Understanding Euclidean Quantum Mechanics and the Path Integral of Quantum Force Fields

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Abstract

Quantum mechanics, as historically formulated, rests on a mathematical shortcut introduced by Weyl in 1918. Weyl attempted to "quantize" the gravitational metric by replacing it with Euler's complex phase $\exp(iX)$, a move later adopted by Schrödinger in 1926 to model atomic spectra. Although Weyl's unification of electromagnetism and gravity was physically incorrect and rejected by Einstein, its mathematical residue — the use of complex configuration space and a single complex wavefunction — became entrenched in quantum mechanics. This "groupthink" formalism obscures the true dynamical mechanism of quantum phenomena.

Feynman's path integral formulation reveals the correct relativistic structure: quantum behaviour arises from multipath interference, not from a single complex wavefunction. Euler's identity,

$$\exp(iS) = i \sin S + \cos S,$$

shows that the complex phase is merely a compressed encoding of a deeper real structure. When multipath interference is treated correctly, the imaginary term $i \sin S$ is unnecessary; the real term $\cos S$ suffices. This is directly analogous to HF radio multipath interference, where all possible paths contribute to the resultant signal. Thus quantum mechanics and quantum field theory can be formulated entirely in Euclidean spacetime using real interference kernels.

The fundamental dynamical object is the real interference functional, *IF[x,y],* over pairs of Euclidean histories,

$$IF[x, y] = \cos\left(\frac{S_E[x] - S_E[y]}{\hbar}\right),$$

from which all transition probabilities are constructed. The usual complex phase $\exp(iS/\hbar)$ is shown to be a compressed representation of this real structure. A Euclidean Schrödinger equation emerges as a coarse-grained approximation to the double-path interference dynamics. The hydrogen atom spectrum is derived within this framework, demonstrating full equivalence with standard quantum mechanics while eliminating complex numbers from the axioms.

We then extend the Euclidean interference principle to quantum field theory, introducing a mechanistic SU(2) electromagnetic force-field model. Massless charged spin-1

bosons cannot propagate in one direction due to infinite magnetic self-inductance; only bidirectional exchange equilibria are dynamically allowed. The photon emerges as a real two-history excitation, half propagating forward and half backward, with magnetic curls cancelling exactly as in two-conductor transmission lines. Spin-torque transfer provides a physical origin for magnetic fields, recovering Maxwell's original mechanical intuition.

This unified Euclidean framework replaces complex Hilbert space with real geometry, replaces Lorentzian structure with emergent effective dynamics, and reveals the simple underlying machinery that Feynman anticipated: quantum phenomena arise from real multipath interference, not from complex wavefunctions.

The group SU(4) contains the Standard Model gauge groups as subgroups:

$$\mathrm{SU}(4) \supset \mathrm{SU}(3) \times \mathrm{U}(1)_G \times \mathrm{SU}(2).$$

This decomposition provides the geometric origin of:

- color charge (from SU(3)),
- gravitational/dark-energy charge (from $\mathrm{U}(1)_G$),
- weak isospin (from SU(2)).

The $\mathrm{U}(1)_G$ factor is the repulsive spin-1 field responsible for both cosmological acceleration and gravity. The SU(2) factor produces weak-isospin doublets and explains why only down-type quarks undergo beta decay. The SU(3) factor produces color confinement and the shell hierarchy of baryon masses.

Contents

1 Historical Foundations: The Failure of Complex Quantum Mechanics

The conventional formulation of quantum mechanics is historically rooted not in physical necessity, but in a mathematical detour introduced by Hermann Weyl in 1918. In his attempt to unify electromagnetism with gravity, Weyl proposed a "gauge" theory in which the gravitational metric was multiplied by Euler's complex phase $\exp(iX)$. Einstein immediately rejected Weyl's theory, pointing out that it predicted unphysical changes in atomic spectral lines under parallel transport. Although Weyl's unification was physically incorrect, the mathematical residue — the use of complex configuration space and complex phases — survived.

In 1926, Schrödinger adopted Weyl's complex phase $\exp(iX)$ in his wave equation, thereby importing Weyl's mathematical device into quantum mechanics. This move introduced complex Hilbert space into the axioms of quantum theory, even though the underlying physical motivation (Weyl's failed gravitational unification) was false. The resulting formalism became entrenched through historical inertia rather than physical necessity.

Richard Feynman repeatedly emphasized that the true mechanism of quantum behaviour is *multipath interference*, not the manipulation of a single complex wavefunction. In *QED: The Strange Theory of Light and Matter* (Penguin, 1990), pp. 55–56 and 84–85, Feynman writes:

> "I would like to put the uncertainty principle in its historical place: when the revolutionary ideas of quantum physics were first coming out, people still tried to understand them in terms of old-fashioned ideas. But at a certain point the old-fashioned ideas would begin to fail, so a warning was developed that said, in effect, 'Your old-fashioned ideas are no damn good when ...'. If you get rid of all the old-fashioned ideas and instead use the ideas that I'm explaining in these lectures — adding arrows [wavefunction phase amplitudes for each possible event] for all the ways an event can happen — there is no need for an uncertainty principle! On a small scale, such as inside an atom, the space is so small that there is no main path, no 'orbit'; there are all sorts of ways the electron could go, each with an amplitude. The phenomenon of interference becomes very important, and we have to sum the arrows to predict where an electron is likely to be."

In *Surely You're Joking, Mr. Feynman!* (1988), Feynman describes the sociological entrenchment of the complex-wavefunction formalism:

> "It was a kind of one-upmanship, where nobody knows what's going on, and they'd put the other one down as if they did know. They all fake that they

> know, and if one student admits for a moment that something is confusing by asking a question, the others take a high-handed attitude, acting as if it's not confusing at all, telling him that he's wasting their time. I couldn't see how anyone could be educated by this self-propagating system in which people pass exams, and teach others to pass exams, but nobody knows anything."

In *QED* (Penguin, 1990), p. 142, Feynman criticizes the patchwork nature of the Standard Model:

> "Stephen Weinberg and Abdus Salam tried to combine quantum electrodynamics with what's called the 'weak interactions' (interactions with W's) into one quantum theory, and they did it. But if you just look at the results they get you can see the glue, so to speak. It's very clear that the photon and the three W's are interconnected somehow, but ... you can still see the 'seams' in the theories; they have not yet been smoothed out so that the connection becomes ... more correct."

In his 1964 Cornell Messenger Lectures, published as *The Character of Physical Law*, Feynman expresses his dissatisfaction with the complexity of the conventional formalism:

> "It always bothers me that, according to the laws as we understand them today, it takes a computing machine an infinite number of logical operations to figure out what goes on in no matter how tiny a region of space, and no matter how tiny a region of time. How can all that be going on in that tiny space? Why should it take an infinite amount of logic to figure out what one tiny piece of spacetime is going to do? So I have often made the hypothesis that ultimately physics will not require a mathematical statement, that in the end the machinery will be revealed, and the laws will turn out to be simple, like the chequer board with all its apparent complexities."

These quotations reveal a consistent theme: the conventional complex-wavefunction formalism is not the fundamental machinery of quantum theory. The true mechanism is multipath interference.

Euler's identity,

$$\exp(iS) = i \sin S + \cos S,$$

shows that the complex phase is a compressed encoding of a deeper real structure. When multipath interference is treated correctly, the imaginary term $i \sin S$ is unnecessary; the real term $\cos S$ suffices. This is directly analogous to HF radio multipath interference in the 1960s, where signals propagated along all possible ionospheric paths (D, E, and F layers), and the resultant amplitude was the real superposition of all contributions.

Thus quantum mechanics and quantum field theory can be formulated entirely in Euclidean spacetime using real interference kernels. The fundamental dynamical object is the real interference functional over pairs of Euclidean histories,

$$IF[x, y] = \cos\left(\frac{S_E[x] - S_E[y]}{\hbar}\right), \tag{1}$$

from which all transition probabilities are constructed. The usual complex phase $\exp(iS/\hbar)$ is shown to be a compressed representation of this real structure.

This real kernel is the Euclidean analogue of the Gell-Mann–Hartle decoherence functional, which also compares pairs of histories. The cosine structure is physically meaningful: the real amplitude is the equal-weight interference of a path with its time-reversed twin. This is precisely the structure Feynman used to interpret antiparticles as particles moving backward in time.

The Euclidean interference principle allows us to derive the hydrogen spectrum, Maxwell's equations, and the SU(2) electromagnetic force field without complex numbers in the axioms. Lorentzian structure and complex Hilbert space emerge as effective descriptions rather than foundational ingredients.

2 Euclidean Spacetime and the Real Interference Principle

2.1 Physical Motivation: The Double–Slit Experiment

Before introducing the Euclidean formalism, it is essential to recall the physical origin of quantum interference as emphasized by Feynman. In Volume III of the *Feynman Lectures on Physics* (1965), Feynman demonstrates the double–slit experiment using photons, electrons, water waves in a tank, and even classical particles. His purpose was to show that interference is not an abstract mathematical construction but a universal physical phenomenon.

In the double–slit experiment, the probability of detection at a point on the screen is not determined by a single "wavefunction" but by the *interference* of contributions from at least two distinct paths: one through slit 1 and one through slit 2. Feynman writes:

> "When an event can happen in several different ways, the total amplitude is the sum of the amplitudes for each way."

This is the simplest nontrivial example of multipath interference. The experiment requires at least two histories, x_1 and x_2, whose amplitudes interfere. The interference pattern disappears when one slit is closed, demonstrating that quantum behaviour arises from the comparison of *pairs* of histories.

Feynman repeatedly emphasized that the double–slit experiment contains the entire mystery of quantum mechanics. In *QED: The Strange Theory of Light and Matter* (1985), he writes:

> "The double–slit experiment contains the only mystery. We cannot make the mystery go away by explaining how it works. We will just tell you how it works."

2.2 Why Two Histories Suffice

The double–slit experiment shows that quantum interference requires at least two histories. Feynman's path integral formalism sums amplitudes over all possible paths, but the essential interference structure is already present in the simplest nontrivial case: a pair of histories.

This is why the Gell–Mann–Hartle decoherence functional compares *pairs* of histories, not single histories.

In many physical systems, including the hydrogen atom, the dominant quantum behaviour arises from the interference between two effective histories:

- a forward–time history $x(T)$,
- and a backward–time (time–reversed) history $y(T)$.

Euler's identity,

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shows that the real amplitude $\cos S$ is the equal–weight interference of a path with its time–reversed twin. This is precisely the structure Feynman used to interpret antiparticles as particles moving backward in time.

Thus, the Euclidean interference principle begins with the simplest nontrivial quantum system: a pair of histories whose action difference determines the interference. This is not an abstraction; it is the direct generalization of the double–slit experiment.

2.3 Euclidean Spacetime

We now introduce the Euclidean formalism. Spacetime is taken to be Euclidean with coordinates $(T, \mathbf{x})$ and metric

$$ds^2 = dT^2 + d\mathbf{x}^2. \tag{2}$$

A history $x(T)$ carries Euclidean action

$$S_E[x] = \int dT \, \left[\frac{m}{2} \dot{x}^2 + V(x) \right]. \tag{3}$$

Quantum behaviour arises from interference between *pairs* of histories. Define the real interference functional

$$IF[x, y] = \cos\left(\frac{S_E[x] - S_E[y]}{\hbar} \right). \tag{4}$$

This replaces the single complex wavefunction with a real kernel comparing two histories. The Born rule already contains this structure: probabilities require two copies of the wavefunction amplitude,

$$p = \Psi^* \Psi.$$

Thus even non-relativistic QM secretly depends on a two-amplitude structure. The Euclidean interference principle makes this explicit.

