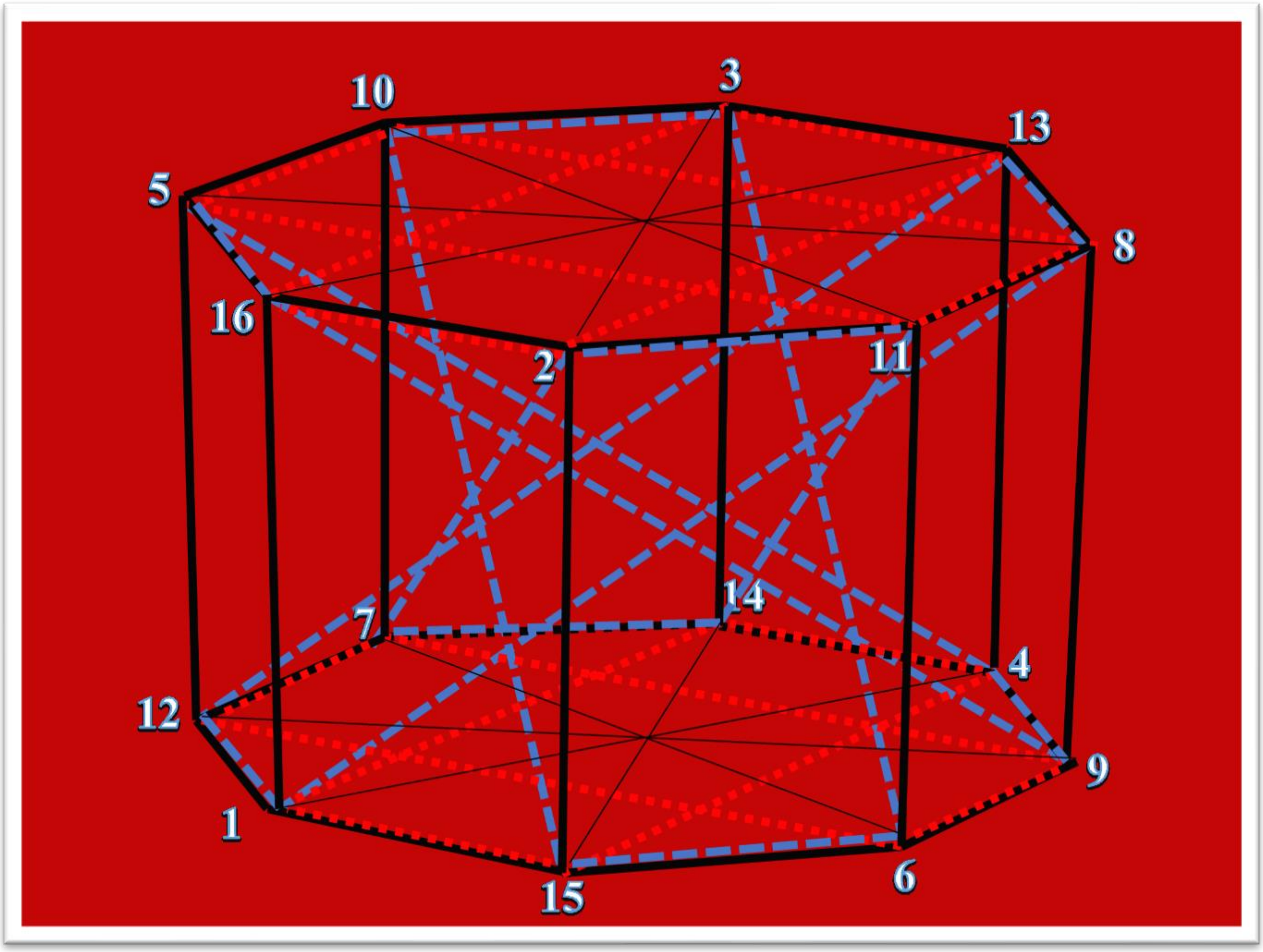

Figure 5 – Dürer's 4×4 magic square projected onto the vertices of an octagonal prism with internal spatial connections

Such a geometric and topological structure (Figure 5) can find applications in the following high-tech areas:

Supercomputer Architecture ("Hypercube" Topology):
• Parallel computing. This scheme is used for the physical connection of processors (nodes) in supercomputers.
• Efficient routing. In such a network, each of the 16 processors is connected to exactly four neighboring nodes. The maximum number of steps (hops) to transfer data between any two most distant nodes is only 4, which minimizes latencies.
Quantum Computing and Simulations:
• 4-qubit registers. A system of 4 qubits has exactly 16 pure states (from $|0000\rangle$ to $|1111\rangle$). Each vertex of this construction corresponds to a single state.
• Quantum algorithms. The edges and internal diagonals (blue dashed lines) indicate the allowed transitions between states under the action of quantum gates (e.g., the controlled-NOT operator—CNOT).
Digital Electronics and Circuit Synthesis:
• Four-dimensional Karnaugh maps. In chip design, logic functions of 4 variables are deployed precisely according to this topology.
• Gray codes. Moving along the edges of this figure corresponds to a change of exactly one bit in a four-digit binary code (e.g., from 0000 to 0001), which prevents glitches and race conditions in processors.
Coding Theory and Noise Immunity:
• Hamming code (8,4). Data vectors are distributed across the vertices of this 4D figure. The distance between the vertices (the number of edges between them) corresponds to the "Hamming distance." This allows communication systems (e.g., satellite TV or Wi-Fi) to automatically detect and correct errors caused by airborne interference.
Network Neuromatrices:
• Multidimensional associative neural networks. In artificial intelligence, such graphs are used to organize memory, where highly correlated concepts are located on adjacent vertices, while contrasting ones are on diametrically distant ones (e.g., pairs 1 and 16, 2 and 15).

3 Conclusion

1. The study demonstrates that projecting order-4 and order-8 magic squares onto a tesseract preserves their fundamental numerical invariants, exhibiting high efficiency of these structures in cryptography and high-tech system optimization. Similarly, the 4×4 Dürer's magic square is projected onto the vertices of an octagonal prism with internal spatial connections.
2. The results obtained in this study open up prospects for utilizing multidimensional geometry to develop new data distribution algorithms. The presented mathematical symmetries can find practical applications in high-tech and theoretical disciplines.

References

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