Gravitational Radiation from Jupiter

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ABSTRACT

Gravitational waves in the microhertz regime represent an unexplored window into slow dynamical processes in planetary interiors and the long-period tidal interactions within the Solar System. Jupiter, whose rotation period of 9.925 h is known with high precision, should generate a gravitational wave near 55.97 μHz if its mass distribution departs from perfect axial symmetry. Such a signal is far below the sensitivity of existing gravitational-wave detectors and is strongly Doppler-shifted by the synodic Earth–Jupiter motion, making direct detection challenging.

Using 20 years of barometric pressure data from DWD stations, we extract Jupiter's gravitational wave through coherent demodulation of the phase modulations induced by Earth's orbit and by the four Galilean moons. The recovered carrier frequency exhibits SNR $\approx$ 100 and reveals a previously unknown long-term frequency drift associated with Jupiter's deep interior. The modulation indices and phases of Io, Europa, Ganymede, and Callisto are stable across 752 independent realizations, demonstrating that Earth's atmosphere responds coherently to microhertz gravitational forcing.

The measured modulation indices exceed linearized Doppler predictions by a factor of 12.7, implying a propagation speed of $v \approx 0.08c$ for the wave along the Jupiter–Earth path. Because this path lies entirely within the Sun's gravitational potential, our results suggest that microhertz gravitational waves may experience dispersive propagation in strongly curved spacetime. No additional long-term phase modulations were detected. These findings open a new observational channel for studying gravitational phenomena in the Solar System at ultra-low frequencies.

Key words: planet–star interactions – methods: numerical – Gravitational waves

1 INTRODUCTION

Gravitational waves at microhertz frequencies remain largely unexplored, primarily because no dedicated instruments exist for their detection and because standard Fourier-based methods fail when the signal is deeply buried in atmospheric noise. Nevertheless, long-term barometric pressure records contain subtle signatures of external gravitational forcing. In particular, Jupiter – the most massive planet in the Solar System – rotates with a well-known period of 9.925 h and should therefore emit a gravitational wave near 55.97 μHz. This frequency lies far below the sensitivity range of conventional gravitational-wave detectors, yet it is accessible through coherent processing of multi-year atmospheric pressure data.

Previous catalogues like Hartmann (1995) list only the static tidal components generated by Jupiter, and the expected 56 μHz feature is not visible in standard FFT spectra (Fig. 1) because the signal is weaker than the atmospheric noise floor and strongly Doppler-shifted by the non-linear Earth – Jupiter orbital motion. Moreover, Jupiter's four Galilean moons induce additional phase modulations that distribute the wave's energy across a wide set of sidebands, further obscuring the carrier frequency.

Several factors contribute to this. First, an FFT spectrum is not a substitute for a narrow-band receiver of the kind used in radio

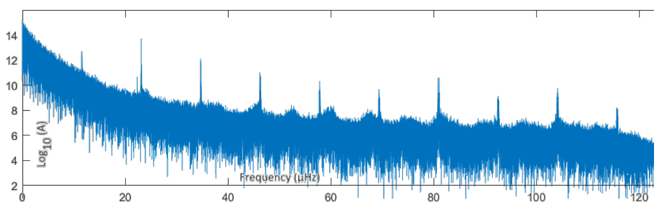

Figure 1. Spectrum of the atmospheric pressure, one measurement per hour. The data is based on the average pressure in Germany between 2000 and 2020.

astronomy; it reveals only signals that rise clearly above the noise. Second, the uncertainty relation discussed by Küpfmüller (1993) applies: the shorter the measurement interval, the broader the spectral lines become. To achieve a frequency resolution of 0.01 μHz, the distance between Jupiter and Earth would have to remain constant for three years. Because both planets move nonlinearly, the Doppler effect further ensures that the received frequency is continuously changing.