The transition probability between boundary points a and b is

$$P(b, a) = N \int_{x(a)=y(a)=a}^{x(b)=y(b)=b} Dx \, Dy \, \cos\left(\frac{S_E[x] - S_E[y]}{\hbar} \right), \tag{5}$$

with normalization N fixed by total probability.

This real kernel is the fundamental dynamical object. The usual complex phase $\exp(iS/\hbar)$ is shown to be a compressed encoding of this real structure.

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with normalization N fixed by total probability.

This real kernel is the fundamental dynamical object. The usual complex phase $\exp(iS/\hbar)$ is shown to be a compressed encoding of this real structure.

3 Emergent Euclidean Schrödinger Equation

Define a coarse-grained real amplitude

$$\Psi(x, T) = \int Dy\, \cos\left(\frac{S_E[x] - S_E[y]}{\hbar}\right). \tag{10}$$

Under locality and Markovian coarse-graining, Ψ satisfies the Euclidean Schrödinger equation

$$\hbar\, \partial_T \Psi = \left(\frac{\hbar^2}{2m}\nabla^2 - V\right)\Psi. \tag{11}$$

Analytic continuation $T \to it$ yields the usual Schrödinger equation,

$$i\hbar\, \partial_t \Psi = \left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\Psi. \tag{12}$$

Thus the complex structure is emergent rather than fundamental.

4 Positronium as a Direct Example of Euclidean Two Wavefunction Amplitudes Interference

Positronium is the simplest physical system in which the Euclidean interference principle can be interpreted literally. It consists of an electron and an anti–electron (positron) bound together in a hydrogen–like state. The system is unstable and annihilates into gamma radiation with a mean lifetime of 0.125 ns for the singlet (anti–parallel spins) and 142 ns for the triplet (parallel spins). These lifetimes are experimentally measured and provide a direct window into the interference of forward–time and backward–time histories.

4.1 Euler's Identity and Forward/Backward Histories

Euler's identity,

$$\cos S = \frac{1}{2}\left(e^{iS} + e^{-iS}\right), \tag{13}$$

shows that the real amplitude $\cos S$ is the equal–weight interference of a path with its time–reversed twin. In the Euclidean formulation, the two exponentials correspond to:

- e^{iS}: the forward–time history of the electron,
- e^{-iS}: the backward–time history of the positron.

This is not a metaphor. It is precisely the structure Feynman used in his 1942 work with Wheeler, where antiparticles were interpreted as particles moving backward in time in the path integral.

Thus the Euclidean interference kernel,

$$IF[x,y] = \cos\left(\frac{S_E[x] - S_E[y]}{\hbar}\right), \tag{14}$$

compares:

- $x(T)$: the electron's forward–time history,
- $y(T)$: the positron's backward–time history.

Positronium is therefore the simplest physical realization of the Euclidean interference principle.

4.2 Two–Body Euclidean Action

Let $x(T)$ be the electron history and $y(T)$ the positron history. The Euclidean action for the two–body system is

$$S_E[x,y] = \int dT\ \left[\frac{m}{2}\dot{x}^2 + \frac{m}{2}\dot{y}^2 - \frac{e^2}{4\pi\varepsilon_0|x-y|}\right]. \tag{15}$$

The interference functional is

$$IF[x,y] = \cos\left(\frac{S_E[x] - S_E[y]}{\hbar}\right), \tag{16}$$

which is the real amplitude for the two–body system.

4.3 Bound State as Interference of Two Histories

The bound state arises from the interference of the two histories. The Euclidean Schrödinger equation for the relative coordinate $r = |x - y|$ is identical to the hydrogen atom equation, with reduced mass $m/2$:

$$\hbar\,\partial_T \Psi = \left(\frac{\hbar^2}{m} \nabla^2 - V(r) \right) \Psi. \tag{17}$$

Thus the positronium energy levels are

$$E_n = -\frac{1}{4} \frac{me^4}{(4\pi\varepsilon_0)^2 \hbar^2} \frac{1}{n^2}, \tag{18}$$

which is exactly half the hydrogen binding energy, as observed.

4.4 Spin Structure and Annihilation

The annihilation lifetimes,

$$\tau_{\text{singlet}} = 0.125\,\text{ns}, \tag{19}$$
$$\tau_{\text{triplet}} = 142\,\text{ns}, \tag{20}$$

arise from the interference of the two histories with different spin alignments. The Euclidean kernel naturally incorporates this through the spin–dependent action difference.

4.5 Interpretation

Positronium demonstrates explicitly that:

- quantum interference is the interference of two histories,
- the real amplitude $\cos S$ is the interference of a path with its time–reversed twin,
- antiparticles are backward–time histories,
- bound states arise from Euclidean interference,
- the hydrogen atom is fundamentally a two–particle system (electron + proton),
- the proton behaves like a massive backward–time history (positron analogue).

Thus positronium is the simplest and clearest physical example of the Euclidean interference principle.

5 Hydrogen Atom as a Two–History Euclidean Interference System

Positronium demonstrates explicitly that quantum bound states arise from the interference of two histories: a forward–time history and a backward–time history. The hydrogen atom is structurally identical, except that the backward–time history corresponds to a massive positively charged particle (the proton) rather than a positron. Thus hydrogen is a direct analogue of positronium, with the proton acting as a heavy backward–time history.

5.1 Euclidean Action for the Electron–Proton System

Let $x(T)$ be the electron history and $y(T)$ the proton history. The Euclidean action is

$$S_E[x, y] = \int dT \left[\frac{m_e}{2}\dot{x}^2 + \frac{m_p}{2}\dot{y}^2 - \frac{e^2}{4\pi\varepsilon_0 |x - y|} \right]. \tag{21}$$

Since $m_p \gg m_e$, the proton's motion contributes negligibly to the kinetic term. Thus the proton behaves like a massive backward–time history, exactly as in Feynman's interpretation of antiparticles.

5.2 Reduction to the Relative Coordinate

Define the relative coordinate

$$r = |x - y|. \tag{22}$$

The reduced mass is

$$\mu = \frac{m_e m_p}{m_e + m_p} \approx m_e. \tag{23}$$

The Euclidean Schrödinger equation for the relative coordinate is

$$\hbar\, \partial_T \Psi = \left(\frac{\hbar^2}{2\mu}\nabla^2 - V(r) \right) \Psi, \tag{24}$$

with

$$V(r) = -\frac{e^2}{4\pi\varepsilon_0 r}. \tag{25}$$

5.3 Stationary Solutions

Assume a stationary solution

$$\Psi(r, \theta, \phi, T) = e^{-ET/\hbar}\, R(r)\, Y_{\ell m}(\theta, \phi). \tag{26}$$

Substituting into the Euclidean Schrödinger equation yields the radial equation

$$\frac{d^2R}{dr^2} + \frac{2}{r}\frac{dR}{dr} - \frac{\ell(\ell+1)}{r^2}R + \frac{2\mu}{\hbar^2}\left(E + \frac{e^2}{4\pi\varepsilon_0 r} \right) R = 0. \tag{27}$$

5.4 Quantization

Define the Bohr radius

$$a_0 = \frac{4\pi\varepsilon_0\hbar^2}{\mu e^2}, \tag{28}$$

and the dimensionless variable

$$\rho = \frac{2r}{na_0}. \tag{29}$$

Writing

$$R(r) = \rho^\ell e^{-\rho/2} L(\rho), \tag{30}$$

yields the associated Laguerre equation, whose polynomial solutions require

$$n = 1, 2, 3, \ldots \tag{31}$$

and give the hydrogen spectrum

$$E_n = -\frac{1}{2}\frac{\mu e^4}{(4\pi\varepsilon_0)^2\hbar^2}\frac{1}{n^2}. \tag{32}$$

5.5 Interpretation

Hydrogen is fundamentally a two–history Euclidean interference system:

- the electron provides the forward–time history,
- the proton provides the backward–time history (massive analogue of the positron),
- the bound state arises from the interference of these two histories,
- the Euclidean Schrödinger equation emerges from coarse–graining the interference kernel,
- the hydrogen spectrum follows without complex numbers in the axioms.

Thus the hydrogen atom is not a one–particle system with a mysterious "wavefunction." It is a two–particle interference system, exactly like positronium, with the proton acting as a massive backward–time history. This resolves the "mystery of quantum mechanics" in the hydrogen atom.

5.6 Why Hydrogen is Fundamentally a Two Wavefunction Amplitudes i.e. Two Paths Interference System

The conventional textbook treatment of the hydrogen atom presents it as a single particle (the electron) moving in a classical Coulomb potential generated by a fixed proton. This picture is historically convenient but physically misleading. It obscures the fact that the Coulomb potential itself is a *two–body quantum interference effect.* In the Euclidean formulation, this becomes transparent.

(1) The Coulomb potential is not classical. In standard QM, the Coulomb term $-e^2/(4\pi\varepsilon_0 r)$ is inserted by hand as a classical field. But in quantum field theory, the Coulomb interaction arises from the exchange of virtual photons between two charged particles. It is therefore a *two–particle quantum effect*, not a one–particle potential.

In the Euclidean interference formulation, the Coulomb term arises naturally from the action difference between the electron's forward–time history and the proton's backward–time history. Thus the Coulomb interaction is explicitly a two–history interference phenomenon.

(2) The proton behaves like a massive backward–time history. In positronium, the backward–time history is literally the positron. In hydrogen, the proton plays the same role, except that its mass is 1836 times larger. In the Euclidean action,

$$S_E[x,y] = \int dT \left[\frac{m_e}{2}\dot{x}^2 + \frac{m_p}{2}\dot{y}^2 - \frac{e^2}{4\pi\varepsilon_0|x-y|}\right],$$

the proton's kinetic term is suppressed by the large mass m_p. Thus the proton's history $y(T)$ is effectively static, but it still contributes as the backward–time partner in the interference kernel

$$IF[x,y] = \cos\left(\frac{S_E[x] - S_E[y]}{\hbar}\right).$$

(3) The hydrogen wavefunction is the interference of two histories. The Euclidean Schrödinger equation for the relative coordinate $r = |x - y|$ emerges from coarse–graining the interference kernel. The hydrogen wavefunction $\Psi(r)$ is therefore not a mysterious single–particle object. It is the coarse–grained representation of the interference between:

- the electron's forward–time history $x(T)$,
- and the proton's backward–time history $y(T)$.

This is structurally identical to positronium, except for the mass ratio.

(4) The "mystery of QM" disappears. The conventional single–particle Schrödinger equation obscures the physical origin of the hydrogen atom's quantization. It appears mysterious because it treats the electron as a lone particle with a wavefunction that somehow "knows" about the proton's presence through a classical potential.

In the Euclidean interference formulation:

- the hydrogen atom is a two–body system,
- the bound state arises from interference of two histories,
- the Coulomb potential is the Euclidean action difference between these histories,
- the wavefunction is the coarse–grained interference amplitude,
- and quantization arises from the structure of the interference kernel.

Thus the hydrogen atom is not a one–particle system with a mysterious wavefunction. It is a two–particle interference system, exactly like positronium, with the proton acting as a massive backward–time history. This resolves the "mystery of quantum mechanics" in the hydrogen atom: quantization arises from the interference of two histories, not from a single complex wavefunction evolving in a classical potential.

5.7 Mathematical Demonstration: Hydrogen as a Two–History Euclidean Interference System

We now make explicit the mathematical structure that proves hydrogen is fundamentally a two–history interference system. The key object is the Euclidean interference kernel

$$IF[x,y] = \cos\left(\frac{S_E[x] - S_E[y]}{\hbar}\right), \tag{33}$$

which compares the electron's forward–time history $x(T)$ with the proton's backward–time history $y(T)$.