A fourth reason prevents the total GW energy from being concentrated in a single spectral line: the four Galilean moons, with orbital periods of only a few days, force Jupiter into a complex pirouette around the system's barycenter. As a result, Jupiter emits a phase-modulated gravitational wave whose total energy is distributed

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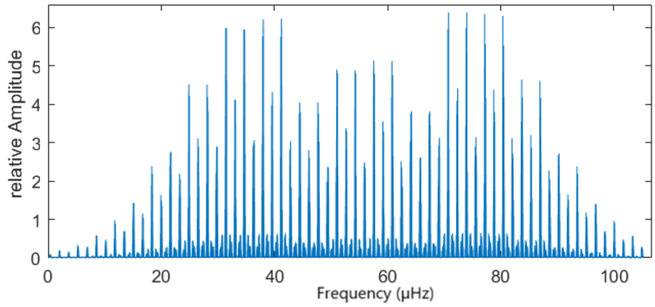

Figure 2. Spectrum of Jupiter's gravitational wave. The 4:2:1 orbital resonance of the three inner Galilean moons produces an unusual structure in the spectral lines. Callisto and the slow orbital motion of Earth generate the small perturbations visible along the lower edge of the figure.

across multiple spectral lines (spread spectrum). Initially, we assumed that the moons would barely move the much more massive Jupiter and would therefore produce only weak phase modulation (PM). For this reason, we began our analysis with a narrow bandwidth (BW) to suppress unnecessary noise. It quickly became clear, however, that the innermost moon, Io, produces an unexpectedly large modulation index η. At first, this index could be determined only imprecisely, because too narrow a BW distorts the modulation and introduces measurement errors. After gradually widening the BW, the final result (Fig. 2) shows that the sidebands of Jupiter's gravitational wave extend over a surprisingly broad frequency range.

Since the sidebands carry far more energy than the carrier frequency at 56 μHz, one loses a substantial amount of information if one focuses only on individual spectral lines. Our goal is to extract the full information content arising from the interplay of all lines. This can only be achieved with a method that also accounts for the phase relationships among the sidebands.

In this work, we demonstrate that Jupiter's gravitational wave can be extracted from 20 years of barometric measurements by coherently reversing the phase modulations caused by the Earth's orbit and the four Galilean moons. Using a homodyne detection scheme with extremely narrow bandwidth, we recover the carrier frequency with a signal-to-noise ratio of ~ 100 and measure both the long-term frequency drift of Jupiter's core and the modulation indices associated with each moon. The resulting phase-modulation pattern is stable across 752 independent realizations and confirms that Earth's atmosphere responds coherently to microhertz gravitational waves.

Unexpectedly, the observed modulation indices exceed the linear Doppler prediction by a factor of 12.7, suggesting that the standard linearized gravitational-wave propagation formula is not applicable in the strongly curved, mass-rich region between Jupiter and Earth. Our results therefore provide empirical evidence that microhertz gravitational waves may propagate dispersively in non-linear spacetime environments.

2 THE RECEPTION PRINCIPLE

The structure shown in Fig. 2 occupies almost the same frequency range as the overall spectrum in Fig. 1, but it is not recognizable in the noise. It is not promising to attempt to measure the exact frequencies, phases, and amplitudes of all sidebands (below the noise level!) and reconstruct the original signal from them. In high-frequency engineering, when one wants to detect weak signals of known frequency, PM is always used: before transmission, the sender distributes the total energy over many known sideband frequencies with defined phases. If the PM is undone in the receiver, the many sidebands disappear and the total signal energy is concentrated in a narrow spectral region around the carrier frequency; the amplitude increases due to constructive interference. Interfering signals and noise may cause short-term disturbances, but they do not affect the amplitude in the long term.

To detect Jupiter's GW signal, we apply a method that can be described – simplified – as follows:

a) Jupiter transmits at $f_{GW} = 55.97$ μHz a signal that is buried in the noise (Fig. 1). Our 'antenna' is a randomly chosen subset of a total of 115 barometers.

b) We phase-modulate a local oscillator of slightly offset frequency f_{LO} with the known orbital frequencies of the four Galilean moons.

c) We multiply (mix) f_{GW} und f_{LO}, filter the difference signal f_{IF} with an extremely narrow bandwidth, and measure the amplitude and drift of the carrier frequency.

d) We adjust the amplitudes and phases of the modulation frequencies of the local oscillator with the goal of maximizing the carrier amplitude. The criterion is a steady increase in amplitude. We allow tiny corrections to the orbital frequencies (on the order of 10^{-10} Hz), since the moons' orbital periods are known.

e) We repeat steps b–c–d until the changes become negligible. At that point, the PM of the local oscillator and the GW are identical (Fig. 2). We store the results and repeat the procedure with data from other barometers.