(1) The Coulomb potential emerges from the action difference. The Euclidean action difference is

$$S_E[x] - S_E[y] = \int dT \left[\frac{m_e}{2}\dot{x}^2 - \frac{m_p}{2}\dot{y}^2 - \frac{e^2}{4\pi\varepsilon_0 |x-y|}\right]. \tag{34}$$

Since $m_p \gg m_e$, the proton's kinetic term is negligible:

$$\frac{m_p}{2}\dot{y}^2 \approx 0. \tag{35}$$

Thus the action difference reduces to

$$S_E[x] - S_E[y] = \int dT \left[\frac{m_e}{2}\dot{x}^2 - \frac{e^2}{4\pi\varepsilon_0 r}\right], \qquad r = |x-y|. \tag{36}$$

The Coulomb potential therefore arises directly from the Euclidean action difference between the two histories. It is not inserted by hand. It is the interference term.

(2) The Euclidean kernel produces the radial Schrödinger equation. Define the coarse–grained amplitude

$$\Psi(r,T) = \int Dy \, \cos\left(\frac{S_E[x] - S_E[y]}{\hbar}\right). \tag{37}$$

Under locality and Markovian coarse–graining, Ψ satisfies

$$\hbar\, \partial_T \Psi = \left(\frac{\hbar^2}{2\mu}\nabla^2 - V(r)\right)\Psi, \tag{38}$$

with reduced mass $\mu \approx m_e$ and

$$V(r) = -\frac{e^2}{4\pi\varepsilon_0 r}. \tag{39}$$

Thus the Schrödinger equation is not fundamental; it is the diffusion equation for the interference kernel.

(3) The Laguerre structure arises from the interference kernel. Assume a stationary solution

$$\Psi(r,\theta,\phi,T) = e^{-ET/\hbar} R(r) Y_{\ell m}(\theta,\phi). \tag{40}$$

Substituting into the Euclidean equation yields

$$\frac{d^2R}{dr^2} + \frac{2}{r}\frac{dR}{dr} - \frac{\ell(\ell+1)}{r^2}R + \frac{2\mu}{\hbar^2}\left(E + \frac{e^2}{4\pi\varepsilon_0 r}\right)R = 0. \tag{41}$$

Define

$$\rho = \frac{2r}{na_0}, \qquad a_0 = \frac{4\pi\varepsilon_0\hbar^2}{\mu e^2}. \tag{42}$$

Writing

$$R(r) = \rho^{\ell} e^{-\rho/2} L(\rho) \tag{43}$$

yields the associated Laguerre equation

$$\frac{d^2L}{d\rho^2} + \left(\frac{2\ell+2}{\rho} - 1\right)\frac{dL}{d\rho} + (n-\ell-1)L = 0. \tag{44}$$

This polynomial structure arises directly from the Euclidean interference kernel, not from a single wavefunction evolving in a classical potential.

(4) The hydrogen spectrum is the interference spectrum. Polynomial solutions require

$$n = 1, 2, 3, \ldots \tag{45}$$

and give

$$E_n = -\frac{1}{2}\frac{\mu e^4}{(4\pi\varepsilon_0)^2\hbar^2}\frac{1}{n^2}. \tag{46}$$

Thus the hydrogen spectrum is the spectrum of the Euclidean interference kernel.

(5) The proton's backward–time history is mathematically equivalent to the positron's. In positronium, the backward–time history is the positron. In hydrogen, the backward–time history is the proton.

Mathematically, both appear in the interference kernel as

$$e^{-iS} \quad \text{(backward–time history)}. \tag{47}$$

The only difference is the mass term in the kinetic action. Thus hydrogen is positronium with a heavy backward–time history.

(6) The hydrogen wavefunction is the coarse–grained interference amplitude. The hydrogen wavefunction $\Psi(r)$ is not a mysterious single–particle object. It is the coarse–grained representation of the interference between:

- the electron's forward–time history $x(T)$,
- and the proton's backward–time history $y(T)$.

Conclusion. Hydrogen is a two–history Euclidean interference system. The Coulomb potential, the Schrödinger equation, the Laguerre structure, and the hydrogen spectrum all arise from the interference kernel. Nothing in the hydrogen atom requires a single complex wavefunction or a classical potential.

This resolves the "mystery of quantum mechanics" in the hydrogen atom.

5.8 Approximate Euclidean Prediction of Stability: Hydrogen versus Positronium

In the Euclidean two–history framework, the stability of a bound state is controlled by how efficiently the interference kernel can couple to radiative (photon) histories. The key quantity is the acceleration of each constituent, since the radiative electric field in the Feynman/Jefimenko form,

$$E_{\text{rad}} \sim \frac{q\,a\sin\theta}{Rc^2}, \tag{48}$$

is proportional to the acceleration a.

(1) Accelerations and radiation strength. For a given electromagnetic force F,

$$a_e = \frac{F}{m_e}, \qquad a_p = \frac{F}{m_p}, \qquad a_{\text{pos}} = \frac{F}{m_e}, \tag{49}$$

so

$$\frac{a_p}{a_e} = \frac{m_e}{m_p} \approx \frac{1}{1836}. \tag{50}$$

In positronium, both constituents have mass m_e, so both histories have large acceleration a_{pos}. In hydrogen, only the electron has large acceleration; the proton's acceleration is suppressed by 1/1836.

Thus the radiative field strengths scale as

$$E_{\text{rad}}^{\text{Ps}} \sim \frac{q\,a_{\text{pos}}}{Rc^2}, \qquad E_{\text{rad}}^{\text{H}} \sim \frac{q\,a_e}{Rc^2} + \frac{q\,a_p}{Rc^2} \approx \frac{q\,a_e}{Rc^2}\left(1 + \frac{1}{1836}\right). \tag{51}$$

But in positronium, *both* histories radiate strongly and symmetrically; in hydrogen, only one does.

(2) Euclidean decay functional and lifetime scaling. Model the decay rate Γ as proportional to the squared radiative field integrated over the two–history interference kernel:

$$\Gamma \propto \int Dx\,Dy\,\left(E_{\text{rad}}[x,y]\right)^2 \cos\!\left(\frac{S_E[x]-S_E[y]}{\hbar}\right). \tag{52}$$

For positronium, both histories contribute equally:

$$\Gamma_{\text{Ps}} \propto 2\,\left(E_{\text{rad}}^{\text{pos}}\right)^2 \sim 2\left(\frac{q\,a_{\text{pos}}}{Rc^2}\right)^2. \tag{53}$$

For hydrogen, the proton's contribution is suppressed:

$$\Gamma_{\rm H} \propto (E^e_{\rm rad})^2 + (E^p_{\rm rad})^2 \sim \left(\frac{q\,a_e}{Rc^2}\right)^2 \left(1 + \frac{1}{1836^2}\right) \approx \left(\frac{q\,a_e}{Rc^2}\right)^2. \tag{54}$$

However, the crucial difference is that in positronium the two histories can annihilate directly into on–shell photon histories (electron + positron $\to 2\gamma, 3\gamma$), whereas in hydrogen the proton's internal structure and large mass suppress such channels. In the Euclidean kernel, this appears as a suppression of the measure of annihilating two–history paths for hydrogen relative to positronium.

We can therefore write schematically

$$\Gamma_{\rm Ps} \sim \alpha^2\,\mu_{\rm Ps}\,c^2, \qquad \Gamma_{\rm H} \sim \alpha^2\,\mu_{\rm H}\,c^2\,\epsilon, \tag{55}$$

with $\mu_{\rm Ps} = m_e/2$, $\mu_{\rm H} \approx m_e$, and $\epsilon \ll 1$ a suppression factor encoding the proton's large mass and internal structure. The lifetimes are

$$\tau_{\rm Ps} \sim \frac{1}{\Gamma_{\rm Ps}}, \qquad \tau_{\rm H} \sim \frac{1}{\Gamma_{\rm H}} \gg \tau_{\rm Ps}. \tag{56}$$

Thus, within the Euclidean two–history framework, positronium is predicted to be short–lived, while hydrogen is effectively stable, because the symmetric light–light system couples strongly to radiative two–history photon channels, whereas the asymmetric light–heavy system does not.

Conclusion. The large mass difference between proton and positron enters the Euclidean interference kernel through the acceleration and action terms, and naturally explains why positronium rapidly decays while hydrogen is stable. Stability and instability are therefore interference–kernel properties, not mysterious features of a single–particle wavefunction.

6 SU(2) Electromagnetic Force Field in Euclidean Quantum Field Theory

In this section we introduce an SU(2) force-field structure in which the fundamental carriers of electromagnetic interaction are massless charged spin–1 bosons. These bosons cannot propagate unidirectionally due to infinite magnetic self–inductance; instead, they exist only in bidirectional exchange equilibria. The photon emerges as a real two–history excitation of this SU(2) exchange field.

The Euclidean interference principle provides the natural foundation for a mechanistic electromagnetic force field consistent with the axiomatic structure developed in vixra: 2606.0090 and the unified SO(3,3) → SU(4) → SU(2)×U(1) framework of vixra: 2606.0012. In the axiomatic Euclidean-first formulation, all propagators arise from Laplace transforms of real-space potentials, and all gauge symmetries emerge from vacuum polarization and vacuum-exchange equilibrium rather than being postulated. Within this structure, the electromagnetic interaction must be mediated by massless spin–1 excitations whose Euclidean propagator is the unique Laplace-transform solution of the Coulomb potential. The requirement of isotropic vacuum exchange (Axiom 4) and the perfect-fluid vacuum below the IR cutoff (Axiom 3) forces the gauge symmetry to be non-Abelian at short distances, with SU(2) emerging as the minimal group supporting bidirectional exchange equilibria.

In this mechanistic picture, the fundamental carriers of electromagnetic interaction are massless charged SU(2) spin–1 bosons. A single charged boson cannot propagate unidirectionally because its magnetic self-inductance diverges, as shown by the Euclidean Feynman–Jefimenko field and the Laplace-propagator structure. Only bidirectional exchange equilibria cancel the infinite self-inductance, producing a finite, stable two-history field configuration. The photon then emerges as a real two-history excitation of this SU(2) exchange field: half forward-time, half backward-time, exactly mirroring the Euclidean interference kernel $\cos[(S_E^{(+)} - S_E^{(-)})/\hbar]$ and the forward/backward propagator duality derived in the axiomatic framework. Thus the SU(2) electromagnetic force field is not an ad hoc gauge choice but the unique mechanistic structure compatible with Euclidean propagators, vacuum polarization, and the emergent gauge symmetry of the unified SO(3,3) $\to$ SU(4) theory.

6.1 Euclidean Double–Field Action for Charged Spin–1 Bosons

Let $A_\mu^{(+)}(x)$ and $A_\mu^{(-)}(x)$ denote forward–time and backward–time histories of a massless charged spin–1 boson. The Euclidean double–field action is

$$S_E[A^{(+)}, A^{(-)}] = \int d^4x \left[\frac{1}{4}F_{\mu\nu}^{(+)}F_{\mu\nu}^{(+)} + \frac{1}{4}F_{\mu\nu}^{(-)}F_{\mu\nu}^{(-)} + J_\mu^{(+)}A_\mu^{(+)} + J_\mu^{(-)}A_\mu^{(-)}\right], \tag{57}$$

with

$$F_{\mu\nu}^{(\pm)} = \partial_\mu A_\nu^{(\pm)} - \partial_\nu A_\mu^{(\pm)}. \tag{58}$$

The Euclidean interference functional is

$$IF[A^{(+)}, A^{(-)}] = \cos\left(\frac{S_E[A^{(+)}] - S_E[A^{(-)}]}{\hbar}\right). \tag{59}$$

This is the field–theoretic analogue of the two–history kernel used for positronium and hydrogen.

6.2 Infinite Magnetic Self–Inductance Forbids One–Way Propagation

Consider a massless charged spin–1 boson propagating in one direction. Its magnetic field is given by the Euclidean analogue of the Feynman/Jefimenko expression,

$$\mathbf{B} \sim \frac{q\,\mathbf{v}\times\hat{\mathbf{R}}}{R^2 c} + \frac{q\,\mathbf{a}\times\hat{\mathbf{R}}}{Rc^2}. \tag{60}$$

For a massless charged boson, $|\mathbf{v}| = c$ and the acceleration term dominates. The magnetic self–inductance L of a single conductor carrying a current I is

$$L \sim \mu_0 \int \frac{I}{R}\, d\ell, \tag{61}$$

which diverges logarithmically as $R \to 0$. For a massless charged boson, the effective current is $I \sim qc$, so

$$L_{\text{self}} \to \infty. \tag{62}$$

Thus a single charged boson cannot propagate unidirectionally: the magnetic self–inductance is infinite.

This is the key dynamical constraint that forces SU(2) exchange.

6.3 Bidirectional Exchange Equilibria

To cancel the infinite self–inductance, two oppositely directed bosons must propagate simultaneously. Let $A_\mu^{(+)}$ propagate forward and $A_\mu^{(-)}$ propagate backward. Their magnetic fields cancel:

$$\mathbf{B}_{\text{net}} = \mathbf{B}[A^{(+)}] + \mathbf{B}[A^{(-)}] \approx 0. \tag{63}$$

This is exactly the same cancellation mechanism as in a two–conductor transmission line, where the magnetic fields of the forward and return currents cancel outside the line.

Thus the SU(2) electromagnetic field is a two–history exchange equilibrium:

$$A_\mu = A_\mu^{(+)} - A_\mu^{(-)}. \tag{64}$$

6.4 Photon as a Two–History Excitation

A photon corresponds to a disturbance in the exchange equilibrium:

$$\delta A_\mu = \delta A_\mu^{(+)} - \delta A_\mu^{(-)}. \tag{65}$$

The Euclidean interference functional for the photon is

$$IF_\gamma = \cos\left(\frac{S_E[\delta A^{(+)}] - S_E[\delta A^{(-)}]}{\hbar}\right). \tag{66}$$

Thus the photon is a real two–history excitation, half forward–time and half backward–time. This explains why the photon has no magnetic self–inductance: the forward and backward histories cancel their magnetic curls.

6.5 Emergence of Maxwell's Equations

Stationary phase of the interference kernel yields

$$\frac{\delta S_E[A^{(+)}]}{\delta A_\mu^{(+)}} = 0, \qquad \frac{\delta S_E[A^{(-)}]}{\delta A_\mu^{(-)}} = 0. \tag{67}$$

These are the Euclidean Maxwell equations for $A^{(+)}$ and $A^{(-)}$. Subtracting them yields the Lorentzian Maxwell equations for the physical field A_μ after analytic continuation $T \to it$.

Thus Maxwell's equations emerge from the SU(2) exchange equilibrium.

6.6 Spin–Torque Origin of Magnetism

The magnetic field in this SU(2) model arises from spin–torque transfer between the forward–time and backward–time histories. The torque density is

$$\boldsymbol{\tau} = \mathbf{A}^{(+)} \times \mathbf{A}^{(-)}. \tag{68}$$

This reproduces the structure of the magnetic field:

$$\mathbf{B} \sim \nabla \times \mathbf{A} \sim \mathbf{A}^{(+)} \times \mathbf{A}^{(-)}. \tag{69}$$

Thus magnetism is not a mysterious relativistic correction; it is a spin–torque effect arising from the two–history SU(2) exchange.

6.7 Interpretation

The SU(2) electromagnetic force field is a Euclidean two–history exchange system:

- massless charged spin–1 bosons cannot propagate unidirectionally,
- infinite magnetic self–inductance forbids one–way propagation,
- bidirectional exchange equilibria cancel magnetic curls,
- the photon is a two–history excitation of this equilibrium,
- Maxwell's equations emerge from the Euclidean interference kernel,
- magnetism arises from spin–torque transfer between histories.

This provides a mechanistic foundation for electromagnetism that is fully compatible with the Euclidean interference principle and resolves the conceptual puzzles of the conventional complex–Hilbert–space formulation.

7 Photon as a Real Two–History SU(2) Exchange Excitation

In the axiomatic Euclidean-first formulation (vixra:2606.0090), all propagators arise from Laplace transforms of real-space potentials, and all gauge symmetries emerge from vacuum polarization and vacuum-exchange equilibrium rather than being postulated. Section 6 showed that the electromagnetic interaction must be mediated by massless charged SU(2) spin–1 bosons whose forward-time and backward-time histories form a bidirectional exchange equilibrium. The photon is the unique excitation of this equilibrium.

7.1 Forward/Backward Histories and the Euclidean Interference Kernel

The Euclidean interference kernel for gauge fields is

$$IF[A^{(+)}, A^{(-)}] = \cos\left(\frac{S_E[A^{(+)}] - S_E[A^{(-)}]}{\hbar}\right), \tag{70}$$

where $A_\mu^{(+)}$ and $A_\mu^{(-)}$ are the forward-time and backward-time histories of the SU(2) gauge field. This structure is identical to the two-history kernel used for positronium and hydrogen, and it reflects the forward/backward duality of Laplace-transform propagators derived in Axiom 2.

A photon corresponds to a small disturbance of the SU(2) exchange equilibrium:

$$\delta A_\mu = \delta A_\mu^{(+)} - \delta A_\mu^{(-)}. \tag{71}$$

The Euclidean interference functional for this disturbance is

$$IF_\gamma = \cos\left(\frac{S_E[\delta A^{(+)}] - S_E[\delta A^{(-)}]}{\hbar}\right). \tag{72}$$

Thus the photon is a real two-history excitation: half forward-time, half backward-time.

7.2 Cancellation of Magnetic Self-Inductance

As proved in the previous section, a single charged spin–1 boson cannot propagate unidirectionally because its magnetic self-inductance diverges:

$$L_{\text{self}} \to \infty. \tag{73}$$

Only bidirectional propagation cancels the magnetic curls:

$$\mathbf{B}_{\text{net}} = \mathbf{B}[A^{(+)}] + \mathbf{B}[A^{(-)}] \approx 0. \tag{74}$$

A photon is precisely the finite, curl-cancelled excitation of this SU(2) equilibrium.

7.3 Laplace Propagator and Gauge Symmetry

In the Euclidean-first axiomatic framework, the photon propagator is the Laplace transform of the Coulomb potential:

$$V(r) = \frac{1}{r} \quad \Rightarrow \quad V(k) = \frac{4\pi}{k^2}. \tag{75}$$

This is the unique massless propagator compatible with:

- the Euclidean field equation,
- reflection positivity,
- vacuum isotropy,
- and SU(2) gauge symmetry emerging from vacuum polarization.

The photon is therefore the unique massless SU(2) excitation whose propagator is the Laplace-transform solution of the Coulomb field.

7.4 Emergent U(1) from SU(2) Exchange

In the unified SO(3,3) → SU(4) → SU(2)×U(1) structure (vixra:2606.0012), the physical electromagnetic U(1) symmetry emerges as the diagonal subgroup of SU(2) exchange:

$$U(1)_{\text{em}} = \text{diag}(SU(2)). \tag{76}$$

The photon is the corresponding diagonal excitation of the SU(2) exchange field.

Thus U(1) electromagnetism is not fundamental; it is the emergent Abelian sector of the SU(2) bidirectional exchange equilibrium.

7.5 Interpretation

The photon is:

- a real two-history excitation of the SU(2) exchange field,
- half forward-time and half backward-time,
- stabilized by cancellation of magnetic self-inductance,
- governed by the Laplace-transform massless propagator,
- and the diagonal U(1) excitation of the SU(2) subgroup of SU(4).

This mechanistic picture replaces the abstract complex-wavefunction description with a Euclidean two-history field excitation fully consistent with the axiomatic QFT and the SO(3,3)/SU(4) unified structure.

8 Spin–Torque Origin of Magnetism

In the SU(2) exchange-equilibrium framework developed in Sections 6 and 7, the magnetic field is not a primitive geometric object nor a relativistic correction. Instead, it arises mechanically from spin–torque transfer between the forward-time and backward-time histories of the SU(2) gauge field. This provides a physical origin for magnetism fully consistent with the Euclidean-first axiomatic QFT (vixra:2606.0090) and the unified SO(3,3) → SU(4) structure (vixra:2606.0012).

8.1 SU(2) Exchange and Internal Spin Structure

Let $A_\mu^{(+)}$ and $A_\mu^{(-)}$ denote the forward-time and backward-time histories of the SU(2) gauge field. Each history carries an internal spin–1 degree of freedom. In the bidirectional exchange equilibrium, these spins interact through the SU(2) commutator:

$$[A_\mu^{(+)}, A_\nu^{(-)}] = i\,\epsilon^{abc} A_\mu^{(+)a} A_\nu^{(-)b} T^c, \tag{77}$$

where T^c are the SU(2) generators. This commutator encodes the internal torque between the two histories.

8.2 Torque Density and Magnetic Field

Define the spin–torque density

$$\boldsymbol{\tau} = \mathbf{A}^{(+)} \times \mathbf{A}^{(-)}. \tag{78}$$

This torque density is the physical origin of the magnetic field. The Euclidean analogue of the Maxwell relation is

$$\mathbf{B} = \nabla \times \mathbf{A} \sim \mathbf{A}^{(+)} \times \mathbf{A}^{(-)}. \tag{79}$$

Thus the magnetic field is the coarse-grained spin–torque interaction between the two histories of the SU(2) exchange field.

8.3 Cancellation of External Magnetic Curls

In the bidirectional equilibrium, the external magnetic curls cancel:

$$\mathbf{B}_{\text{ext}} = \mathbf{B}[A^{(+)}] + \mathbf{B}[A^{(-)}] \approx 0. \tag{80}$$

This cancellation is the mechanism that prevents infinite magnetic self-inductance and stabilizes the SU(2) exchange field. Only internal torque survives, producing the physical magnetic field.

8.4 Emergent Maxwell Structure

The torque density satisfies the Euclidean Maxwell equation

$$\nabla \times \mathbf{B} = \partial_T \mathbf{E} + \mathbf{J}, \tag{81}$$

where $\mathbf{E}$ and $\mathbf{J}$ arise from the forward/backward exchange currents. After analytic continuation $T \to it$, this becomes the Lorentzian Maxwell equation.

Thus Maxwell's equations emerge from the SU(2) spin–torque structure, not from a postulated U(1) gauge symmetry.

8.5 Interpretation

Magnetism is:

- a spin–torque interaction between forward-time and backward-time SU(2) histories,
- stabilized by cancellation of external magnetic curls,
- encoded in the commutator structure of SU(2),
- and emerging as the coarse-grained field $\mathbf{B} = \nabla \times \mathbf{A}$.

This mechanistic origin of magnetism recovers Maxwell's original intuition that magnetic fields arise from rotational stresses in the underlying medium. In the Euclidean-first framework, the "medium" is the SU(2) vacuum-exchange equilibrium, and the rotational stress is the spin–torque interaction between its two histories.

9 Weak Interaction as an SU(2) Vacuum–Polarization Branch

In the unified SO(3,3) → SU(4) framework (vixra:2606.0012), the weak interaction is not an independent gauge sector inserted by hand, nor does it require the historical Weinberg–Salam construction with arbitrary hypercharge assignments. Instead, the weak interaction emerges naturally as the first vacuum–polarization branch of the SU(4) structure, generated by the same Euclidean interference principle that produces electromagnetism in Section 6.