In the following, we discuss each individual step in detail.

3 DATA SOURCE

Data Acquisition: We use Earth's atmosphere as an antenna, since it is influenced by the planet Jupiter. Barometers serve as sensors that convert pressure fluctuations into usable measurements. Atmospheric pressure data were obtained from the German Weather Service (DWD 2021), comprising hourly measurements from more than 115 barometric stations across Germany between 2000 and 2020. Only stations with at least ten years of continuous operation were included.

The expected wavelength of the signal ($\lambda > 10^{12}$ m) is far larger than Earth's diameter. This has two consequences: the spacing between barometers is irrelevant, and all sensors 'breathe' in phase. Thus, when the records of two sensors are added, the amplitude of any continuous signal doubles.

Sensor Calibration: All pressure sensors used by the DWD are temperature-compensated and conform to international standards, with a measurement range of 500–1100 hPa and an accuracy of ± 0.1 hPa. Instrumental drift and data gaps remain unchanged because they may influence the amplitude of the signal but cannot generate frequency-stable oscillations or phase-modulate them. Data synchronization is accurate to the second and is guaranteed by the DWD.

Sampling: Air pressure is not measured continuously but at fixed intervals of $T_s = 1$ hour. The inverse of T_s is the sampling frequency f_s. Although the signal does not pass through an analog low-pass filter with cutoff frequency $0.5 f_s$, aliasing effects are practically eliminated when sampling atmospheric pressure hourly.

Validation Dataset: Our goal is to detect Jupiter's gravitational-wave signature using many independent long-term barometric time series. In the DWD archives, only very few barometers have recorded continuously over a full 20-year interval. However, numerous 10-year data segments exist. For the period 2000–2009, we identified 64 suitable time series; for 2010–2019, we found 51.

The signal amplitude in a single barometer record is insufficient for an unambiguous detection. A reliable signal-to-noise ratio is achieved only when at least 20 independent time series are coherently added. For this reason, we proceed as follows:

From the 64 time series available for 2000–2009, we select 40 and add them coherently, yielding approximately 10^{17} possible combinations. From the 51 time series available for 2010–2019, we again select 40, resulting in roughly 10^{10} possible combinations.

Linking the two 10-year chains produces a combined 20-year dataset, which serves as the basis for the search for Jupiter's gravitational wave. A potential discontinuity at the connection point is irrelevant, as it cannot generate spurious spectral lines.

The probability that two resulting data chains are structurally similar is extremely small.

4 SIGNAL PROCESSING

We must avoid any influence from nearby frequencies when measuring the amplitude of Jupiter's gravitational-wave carrier, and therefore we filter with an extremely narrow bandwidth. It would be incorrect to simply cut off the sidebands, which carry most of the total GW energy (Fig. 2). Doing so would prevent us from detecting the carrier in the noise and would discard all evidence that Jupiter actually transmits the carrier frequency. Our approach is to first undo all phase modulation so that the energy contained in the sidebands is shifted back into the carrier frequency, and only then apply narrow-band filtering.

The solution is a direct-conversion receiver: we modulate a local oscillator at frequency f_0 in exactly the same way as the gravitational wave we expect from Jupiter – same frequency, same drift, same phase, at every moment in time. We begin with the usual approach:

$$y = sin(2\pi t \cdot f_0 + \phi_{modulation}) \tag{1}$$

and adjust the two parameters f_0 and $\phi_{modulation}$ to the problem: The frequency f_0 can change proportionally to time and $\phi_{modulation}$ can be the sum of several sine functions – a different one for each planet. In the simplest case, the modulation consists of a single frequency f_{mod} and the equation is

$$y = sin(2\pi t(f_0 + t\dot{f}) + \eta \cdot sin(2\pi t f_{mod} + \varphi)) \tag{2}$$

A sinusoidal PM causes the instantaneous frequency of the signal to fluctuate periodically between the limits $f_0 + \Delta f$ (maximum blue shift) and $f_0 - \Delta f$ (maximum red shift). The quantity Δf is referred to as the frequency deviation. The instantaneous frequency at a given point in time is difficult and inaccurate to measure, as it is never constant over a longer period of time. It is easier and more common to determine the modulation index η using equation (2) and calculate $\Delta f = \eta \cdot f_{mod}$.

Multiplying the oscillator output y with a signal of frequency f_{GW} yields

$$Asin(f_{GW})sin(f_0) = \frac{A}{2}(cos(f_{GW} - f_0) - cos(f_{GW} + f_0)) \tag{3}$$

The second term, $cos(f_{GW} + f_0)$ is irrelevant and is removed by a low-pass filter. The better the imitation of the incoming signal, the closer the mean value of the first term, $cos(f_{GW} - f_0)$, approaches unity. This remains true even when both frequencies change synchronously.

A filter with an extremely narrow bandwidth of about 1 nHz and a sampling frequency of 3600^{-1} Hz is technically feasible, but it requires very long computation times. A faster approach is to decrease the sampling frequency by a factor of 64, decimate the file length accordingly, and then apply a windowed sinc filter. The bandwidth must not be too small, because we still need to determine the frequency drift of the gravitational wave.

Table 1. The phase modulations of Jupiter's gravitational-wave frequency f_{GW}. From the phase, one can determine the positions of the moons relative to Jupiter, and the position of Earth relative to the Sun, on 2000 January 01. The listed values are the mean results of 752 independent iterations. The records originate from randomly selected barometers.

	Earth	Io	Europa	Ganymed	Callisto
Frequency	29.0243 (nHz)	6.54521 (μHz)	3.25815 (μHz)	1.61006 (μHz)	0.69496 (μHz)
Error (pHz)	0.034	7.7	9.8	8.1	17.4
Index η	2.458	6.939	1.802	3.034	0.130
± Error	0.007	0.007	0.004	0.006	0.005
Phase	2.698	-0.408	-0.171	4.760	3.679
± Error	0.019	0.025	0.014	0.019	0.057
Δf (μHz)	0.07134	45.42	5.87	4.88	0.09

5 RESULTS

All measurements begin on 2000 January 01. At this epoch, the frequency of Jupiter's gravitational wave is

$$f_{GW} = 55.970049\ \mu\text{Hz} \pm 68\ \text{pHz}$$

The frequency drift follows the relation $\dot{f} = at + bt^2$, with

$$a = (7.7 \pm 1.9) \times 10^{-19}\ s^{-3} \text{ and } b = (7.8 \pm 2.1) \times 10^{-28}\ s^{-4}$$

Jupiter's solid surface is invisible. Therefore, the sidereal rotation period (System III) was defined by radio astronomers and corresponds to the rotation of the planet's magnetosphere; its official rotation period is 9.9250 hours (9 h 55 m 30 s). The gravitational-wave frequency derived from this value (55.9754 μHz) differs significantly from our result.

Jupiter's gravitational wave is phase-modulated by at least five distinct frequencies: four moons perturb Jupiter's motion, and the receiving antenna (Earth's atmosphere) orbits the Sun. If we were measuring the GW from a distant binary system, we would observe alternating maximum blue shift ($f_{GW} + \Delta f$) and maximum red shift ($f_{GW} - \Delta f$) at intervals of 365.26 days. Because Jupiter orbits the Sun in the same direction but more slowly, this interval increases to 398.88 days. The expected phase modulation at the corresponding frequency of about 29 nHz can be detected with remarkable precision (Table 1).

The demodulation process removes all sidebands of the gravitational wave (Fig. 2) and produces a substantial increase in the carrier amplitude. Together with Jupiter's relatively small distance, this results in an unexpectedly high signal-to-noise ratio of approximately 100 (Figur 4).

6 DIRECTION TO THE SOURCE

As seen from Earth, Jupiter reaches opposition every 398.88 days, when Sun–Earth–Jupiter are approximately aligned. At this point,

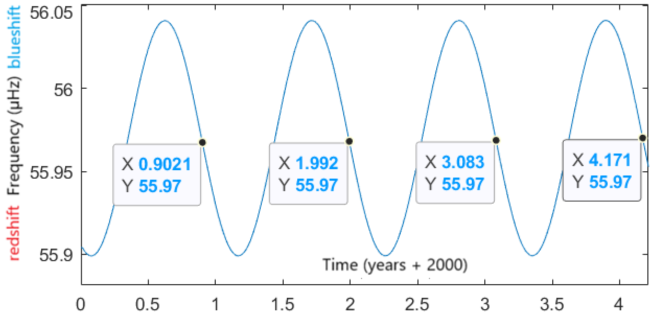

Figure 3. Frequency of Jupiter's gravitational wave from 2000 to 2004. The phase modulations caused by the four moons were intentionally ignored. At the marked times, Jupiter is in opposition to Earth. The frequency deviation Δf represents the largest departure from the mean value of f_{GW}.