9.1 SU(4) Decomposition and the Origin of SU(2)$_{\text{weak}}$

The spin group of SO(3,3) is SU(4). Its maximal subgroups include

$$SU(4) \supset SU(2) \times SU(2) \times U(1). \tag{82}$$

The electromagnetic SU(2) of Section 6 occupies one of these SU(2) factors. The second SU(2) factor corresponds to the weak interaction:

$$SU(2)_{\text{weak}} \subset SU(4). \tag{83}$$

This identification is not arbitrary. It follows from:

- the Euclidean Laplace-propagator structure (Axiom 2),
- the perfect-fluid vacuum below the IR cutoff (Axiom 3),
- vacuum-exchange equilibrium (Axiom 4),
- and the SU(4) representation theory of vixra:2606.0012.

Thus the weak interaction is the second SU(2) branch of the SU(4) vacuum-polarization structure.

9.2 Vacuum Polarization and the Weak Bosons

In the Euclidean-first axiomatic QFT (vixra:2606.0090), vacuum polarization modifies the propagator of a gauge field according to

$$D_{\mu\nu}(p) = \frac{\delta_{\mu\nu}}{p^2 + \Pi(p)}, \tag{84}$$

where $\Pi(p)$ is the vacuum-polarization scalar. For the electromagnetic SU(2) branch, $\Pi(p)$ vanishes at $p^2 = 0$, producing massless photons.

For the weak SU(2) branch, the vacuum polarization does not vanish:

$$\Pi_{\text{weak}}(0) = M_W^2, \tag{85}$$

yielding massive weak bosons. In the Euclidean formulation, this mass arises from the self-energy of the SU(2)$_{\text{weak}}$ exchange field, not from a Higgs mechanism.

Thus the weak bosons are massive because their vacuum-polarization branch is not curl-cancelled by a backward-time partner.

9.3 Forward/Backward Asymmetry and Weak Mass Generation

The electromagnetic SU(2) branch is stabilized by bidirectional exchange:

$$A_\mu = A_\mu^{(+)} - A_\mu^{(-)}. \tag{86}$$

This cancels magnetic self-inductance and produces massless photons.

The weak SU(2) branch lacks this symmetry. Its forward-time and backward-time histories do not cancel:

$$B_\mu^{(+)} \neq B_\mu^{(-)}. \tag{87}$$

The resulting uncancelled self-inductance produces a finite Euclidean self-energy:

$$M_W^2 \sim \Pi_{\text{weak}}(0). \tag{88}$$

Thus weak boson masses arise mechanistically from vacuum polarization, not from symmetry breaking.

9.4 Emergent Weak Interaction from SU(4) Vacuum Structure

The SU(4) vacuum structure contains two SU(2) branches:

$$SU(4) \supset SU(2)_{\text{em}} \times SU(2)_{\text{weak}} \times U(1). \tag{89}$$

The electromagnetic branch is curl-cancelled and massless. The weak branch is not curl-cancelled and therefore massive.

This reproduces the observed structure of the electroweak interaction without invoking a Higgs field, Yukawa couplings, or spontaneous symmetry breaking.

9.5 Interpretation

The weak interaction is:

- the second SU(2) vacuum-polarization branch of SU(4),
- generated by the Euclidean interference principle,
- massive due to uncancelled vacuum self-inductance,
- and unified with electromagnetism through the SU(4) structure.

Thus the weak interaction is not a separate force with arbitrary gauge assignments. It is the natural second branch of the SU(4) vacuum-polarization structure, emerging directly from the Euclidean-first axiomatic QFT and the SO(3,3) geometry.

10 Strong Interaction from SU(3) Vacuum Shells

The SU(3) sector of the unified SO(3,3) $\to$ SU(4) framework carries a simple and empirically forced physical meaning: it encodes the vacuum–polarization shell structure responsible for fractional quark charges. As shown in the mechanistic derivation of vixra: 2511.0122v2, all fermions contain a universal core charge $Q_{\text{core}} = 1/\alpha$, and confinement redistributes the bare Coulomb field energy of this core into three possible components: electric, weak–isospin, and strong (colour). Down–type quarks partition their field energy into three equal thirds (EM/weak/strong), yielding an observable electric charge $-1/3$. Up–type quarks partition only into EM/strong, with no weak–isospin component, yielding $+2/3$. This partitioning is not optional: it is forced by vacuum polarization, SU(2) weak–isospin bookkeeping, and the empirical charge of the Ω^- baryon, which demonstrates that each quark core contributes $1/\alpha$ and that the observable charge is the shielded fraction of this core. Thus the SU(3) sector is the vacuum–shell symmetry governing how the core field energy is redistributed under confinement, and the familiar 1/3 and 2/3 quark charges arise directly from this mechanistic shell structure.

In the unified SO(3,3) $\to$ SU(4) framework (vixra:2606.0012), the strong interaction emerges as the SU(3) vacuum-shell branch of SU(4). Unlike the electroweak SU(2) branches, which arise from forward/backward exchange equilibria, the SU(3) sector originates from radial vacuum-polarization shells surrounding fermionic cores. These shells are generated by the Euclidean Laplace-propagator structure (vixra:2606.0090) and the energy-redistribution principle of the unified theory.

10.1 SU(4) Decomposition and the Origin of SU(3)$_{\text{strong}}$

The spin group of SO(3,3) is SU(4). Its maximal subgroup structure includes

$$SU(4) \supset SU(3) \times U(1). \tag{90}$$

The SU(3) factor corresponds to the strong interaction. In the unified theory, this SU(3) sector arises from the vacuum-polarization shells surrounding fermions, not from an independent gauge symmetry inserted by hand.

The SU(3) generators are the Euclidean analogue of the colour degrees of freedom in conventional QCD.

10.2 Vacuum Shells from Laplace-Transform Propagators

In the Euclidean-first axiomatic QFT (vixra:2606.0090), the vacuum response to a static charge is determined by the Laplace transform of the Coulomb potential. Integrating the field energy density over radial shells yields the shell-energy hierarchy:

$$E_n = \int_{r_n}^{r_{n+1}} \frac{1}{2}|\mathbf{E}(r)|^2\, 4\pi r^2\, dr, \tag{91}$$

where r_n are the shell radii determined by vacuum polarization.

10.3 Running Coupling from Shell Energy Redistribution

The running of the strong coupling arises from energy redistribution between vacuum shells:

$$\alpha_s(r) \sim \frac{E_{\text{shell}}(r)}{E_{\text{total}}}. \tag{92}$$

As r decreases, more shells contribute, increasing the effective coupling. As r increases, fewer shells contribute, decreasing the coupling.

This mechanism produces asymptotic freedom of quarks:

$$\alpha_s(r) \to 0 \quad \text{as} \quad r \to 0, \tag{93}$$

and confinement:

$$\alpha_s(r) \to \infty \quad \text{as} \quad r \to r_{\text{shell}}. \tag{94}$$

10.4 SU(3) as the Vacuum-Shell Symmetry

The three vacuum shells correspond to the three colour charges:

$$\{r_1, r_2, r_3\} \quad \leftrightarrow \quad \{R, G, B\}. \tag{95}$$

The SU(3) generators act on these shells by mixing their energies:

$$E_i \to E_i + \epsilon^{ijk} E_j T^k. \tag{96}$$

Thus SU(3) is the symmetry group of vacuum-shell energy redistribution.

This mechanistic interpretation replaces the abstract colour-charge picture of QCD with a Euclidean vacuum-shell structure.

10.5 Interpretation

The strong interaction is:

- the SU(3) vacuum-shell branch of SU(4),
- generated by Laplace-transform propagators,
- confined by the linear growth of the Euclidean vacuum potential,
- asymptotically free due to shell-energy redistribution,
- and mechanistically realized as radial vacuum shells around fermions.

Thus the strong interaction is not a separate Yang–Mills sector. It is the SU(3) vacuum-shell structure emerging directly from the Euclidean-first axiomatic QFT and the SO(3,3)/SU(4) geometry.

The SU(3) generators are the Euclidean analogue of the colour degrees of freedom in conventional QCD.

11 The Quark-Charge Rosetta Stone: Vacuum Polarization and Fractional Charges

One of the most radical and empirically defensible results of the Final Theory is the derivation of fractional quark charges from vacuum polarization. In the Standard Model, quark charges $+2/3$ and $-1/3$ are inserted by hand. Here they arise from a physical mechanism: redistribution of the bare Coulomb field energy of a universal fermion core under confinement.

This section presents the Rosetta Stone argument based on the Ω^- baryon, establishes the universal core charge $1/\alpha$, and derives the fractional quark charges from vacuum-polarization shielding and SU(2) weak-isospin bookkeeping. The result is forced by empirical facts and does not violate QCD.

11.1 Universal fermion core charge: $1/\alpha$

In the Final Theory, all fermions contain a universal core charge

$$Q_{\text{core}} = \frac{1}{\alpha}, \tag{97}$$

where α is the fine-structure constant. This core is surrounded by a vacuum-polarization shell whose shielding determines the observable charge. The shielding strength is proportional to the Coulomb field energy, so larger core charges produce stronger vacuum polarization.

11.2 The Ω^- baryon as Rosetta Stone

The Ω^- baryon contains three identical strange quarks. Each quark core contributes $1/\alpha$, so the total core charge is

$$Q_{\text{core}}^{(\Omega^-)} = \frac{3}{\alpha}. \tag{98}$$

A core charge three times larger produces a vacuum-polarization shield three times stronger. The observable charge is therefore reduced by the same factor:

$$Q_{\text{obs}}^{(\Omega^-)} = \frac{3/\alpha}{3/\alpha} = 1. \tag{99}$$

Thus each strange quark contributes

$$Q_s = -\frac{1}{3}. \tag{100}$$

This argument is empirical: it follows directly from Coulomb-powered vacuum polarization and the observed charge of the Ω^-.

11.3 Vacuum-polarization partitioning of field energy

The bare Coulomb field energy of the fermion core is redistributed into three possible components:

- electric (EM) field energy,
- weak-isospin field energy,
- strong (color) field energy.

The observable electric charge is the fraction of the core field energy that remains unscreened by vacuum polarization.

11.4 Down-type quarks: equal partition into thirds

For down-type quarks (d, s, b), the bare Coulomb field energy is partitioned into three equal thirds:

$$\text{EM: } \frac{1}{3}, \qquad \text{weak: } \frac{1}{3}, \qquad \text{strong: } \frac{1}{3}. \tag{101}$$

This partition is the only one consistent with:

- the Ω^- Rosetta Stone (three identical quarks $\to -1/3$ each),
- SU(2) weak-doublet structure,
- the fact that down-type quarks undergo beta decay.

Thus the down quark has electric charge

$$Q_d = -\frac{1}{3}. \tag{102}$$

11.5 Up-type quarks: EM/strong partition

For up-type quarks (u, c, t), the partition is:

$$\text{EM: } \frac{2}{3}, \qquad \text{weak: } 0, \qquad \text{strong: } \frac{1}{3}. \tag{103}$$

This is forced by SU(2) weak-isospin:

$$T_3(u) = +\frac{1}{2}, \qquad T_3(d) = -\frac{1}{2}. \tag{104}$$

The up quark carries no weak-isospin field energy, explaining:

- up quarks do not beta-decay,
- down quarks do,
- protons are stable,
- neutrons are unstable.

Thus the up quark has electric charge

$$Q_u = +\frac{2}{3}. \tag{105}$$

11.6 Weak-isospin bookkeeping and beta decay

The weak-isospin structure of the SU(2) subgroup of SU(4) forces the following identifications:

- down quark $\leftrightarrow$ left-handed electron core (weak-isospin active),
- up quark $\leftrightarrow$ right-handed electron core (weak-isospin inactive).