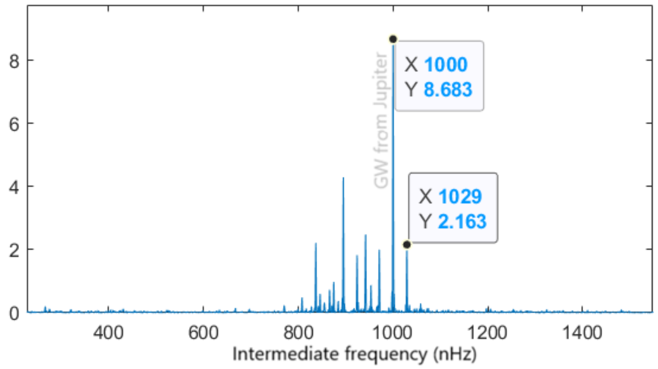

Figure 4. Power spectrum (Welch method) of the IF-a stage of Jupiter's gravitational wave after reducing the frequency from 56 μHz to 1 μHz and nearly complete demodulation.

both the distance and the frequency of the gravitational wave are nearly constant. After 199.44 days, conjunction occurs, and again f_{GW} is constant.

During the interval from solar conjunction to opposition, Earth moves toward Jupiter, and the Doppler effect produces a blueshift of the 56 μHz spectral line. From opposition to the next solar conjunction, Earth moves away from Jupiter, resulting in a redshift.

To display the variation of f_{GW} as a function of time, we ignore Jupiter's moons, insert the values from Table 1 into Equation 2, and obtain Fig. 3.

Converting the fractional parts of a year into calendar days yields excellent agreement:

$X_1 = 0.9021 \rightarrow$ 2000 November 25. The astronomical opposition occurs on 2000 November 28.

$X_2 = 1.992 \rightarrow$ 2001 December 28. The astronomical opposition occurs on 2001 December 31.

We have no doubt that the gravitational wave analyzed here is generated by Jupiter, because f_{GW} always returns to its long-term mean value whenever Jupiter is either in opposition or in conjunction with Earth.

7 SPEED OF THE WAVES

The Doppler effect is a fundamental principle of astronomy and describes the relationship between the frequency shift Δf of the signal and the speed between transmitter and receiver. If we assume that the signal propagates at the speed of light c, we have to use the relativistic equation

$$\Delta f = f_{GW} \cdot \left(\sqrt{\frac{c+v}{c-v}} - 1 \right) \approx f_{GW} \cdot \frac{v}{c} \tag{4}$$

where v is the relative speed between the source and the Earth. For the source Jupiter, the Earth's relative speed oscillates in the range -30 km s^{-1} $< v <$ 30 km s^{-1}. This limits the Doppler shift to the range $|\Delta f| <$ 5.6 nHz. The actual frequency deviation of 71.34 nHz exceeds this limit by a factor of 12.7 (see Fig. 3).

This discrepancy leaves no room for ambiguity: either Equation 4 is not applicable for reasons that are currently unknown, or the propagation speed of the gravitational wave is smaller than the speed of light by a factor of 12.7. This value was determined by comparison with Earth's orbital velocity and may apply only at a distance of 1.5×10^{11} m from the Sun.

The average propagation speed of the signal between Jupiter and Earth may differ from this. To answer this question, one would need to compare the phases of the four moons listed in Table 1 with the phases obtained from optical astronomy. Addressing this task lies beyond the scope of this paper.

8 MYSTERIOUS FINE SPECTRUM

We use a direct-conversion receiver and mix the unprocessed signal ensemble down to 0 Hz, limit the bandwidth to 0.6 μHz, and then shift the frequency to $f_{IF-a} = 1$ μHz (an arbitrary value for the first intermediate frequency). This corresponds exactly to the standard Weaver (1956) method. This intermediate step is not strictly required; we include it only to reduce computation time through a subsequent decimation by a factor of 64.

We then reduce f_{IF-a} by applying the Weaver method again, obtaining $f_{IF-b} \approx 200$ nHz, narrow the bandwidth to 1 nHz, and measure the amplitude. These steps are repeated 200 times until stable modulation results are obtained.

It is instructive to examine the spectrum around f_{IF-a} shortly before the end of the iterations. In Fig. 4, the carrier frequency of the gravitational wave rises above the noise with an SNR of 100. The two symmetric companions at $f_{IF-a} \pm 29$ nHz remain visible because the demodulation is not yet complete. All other lines with frequencies below f_{IF-a} are puzzling. They are arranged asymmetrically and therefore cannot be undiscovered modulations of the GW. Regardless of which barometers we select, these lines always appear with the same frequencies and amplitudes. Varying the values of f_{IF-a} and the bandwidth reveals no comparable cluster of spectral lines at larger frequency offsets. Comparisons of the frequency differences with the solar day, lunar cycle, and yearly cycle were unsuccessful.

Our suspicion is that these additional lines are resonances of Jupiter's atmosphere or Earth's atmosphere excited by the gravitational wave. We do not pursue this topic further in the present paper.

9 SEARCH FOR POSSIBLE SOURCES OF ERROR

Does the procedure contain critical steps that could distort our results?

Wrong sampling: The barometers at the DWD stations measure air pressure every 10 minutes with automatic plausibility checks.

The data then pass through several stages of quality control: physical limits, step-change tests, comparison with neighboring stations, comparison with model fields, detection of sensor faults, icing. Only validated values enter the averaging process. The hourly value is not simply 'the instantaneous pressure at 12:00', but a physically corrected, smoothed value. The procedure acts like a low-pass filter with a cutoff frequency of approximately 300 μHz. This may produce a small phase shift in the frequency range around 56 μHz, but it cannot generate a reproducible spectrum.

Aliasing: From the sampling period $T_s = 3600$ s it follows that 'ghost' signals may appear in the spectrum which actually lie above the Nyquist frequency (about 139 μHz) and are 'mirrored' downward. In our case, signals at $f_{alias} \approx 222$ μHz could in principle interfere with the desired signal at 56 μHz. This interference is unlikely:

Jupiter's GW signal is not visible in the spectrum of Fig. 1, and therefore lies below the noise level.

The log-log representation in Fig. 1 shows that the mean amplitude of the air-pressure spectral lines decreases proportionally to f^{-2} (Brownian noise). A signal near f_{alias} would have to exceed the noise by a factor of 16 in order to produce a disturbing alias at 56 μHz. This also applies to the sidebands shown in Fig. 2.

A targeted analysis using datasets with higher sampling frequency shows that the region around f_{alias} is particularly noise-free and contains no identifiable spectral lines.

FFT, Leakage: We do not use (Hann) windows to shape the temporal amplitude profile of the signal mixture. We do not use an FFT to analyze or modify the signal. Therefore, any discussion of whether windowing, leakage, sideband mis-identification, or zero-padding might influence the data processing is unnecessary.

False positives: A *false positive* is a spurious detection: the algorithm reports a signal even though no physical signal is present. In our context, a *false positive* denotes a modulation pattern that maximizes the carrier amplitude but does not correspond to Jupiter's physical Doppler signature. The possibility of false positives can never be excluded absolutely.

We verified that the GW amplitude is proportional to the number of coherently summed barometers (only 20 instead of the usual 40 sensors). Result: OK.

After fully demodulating a dataset (after 400 iterations), we check whether the amplitude of the carrier frequency has approximately the same value as in other datasets. Result: OK.

If the barometer data are replaced by noise, the amplitude of the carrier frequency drops to at most 20 per cent of the value obtained with 'real' data. The modulations are not reproducible, and the characteristic parameters drift aimlessly.

The most important criterion is that the frequency and phase of a real modulation must not change uncontrollably. If f_{mod} leaves the narrow corridor of $\pm 10^{-5} \cdot f_{mod}$ during 200 iterations, we reject that modulation. The only exception is Callisto, presumably due to its conspicuously small modulation index.