Thus:

- down quarks carry weak-isospin field energy $\rightarrow$ they decay,
- up quarks do not carry weak-isospin field energy $\rightarrow$ they do not decay.

This explains neutron decay and proton stability without free parameters.

11.7 No conflict with QCD

This mechanism does not violate QCD. We do not modify:

- gluon charges,
- color coupling strengths,
- flavor-blindness of QCD.

We describe how the bare Coulomb field energy of the universal fermion core is redistributed under confinement. This is orthogonal to QCD's gauge structure.

11.8 Summary

Fractional quark charges arise from vacuum-polarization redistribution of the bare Coulomb field energy of a universal fermion core. The Ω^- baryon provides the Rosetta Stone demonstrating that each quark core carries $1/\alpha$ and that vacuum polarization shields the observable charge by a factor equal to the number of quarks. Down-type quarks partition their field energy into EM/weak/strong thirds, while up-type quarks partition into EM/ strong with no weak-isospin component. This mechanism explains the fractional charges, proton stability, neutron decay, and SU(2) weak-doublet structure, all without violating QCD. The result is radical but empirically forced.

12 Spin–1 Quantum Gravity and the U(1)_G Vacuum Field

Why Conventional Gauge Theory and the Einstein–Hilbert Lagrangian Fail Mechanistically

The foundational category error of Einstein was the compression of rank-1 Maxwell field-line physics into rank-2 spacetime curvature tensors, instead of using the same physical description (all field lines, or all spacetime curvature) for both fields. There are three key failures of the conventional framework: (1) the mathematical structure of Yang–Mills gauge theory, (2) the contrived nature of the Einstein–Hilbert Lagrangian, and (3) the incompatibility between rank-1 gauge fields and rank-2 curvature.

Standard gauge theory begins by defining the non-Abelian field-strength tensor

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^{abc} A^b_\mu A^c_\nu,$$

and then asserts—purely by mathematical analogy—that this object is like curvature, purely because both are rank-2 equations. Yang became interested in the geometry of fibre bundles after discovering in the 1960s that his Yang–Mills non-Abelian field equation is transformable into the Riemann tensor. This 'transformation' is based on two assumptions: (1) the Christoffel symbol equals the gauge potential, and (2) the field-strength tensor equals the Riemann tensor. These assumptions have no physical justification. See vixra: 1301.0187 for further discussion on the corruption of theoretical physics due to the rank-1 vs rank-2 tensor incompatibility cover-up for obfuscation reasons.

Maxwell's equations remain rank-1 even when embedded in rank-2 tensors

The conventional Yang–Mills formulation of gauge theory is historically elegant but physically incomplete. Its mathematical structure is built on rank-1 field-line dynamics (curl and divergence of E and B) artificially embedded into rank-2 tensors by index-selection tricks. Maxwell's equations arise only after "picking out" components of the field-strength tensor by setting one index to zero, which leads to mathematical obfuscation of the physical basis of different field models, since it is not a true rank-2 spacetime curvature but a disguised rank-1 vector calculus. This mismatch becomes fatal when attempting to unify electromagnetism with gravity: the Ricci tensor genuinely describes curvature of spacetime, not the field line divergences and curls of Einstein's method of using components of a rank-2 compression for rank-1 Maxwell equations.

Conventional Yang–Mills theory embeds Maxwell's rank-1 field lines into a rank-2 tensor but never escapes the underlying 3-space geometry. The fibre bundle "transformation" between Yang–Mills and Riemann tensors proposed in the 1970s is therefore a mathematical sleight-of-hand: it equates two objects that describe fundamentally different physics.

Einstein's "unification" of electromagnetism and gravity relied on embedding Maxwell's equations inside the antisymmetric tensor

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.$$

But as shown in *1301.0187v1.pdf*, this embedding does not convert electromagnetism into curvature. If we want compatibility of electromagnetism and general relativity, we must move away from lines in space and towards spacetime curvature, which is truly rank-2 over spacetime indices, without the mathematical trick of picking out rank-1 equations from a rank-2 tensor by artificially setting one index to zero, as in $E_\mu = F_{\mu 0}$.

The problem is structural:

- Maxwell's equations describe rank-1 field-line divergence and curl. - General relativity describes rank-2 curvature of spacetime. - Embedding rank-1 physics inside a rank-2 tensor does not change its physical content. - The "tensor compression" trick hides field lines inside a larger tensor but does not convert them into curvature.

This is why attempts to unify gravity and electromagnetism by tensor algebra have failed for 100 years: they conflate mathematical form with physical mechanism.

Our framework resolves this by treating all interactions as vacuum polarization or energy exchange processes, not as geometric curvature. Rank-1 and rank-2 descriptions become different projections of the same underlying mechanistic vacuum dynamics.

The Einstein–Hilbert Lagrangian is not a quantum Lagrangian

The Einstein–Hilbert Lagrangian,

$$\mathcal{L}_{\mathrm{EH}} = \frac{R\sqrt{-g}}{16\pi G},$$

is usually assumed to be the quantum gravity Lagrangian. But as shown in vixra: 1301.0188v1, this is mathematically and physically false. While $\mathcal{L}_{\mathrm{EH}}$ appears mathematically incontrovertible, it is not logical or correct physically, because it is contrived to model *only the classical path of least action (the real or onshell path in a path integral)*, unlike quantum field theory Lagrangians, in which all paths are taken by gauge bosons but most produce isotropic forces that cancel, leaving the path of least action asymmetric and therefore misleadingly assumed to be the only path.

Quantum field theory requires a Lagrangian that assigns amplitudes to all possible paths. The Einstein–Hilbert Lagrangian assigns meaning only to the single classical extremal path.

Thus:

- It is not a quantum Lagrangian. - It cannot be used in a path integral. - It cannot describe off-shell graviton exchange. - It cannot describe vacuum polarization. - It cannot describe cosmological acceleration. - It cannot describe gravitational reaction forces (Newton's 3rd law).

The Einstein–Hilbert Lagrangian is therefore not a physical law. It is an epicycle-type misleading mathematical device engineered to reproduce the classical geodesic equation, and thus misleads physicists over dynamics!

Our U(1) quantum gravity Lagrangian replaces this contrivance with a mechanistic energy-exchange that automatically yields the correct weak-field classical general relativity limit, without relying on general relativity's misleading spacetime curvature.

The Einstein–Hilbert Lagrangian is not a quantum Lagrangian at all. It is a classical least-action Lagrangian, contrived specifically so that varying the action produces the classical path of least action. Quantum field theory requires a Lagrangian valid for all paths in the path integral, not merely the classical extremal path. The Einstein–Hilbert term is therefore not a true quantum Lagrangian: it encodes only the classical trajectory, not the off-shell fluctuations that dominate quantum dynamics. This is why attempts to quantize gravity by "quantizing the metric" inevitably fail — the metric is already a classical extremal object.

Consequence: the Standard Model and GR share the same foundational mistake

The three failures above share a single root cause: gauge theory and general relativity mistake mathematical structure for physical mechanism.

Gauge theory assumes that Lie algebra structure constants encode physical charge-transfer dynamics. General relativity assumes that curvature tensors encode physical gravitational forces.

Neither assumption is true.

The mechanistic vacuum-polarization framework replaces these assumptions with a single physical principle: Standard Model forces and gravity are not fundamental but are consequences of the redistribution of the shielded electromagnetic field (running coupling) when energy conservation is imposed on vacuum polarization. I.e., electromagnetic field energy that is shielded is absorbed by virtual pairs, which are then able to contribute short ranged nuclear forces.

This principle:

- reproduces Newtonian gravity,
- reproduces the weak-field limit of GR,
- predicts the cosmological acceleration (confirmed 1998),
- eliminates the need for spin-2 gravitons,
- eliminates renormalization divergences,
- explains mass generation without the Higgs field,
- explains electroweak mixing as a running vacuum-polarization angle,
- explains why SU(2) becomes massless at high energy,
- explains why U(1) gravity has no charge sign and no running coupling.

It is the first fully predictive framework to completely unify gravity, dark energy and all known forces in which all interactions share a single mechanistic origin, and all mixing angles and couplings are predicted from first principles rather than inserted by hand as per Standard Model epicycles!

Summary

The conventional mathematical structures of gauge theory and general relativity are not physical theories. They are algebraic models built on category errors:

- rank-1 vs rank-2 confusion, - curvature vs field-line confusion, - classical vs quantum Lagrangian confusion, - algebraic substitution vs mechanistic dynamics.

The Euclidean-first vacuum-polarization framework corrects these errors and provides the first unified, mechanistic, parameter-free description of all interactions.

The SU(4) structure contains a single abelian factor,

$$SU(4) \supset SU(3) \times U(1)\_G \times SU(2), \tag{106}$$

and this $U(1)\_G$ subgroup plays a dual role: it generates both cosmological acceleration and Newtonian gravity. In the Euclidean interference framework, gravity is not curvature of spacetime and not a spin–2 interaction. It is a reaction force produced by the repulsive spin–1 $U(1)\_G$ vacuum field, whose flux–shadow geometry yields Newton's law. This section integrates the mechanistic derivation of gravity from vixra:2511.0122v2 into the Euclidean path–interference formalism developed in Sections 2–10.

12.1 U(1)_G as the Repulsive Vacuum Field

Every mass–energy source emits $U(1)\_G$ bosons isotropically. These bosons carry repulsive momentum, producing an outward acceleration

$$a = Hc, \tag{107}$$

where H is the vacuum–repulsion rate (the observed Hubble parameter). This acceleration is not geometric; it is the net outward momentum transfer from the $U(1)\_G$ boson bath.

In the Euclidean interference picture, the outward flux corresponds to a forward–time history of the $U(1)\_G$ field, while the inward reaction flux corresponds to its backward–time history. Gravity arises from the interference of these two histories.

12.2 Reaction Flux and the Origin of Gravity

By Newton's third law, the outward acceleration a implies an inward reaction flux of equal magnitude:

$$F_{\text{in}} = ma. \tag{108}$$

This inward flux is isotropic in the absence of other masses and produces no net force. When a second mass M is present at distance R, it shadows a fraction of the inward flux. The shadowed fraction is

$$f_{\text{shadow}} = \frac{\sigma_G M}{4\pi R^2}, \tag{109}$$

where σ_G is the graviton–matter scattering cross–section.

The net force on m toward M is therefore

$$F_{\text{grav}} = F_{\text{in}}\, f_{\text{shadow}} = ma\, \frac{\sigma_G M}{4\pi R^2}. \tag{110}$$

12.3 Matching Newton's Law

To reproduce Newton's law,

$$F_{\text{grav}} = \frac{GMm}{R^2}, \tag{111}$$

we require

$$\frac{ma\,\sigma_G M}{4\pi R^2} = \frac{GMm}{R^2}. \tag{112}$$

Thus

$$\sigma_G = \frac{4\pi G}{a}. \tag{113}$$

This relation is purely mechanistic: it equates the screened fraction of the inward $U(1)\_G$ flux to the Newtonian force.

12.4 Weak–Scale Cross–Section and Horizon–Area Scaling

The graviton–matter cross–section is determined by the weak neutral–current cross–section at the Z–boson mass scale. Because the $U(1)\_G$ coupling is extremely small, the cross–section scales in direct proportion to the square of the coupling:

$$\sigma_G \sim \left(\frac{G_N}{G_F}\right)^2 \sigma_{weak}, \tag{114}$$

where G_F is the Fermi constant. This is the horizon–area scaling: the graviton cross–section is proportional to the square of the Schwarzschild radius.

Substituting this into the flux–shadow relation yields a parameter–free prediction of G consistent with CODATA (vixra:2511.0122v2).

12.5 Euclidean Interference Interpretation

The $U(1)\_G$ field has two histories:

- a forward–time history producing outward repulsion,
- a backward–time history producing inward reaction flux.

Gravity is the interference of these two histories, exactly as electromagnetism arises from the interference of forward/backward SU(2) histories (Sections 6–8).

The Euclidean interference kernel for gravity is therefore

$$IF_G[x,y] = \cos\left(\frac{S_G[x] - S_G[y]}{\hbar}\right), \tag{115}$$

where S_G is the Euclidean action of the $U(1)\_G$ field. The flux–shadow geometry arises from coarse–graining this kernel over the vacuum–polarization background.

12.6 Interpretation

Spin–1 quantum gravity is:

- the $U(1)\_G$ vacuum–polarization branch of SU(4),
- repulsive at the microscopic level,
- attractive macroscopically via flux–shadow geometry,
- mechanistic rather than geometric,
- and fully compatible with Euclidean interference dynamics.

Thus gravity is not curvature of spacetime. It is the screened fraction of an inward isotropic momentum flux generated by the repulsive $U(1)\_G$ field. This completes the unified SU(4) force structure: SU(2) electromagnetism, SU(2) weak interaction, SU(3) strong interaction, and U(1)_G quantum gravity all arise from the Euclidean interference principle.

13 Derivation of G using quantum gravity: mechanistic dark energy modelling by spin-1 exchange repulsion, with empirical evidence

We present a complete mechanistic derivation of Newton's gravitational constant G within a unified Euclidean quantum field theory based on the group chain $\mathrm{SO}(3,3) \cong \mathrm{SU}(4) \supset \mathrm{SU}(3) \times \mathrm{U}(1)_G \times \mathrm{SU}(2)$. All interactions arise from vacuum polarization and momentum exchange of spin-1 gauge bosons. Gravity and dark energy are generated by a single repulsive $U(1)_G$ field whose flux-shadow geometry produces Newton's law. Cosmological evolution is not geometric: the Friedmann–Robertson–Walker metric is replaced by bosonic repulsion, number dilution, and vacuum drag. A fully mechanistic derivation of the density-correction factor e^3 is given, arising from three-dimensional number dilution over one Hubble time. The resulting effective density $\rho_{\text{eff}} = \rho_0 e^3$ inserted into the flux-shadow relation yields a parameter-free prediction of G consistent with CODATA.

A second major result is a radical but empirically defensible derivation of fractional quark charges from vacuum polarization. Using the Ω^- baryon as a Rosetta Stone, we show that quark cores carry a universal charge $1/\alpha$, and fractional quark charges arise from redistribution of bare Coulomb field energy into electric, weak-isospin, and strong (color) components. Down-type quarks partition their field energy into three equal thirds (EM/weak/strong), while up-type quarks partition into EM/strong with no weak-isospin component. This explains the 2/3 and 1/3 charges, proton stability, neutron decay, and the SU(2) weak-doublet structure, all without violating QCD. The result is forced by vacuum polarization and is fully empirical.

This replaces earlier derivations of G by presenting a unified, mechanistic, and empirically grounded quantum-gravity framework.

The Standard Model and general relativity are historically successful but structurally incomplete. The SM contains over twenty free parameters, unexplained charge assignments, and an unphysical Higgs mechanism. GR models gravity as classical curvature, incompatible with quantum field theory and unable to explain cosmological acceleration without an ad hoc cosmological constant.

We construct a final theory in which:

- all forces arise from vacuum polarization and gauge-boson momentum transfer,
- all masses arise from energy conservation in vacuum-polarization shells,
- gravity and dark energy arise from a single $U(1)_G$ vacuum field,
- cosmological evolution is mechanistic, not geometric,
- the gravitational constant satisfies $G(t) \propto t$,
- fractional quark charges arise from vacuum-polarization energy redistribution,
- proton stability and neutron decay follow from weak-isospin bookkeeping.

The theory is based on the Euclidean geometry of $\mathrm{SO}(3,3)$ and its spin group $\mathrm{SU}(4)$.

14 Euclidean SO(3,3) Geometry and the SU(4) Spin Group

The Final Theory does not assume a Lorentzian spacetime metric. Instead, spacetime is fundamentally Euclidean, and all relativistic phenomena arise from dynamical interactions between matter and the vacuum-polarization field. This section establishes the geometric foundation of the theory: the replacement of Lorentzian spacetime by $\mathrm{SO}(3,3)$ Euclidean geometry and its spin group $\mathrm{SU}(4)$.

14.1 Why Euclidean geometry replaces Lorentzian spacetime

In classical relativity, time dilation, length contraction, and curvature are kinematic consequences of a Lorentzian metric. In the Final Theory, these effects are produced instead by dynamical interactions with vacuum field quanta. When a particle accelerates into the vacuum, it experiences increased momentum flux from the spin-1 gauge fields. This flux compresses the particle's internal structure in the direction of motion and slows the rate of internal interactions, producing physical time dilation and length contraction.

Because these effects arise from field dynamics rather than geometry, the spacetime metric need not carry Lorentzian signature. The simplest consistent geometry is Euclidean, with three spatial and three temporal evolution parameters. The kinematic symmetry group is therefore

$$\mathrm{SO}(3,3), \tag{116}$$

the six-dimensional Euclidean rotation group.

14.2 Spin group of SO(3,3): SU(4)

The spin group of $\mathrm{SO}(3,3)$ is

$$\mathrm{Spin}(3,3) \cong \mathrm{SL}(4,\mathbb{R}), \tag{117}$$

but when relativistic effects are produced dynamically by vacuum polarization rather than by Lorentzian geometry, the effective symmetry group becomes

$$\mathrm{Spin}(6) = \mathrm{SU}(4). \tag{118}$$

This is the key geometric insight: the vacuum-polarization engine replaces Lorentzian spacetime with Euclidean $\mathrm{SO}(3,3)$, and the spin group becomes $\mathrm{SU}(4)$.

14.3 Decomposition of SU(4) into Standard Model subgroups

The group $\mathrm{SU}(4)$ contains the Standard Model gauge groups as subgroups:

$$\mathrm{SU}(4) \supset \mathrm{SU}(3) \times \mathrm{U}(1)_G \times \mathrm{SU}(2). \tag{119}$$

This decomposition provides the geometric origin of:

- color charge (from $\mathrm{SU}(3)$),

- gravitational/dark-energy charge (from $U(1)_G$),
- weak isospin (from SU(2)).

The $U(1)_G$ factor is the repulsive spin-1 field responsible for both cosmological acceleration and gravity. The SU(2) factor produces weak-isospin doublets and explains why only down-type quarks undergo beta decay. The SU(3) factor produces color confinement and the shell hierarchy of baryon masses.

14.4 Relativistic effects as dynamical vacuum-polarization responses

In this framework, relativistic effects arise from the following mechanisms:

- **Time dilation**: increased vacuum-polarization drag slows internal interaction rates.
- **Length contraction**: increased momentum flux compresses matter in the direction of motion.
- **Mass increase**: vacuum polarization increases the effective energy stored in the field surrounding a moving particle.
- **Curvature**: anisotropic graviton flux produces an effective metric that matches the Einstein equations in the thermodynamic limit.

None of these require a Lorentzian metric. They arise from the dynamics of spin-1 gauge fields in Euclidean spacetime.

14.5 Summary

The $SO(3,3)$ Euclidean geometry and its spin group SU(4) provide the geometric backbone of the Final Theory. All Standard Model gauge groups arise naturally from the decomposition of SU(4), and all relativistic phenomena arise from dynamical vacuum-polarization interactions rather than from Lorentzian geometry. This sets the stage for the mechanistic derivation of fractional quark charges and the gravitational constant G.

15 Spin-1 Quantum Gravity and Flux-Shadow Geometry

Gravity in the Final Theory is not curvature of spacetime. It is a reaction force produced by the repulsive spin-1 $U(1)_G$ vacuum field. This field generates the observed cosmological acceleration and simultaneously produces gravity as the screened fraction of an inward isotropic momentum flux. The mechanism is fully quantum, fully mechanistic, and fully consistent with the Euclidean $SO(3,3)$ geometry established in Section 2.

15.1 The repulsive $\mathrm{U}(1)_G$ vacuum field

The $\mathrm{U}(1)_G$ subgroup of SU(4) generates a spin-1 gauge field whose quanta (gravitons) carry repulsive momentum. Every mass-energy source emits $\mathrm{U}(1)_G$ bosons isotropically. Because the coupling is extremely small, only the two-vertex tree-level diagram contributes significantly to graviton–matter scattering, exactly as in Møller scattering in QED.

The repulsive flux produces an outward acceleration

$$a = Hc, \tag{120}$$

where H is the vacuum-field repulsion rate (the observed Hubble parameter). This acceleration is not geometric: it is the net outward momentum transfer from the $\mathrm{U}(1)_G$ boson bath.

15.2 Newton's third law and the inward reaction force

A mass m experiences an outward force

$$F_{\text{out}} = ma. \tag{121}$$

By Newton's third law, the vacuum field experiences an equal and opposite reaction force. This reaction manifests as an inward isotropic momentum flux converging on the mass:

$$F_{\text{in}} = ma. \tag{122}$$

This inward flux is the physical origin of gravity.

15.3 Flux-shadow geometry

In the absence of other masses, the inward flux is perfectly isotropic and produces no net force. When a second mass M is present at distance R, it shadows a fraction of the inward flux. The shadowed fraction is the ratio of the graviton scattering cross-section of M to the total area of the sphere of radius R:

$$f_{\text{shadow}} = \frac{\sigma_{gM}}{4\pi R^2}. \tag{123}$$

Thus the net force on m toward M is

$$F_{\text{grav}} = F_{\text{in}}\, f_{\text{shadow}} = ma\, \frac{\sigma_{gM}}{4\pi R^2}. \tag{124}$$

This is the flux-shadow geometry: gravity is the screened fraction of the inward isotropic $\mathrm{U}(1)_G$ momentum flux.

15.4 Matching Newton's law

To reproduce Newton's law,

$$F_{\text{grav}} = \frac{GMm}{R^2}, \tag{125}$$

Eq. (124) requires

$$\sigma_{gM} = \frac{4\pi GM}{a}. \tag{126}$$

This relation is purely mechanistic: it equates the screened fraction of the inward flux to the Newtonian force.

15.5 Weak-scale cross-section and horizon-area scaling

The graviton–matter scattering cross-section is determined by the weak neutral-current cross-section at the Z-boson mass scale. Because the $\mathrm{U}(1)_G$ coupling is extremely small, the cross-section scales as the square of the coupling, exactly as in QED:

$$\sigma_{gM} = \sigma_{\nu p}\left(\frac{G_N}{G_F}\right)^2 = \pi\left(\frac{2GM}{c^2}\right)^2. \tag{127}$$

This is the horizon-area scaling: the graviton cross-section is proportional to the square of the Schwarzschild radius.

15.6 Interpretation

The $\mathrm{U}(1)_G$ field produces:

- cosmological acceleration (outward momentum flux),
- gravity (inward reaction flux),
- Newton's law (flux-shadow geometry),
- the gravitational constant G (via Eq. (126)).

Gravity is therefore a quantum reaction force produced by the repulsive spin-1 $\mathrm{U}(1)_G$ field. It is not curvature of spacetime. It is not a spin-2 interaction. It is the screened fraction of an inward isotropic momentum flux generated by vacuum polarization.

This mechanistic picture sets the stage for the derivation of the density-correction factor e^3 and the gravitational constant G in Sections 5 and 6.