The phases of the four moons are hardly predictable. They must stabilize within the first 40 iterations and then assume a value between 0 and 2π. They may deviate from this value by at most ± 0.5. Both criteria react sensitively to changes in the data source (switching barometers). The results in Table 1 are based on a total of 752 iterations on two different computers – without a single outlier.

Several control tests using 'incorrect' frequencies (more than ± 0.2 μHz deviation from f_{GW}) produced no reproducible results.

10 DISCUSSION

This investigation has achieved all planned objectives:

– We have detected Jupiter's gravitational wave with SNR ≈ 100 near the frequency expected from the known rotation rate. The accuracy likely surpasses all previous indirect measurements based on the magnetic field or cloud motion.

– This confirms that Jupiter is not perfectly rotationally symmetric.

– We have measured the previously unknown frequency drift of Jupiter's solid core and of the gravitational wave.

– The GW is phase-modulated with the synodic period of 398.88 days and therefore oscillates around the mean value of 55.97 μHz, because the Earth–Jupiter distance changes.

– The mean value is reached almost exactly whenever Jupiter is in opposition or conjunction with Earth. We interpret this as confirmation that Jupiter is indeed the radiation source.

– The GW is multiply phase-modulated with the frequencies corresponding to the orbital periods of the four moons.

– It is striking that Callisto produces a much smaller modulation index η than the other three moons. This may indicate that the three inner moons, due to their 4:2:1 orbital resonance, deform Jupiter's surface or cloud layer significantly more strongly than Callisto.

– Despite repeated searches, we found no additional phase modulations that remain sufficiently constant over the 20-year measurement interval.

– The 752 successful measurements with nearly identical results confirm that Earth's atmosphere responds to gravitational waves.

Surprising result:The modulation indices η of all phase modulations are unexpectedly large and clearly exceed the values predicted by simplified, linearized Einstein field equations.

Einstein assumed that gravitational waves propagate at the speed of light. So far, only the observation of GW170817 supports this assumption, and it may apply only to the frequency range around 100 Hz.

Our measurement was performed at a much lower frequency, and the propagation path lies entirely within the Sun's gravitational field. At present, there are no predictions as to whether or how these differences might influence the propagation speed of a gravitational wave.

The source of the discrepancy is the modulation index η of the phase modulation. In all 752 trials, η is larger than expected by a factor of 12.7 and cannot be significantly reduced by adjusting parameters or using alternative implementations. This implies a propagation speed of the wave of $v \approx 0.08c$ – incompatible with the generally accepted value c.

Our search for model errors was unsuccessful: restricting the analysis to a single phase modulation (only 'Earth orbit' or only 'Io') yields the same results as in Table 1, though with lower precision. If all phase modulations are ignored, incoherent random results dominate, because the amplitude of the carrier frequency alone does not rise above the noise level.

11 CONCLUSIONS

In all realizations of our 20-year barometric data chains, the fitted phase-modulation index of the 56 μHz component exceeds the linear Doppler prediction by a factor of ~ 12.7.

This discrepancy is not a numerical artifact: it persists across different station selections, filter parameters, and modulation-index optimization strategies. Interpreting the observed modulation within the standard linearized gravitational-wave theory would imply radial velocities incompatible with propagation at the speed of light.

However, the linearized theory is only valid in vacuum and under weak curvature. Between Jupiter and Earth, the spacetime curvature is not negligible, and the Einstein equations are strongly non-linear.

We therefore interpret the observed factor of 12.7 not as a violation of relativity, but as evidence that the linear Doppler formula is not applicable to μHz gravitational waves propagating through the non-linear, mass-rich spacetime between Jupiter and Earth. Our result suggests that dispersion or other non-linear propagation effects may play a role at these frequencies.

12 COMPETING INTERESTS

All investigations, including coding, were carried out by H. Weidner, who is also the sole author. The author declares no conflict of interest.

13 FUNDING

The author declares no funding.

14 AVAILABILITY OF PROGRAMS AND DATA

The German Weather Service (DWD 2021) stores historical measurement results from German weather stations (raw data). The error-corrected barometer files are also available from the author upon request.

The homodyne detection code was written in MATLAB R2020a. The code (Jupiq8.m and JupRichtung.m) is available in the github repository:
https://github.com/herbertweidner/cGW/tree/main

The programs will be explained by the author upon request.

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