16 Mechanistic Cosmological Expansion: No FRW, No Geometry

In the Final Theory, cosmological expansion is not a geometric stretching of spacetime. There is no Friedmann–Robertson–Walker metric, no scale factor $a(t)$, and no curvature-driven dynamics. Instead, expansion is a direct mechanical consequence of repulsive momentum flux from the spin-1 $\mathrm{U}(1)_G$ vacuum field. This section develops the dynamical expansion law that replaces FRW and sets the stage for the density-correction factor e^3 derived in Section 6.

16.1 Repulsive $\mathrm{U}(1)_G$ flux as the driver of expansion

Every mass-energy source emits $\mathrm{U}(1)_G$ bosons isotropically. These bosons carry repulsive momentum. The net outward momentum flux produces a physical acceleration

$$a = Hc, \tag{128}$$

where H is the observed Hubble parameter, now interpreted as the repulsion rate of the vacuum-polarization engine.

This acceleration is not geometric. It is the direct result of boson emission and momentum transfer.

16.2 Mean separation between sources

Let $L(t)$ denote the mean physical separation between matter sources (baryons, leptons, and dark-energy quanta). The repulsive flux increases $L(t)$ at a constant fractional rate:

$$\frac{1}{L}\frac{dL}{dt} = H. \tag{129}$$

This is the mechanistic replacement for the FRW relation $\dot{a}/a = H$. It arises from boson dynamics, not geometry.

Integrating Eq. (129) gives

$$L(t) = L_0\, e^{H(t-t_0)}, \tag{130}$$

where L_0 is the present mean separation.

16.3 Number-density dilution

Because the vacuum-polarization engine is Euclidean and particulate, the number density of sources scales inversely with the physical volume occupied by a fixed comoving set of sources:

$$n(t) \propto \frac{1}{L(t)^3}. \tag{131}$$

Using Eq. (130),

$$n(t) = n_0\, e^{-3H(t-t_0)}. \tag{132}$$

Thus the matter density is

$$\rho(t) = \rho_0\, e^{-3H(t-t_0)}, \tag{133}$$

where ρ_0 is the present density.

16.4 Continuity equation from boson dynamics

Differentiating Eq. (133) gives

$$\frac{d\rho}{dt} = -3H\rho. \tag{134}$$

This is the continuity equation, but here it arises from:

- three-dimensional number dilution,
- repulsive $\mathrm{U}(1)_G$ momentum flux,
- Euclidean vacuum-polarization dynamics,

not from geometric expansion of spacetime.

16.5 Vacuum drag and graviton redshift

In the Final Theory, redshift is not caused by metric stretching. Instead, gravitons lose momentum as they propagate through the vacuum-polarization field. The momentum-loss rate is proportional to the repulsion rate H:

$$E(t) = E_0 \, e^{-H(t-t_0)}. \tag{135}$$

Thus:

- graviton energy redshifts by $e^{-H(t_0-t)}$,
- graviton arrival rates dilate by $e^{-H(t_0-t)}$,

exactly matching the two redshift factors that appear in the shell integration of Section 6.

16.6 Mechanistic interpretation

The expansion of the universe is therefore a dynamical process governed by:

- repulsive spin-1 $\mathrm{U}(1)_G$ boson flux,
- three-dimensional number dilution,
- vacuum drag and momentum loss,
- Euclidean $\mathrm{SO}(3,3)$ geometry.

No FRW metric is required. No curved spacetime is assumed. All expansion phenomena arise from the dynamics of the vacuum-polarization engine.

16.7 Summary

The mechanistic expansion law

$$L(t) = L_0 e^{H(t-t_0)}, \qquad \rho(t) = \rho_0 e^{-3H(t-t_0)}, \tag{136}$$

replaces the FRW scale-factor formalism. It provides the continuity equation and exponential dilution law needed for the full shell-integration derivation of the density-correction factor e^3.

Full Shell-Integration Derivation of the e^3 Density Correction

This appendix presents the complete, detailed, mechanistic derivation of the dimensionless density-correction factor e^3. The main text (Section 6) summarizes the result. Here we provide the full shell-based calculation, showing explicitly how three-dimensional number dilution, vacuum drag, and horizon-scale graviton propagation combine to produce the factor e^3.

Shell geometry and source density

Consider a thin spherical shell of matter sources at physical radius R from a chosen origin, with thickness dR. The number of sources in the shell is

$$dN(R) = n(R)\, 4\pi R^2\, dR, \tag{137}$$

where $n(R)$ is the number density of sources at radius R.

In the mechanistic vacuum-polarization engine, the repulsive $\mathrm{U}(1)_G$ flux drives sources apart with a constant fractional expansion rate

$$\frac{1}{L}\frac{dL}{dt} = H, \tag{138}$$

where $L(t)$ is the mean separation between sources.

Integrating,

$$L(t) = L_0\, e^{H(t-t_0)}. \tag{139}$$

Thus the number density scales as

$$n(t) = n_0\, e^{-3H(t-t_0)}, \tag{140}$$

and the matter density is

$$\rho(t) = \rho_0\, e^{-3H(t-t_0)}. \tag{141}$$

Emission power from a shell

Let the graviton emission power per unit volume be proportional to the local matter density:

$$P_{\mathrm{em}}(t) \propto \rho(t). \tag{142}$$

The proper volume of the shell at emission time t is

$$dV(t) = 4\pi R^2\, dR. \tag{143}$$

Thus the total emitted power from the shell is

$$dP_{\mathrm{em}}(t) \propto \rho(t)\, dV(t) = \rho_0\, e^{-3H(t-t_0)}\, 4\pi R^2\, dR. \tag{144}$$

Vacuum drag and graviton redshift

As gravitons propagate through the vacuum-polarization field, they lose momentum by vacuum drag. The graviton energy scales as

$$E(t) = E_0\, e^{-H(t-t_0)}. \tag{145}$$

Thus:

- energy redshift contributes a factor $e^{-H(t_0-t)}$,
- arrival-rate dilation contributes another factor $e^{-H(t_0-t)}$.

The observed power from the shell is therefore

$$dP_0(t) = dP_{\rm em}(t)\, e^{-2H(t_0-t)}. \tag{146}$$

Combining density evolution and redshift

Substituting Eq. (144) into Eq. (146) gives

$$dP_0(t) \propto \rho_0\, e^{-3H(t-t_0)}\, e^{-2H(t_0-t)}\, 4\pi R^2\, dR. \tag{147}$$

Simplifying,

$$dP_0(t) \propto \rho_0\, e^{H(t_0-t)}\, 4\pi R^2\, dR. \tag{148}$$

Weighting function for emission times

Equation (148) shows that the contribution of a shell emitted at time t to the present graviton flux is weighted by

$$w(t) \propto \rho_0\, e^{H(t_0-t)}. \tag{149}$$

This factor arises from:

- three powers of $e^{H(t_0-t)}$ from the density at emission,
- two inverse powers from redshift and arrival-rate dilation.

Horizon-scale emission

The graviton bath observed today is dominated by emission from sources near the Hubble horizon scale. The characteristic travel time of such gravitons is of order the Hubble time:

$$\Delta t \equiv t_0 - t_{\rm H} \approx \frac{1}{H}. \tag{150}$$

The matter density at that emission epoch is

$$\rho_{\rm past} = \rho(t_{\rm H}) = \rho_0\, e^{-3H(t_{\rm H}-t_0)} = \rho_0\, e^{3H(t_0-t_{\rm H})} \approx \rho_0\, e^3. \tag{151}$$

A.7 Final result

Thus the effective density seen by the graviton bath is

$$\rho_{\text{eff}} = \rho_0 \, e^3. \tag{152}$$

This is the mechanistic origin of the dimensionless density-correction factor e^3: three powers arise from three-dimensional number dilution over one Hubble time, and the Hubble time $1/H$ is the characteristic propagation time of horizon-scale gravitons in the spin-1 vacuum-polarization engine.

The mechanistic expansion law developed earlier implies that the matter density at earlier emission times is exponentially larger than the present density. Because gravitons lose momentum by vacuum drag and their arrival rates dilate, the observed graviton flux from a shell emitted at time t is weighted by

$$w(t) \propto \rho_0 \, e^{H(t_0 - t)}. \tag{153}$$

Three powers of $e^{H(t_0-t)}$ arise from three-dimensional number dilution, while two inverse powers arise from energy redshift and arrival-rate dilation. The net weighting is therefore a single exponential factor.

Gravitons observed today are dominated by emission from sources near the Hubble horizon scale. The characteristic propagation time of such gravitons is

$$\Delta t = t_0 - t_{\text{H}} \approx \frac{1}{H}. \tag{154}$$

Thus the density at the emission epoch is

$$\rho_{\text{eff}} = \rho_0 \, e^3. \tag{155}$$

This dimensionless factor e^3 is the density correction that must be applied to the graviton bath when deriving Newton's constant G. The full shell-based derivation, including emission geometry, density evolution, vacuum drag, and arrival-rate dilation, is given in Appendix A.

17 Numerical Evaluation of G and Comparison with CODATA

The mechanistic derivation of Newton's constant requires inserting the effective density

$$\rho_{\text{eff}} = \rho_0 e^3 \tag{156}$$

into the flux–shadow relation

$$G = \frac{3H^2}{4\pi \rho_{\text{eff}}}. \tag{157}$$

This section evaluates Eq. (157) numerically and compares the result with the CODATA value of G.

17.1 Cosmological parameters

We use the following observational inputs:

- Hubble parameter:

$$H = 2.27 \times 10^{-18}\ \mathrm{s}^{-1}$$

  corresponding to $H_0 = 70\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$.

- Present matter density:

$$\rho_0 = 9.2 \times 10^{-27}\ \mathrm{kg\,m^{-3}}$$

  including baryonic and dark matter contributions.

- Density-correction factor:

$$e^3 = 20.085.$$

Thus the effective density seen by the graviton bath is

$$\rho_{\text{eff}} = \rho_0 e^3 = (9.2 \times 10^{-27})(20.085) = 1.85 \times 10^{-25}\ \mathrm{kg\,m^{-3}}. \tag{158}$$

17.2 Evaluation of the gravitational constant

Substituting into Eq. (157):

$$G = \frac{3H^2}{4\pi\rho_{\text{eff}}} = \frac{3(2.27 \times 10^{-18})^2}{4\pi(1.85 \times 10^{-25})}.$$

Compute numerator:

$$3H^2 = 3(5.15 \times 10^{-36}) = 1.545 \times 10^{-35}.$$

Compute denominator:

$$4\pi\rho_{\text{eff}} = 12.566(1.85 \times 10^{-25}) = 2.326 \times 10^{-24}.$$

Thus

$$G = \frac{1.545 \times 10^{-35}}{2.326 \times 10^{-24}} = 6.64 \times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}.$$

17.3 Comparison with CODATA

The CODATA 2018 value is

$$G_{\text{CODATA}} = 6.67430(15) \times 10^{-11}\ \text{m}^3\,\text{kg}^{-1}\,\text{s}^{-2}.$$

The mechanistic prediction differs by

$$\frac{|G - G_{\text{CODATA}}|}{G_{\text{CODATA}}} = 0.5\%.$$

Given the uncertainties in ρ_0 and H, this agreement is well within observational error bars.

17.4 Interpretation

The numerical evaluation confirms that:

- the repulsive spin-1 $\mathrm{U}(1)_G$ field,
- the flux–shadow geometry,
- the horizon-area cross-section,
- and the density-correction factor e^3,

combine to produce a parameter-free prediction of G that matches experiment to within one percent.

This is strong empirical evidence that gravity is a quantum reaction force arising from vacuum polarization and repulsive boson exchange, not curvature of spacetime.

References

Cook, N. B., The Final Theory, viXra:2606.0012, 2026.
https://vixra.org/abs/2606.0012