A Dimensionless Gravitational State Parameter from a Planck-Scale $U(1)$ Lattice

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We derive a dimensionless gravitational state parameter R from a discrete $U(1)$ lattice with Planck spacing. From $H = J\sum[1-\cos(\Delta\varphi)]$ with $J = \hbar c/\ell_P$, point defects interact via the 3D Coulomb potential, yielding Newton's law with $n = m/m_P$. The parameter $R \equiv Fr^2/F_P$ ($F_P = c^4/G$) decomposes into a defect rigidity term $R_m = n^2 r/\ell_P$ and the lowest-order compact $U(1)$ vacuum contribution $2\pi\ell_P/r$:

$$R = \left(\frac{m}{m_P}\right)^2 \frac{r}{\ell_P} - 2\pi\frac{\ell_P}{r}. \tag{1}$$

$R > 0$: classical attraction. $R < 0$: repulsion. $R = 0$ defines a critical line $mr = \sqrt{2\pi}\,\hbar/c$. The effective force $F = F_P R/r^2$ acquires a mass-independent r^{-4} correction. At laboratory scales this effective correction is estimated to be $\sim 10^{-130}$ N.

I. INTRODUCTION

Quantum gravity lacks a dimensionless parameter characterizing the net gravitational state of a system. Here we derive such a parameter from a discrete $U(1)$ lattice [3, 4]. The parameter R emerges from the competition between defect phase rigidity and compact phase-cycle response.

II. LATTICE FOUNDATION

Space is a 3D grid of spacing $\ell_P = \sqrt{\hbar G/c^3}$ with $U(1)$ phases and Hamiltonian

$$H = J\sum_{\langle i,j\rangle}[1-\cos(\varphi_i - \varphi_j)], \qquad J = \frac{\hbar c}{\ell_P}. \tag{2}$$

A defect satisfies $\oint d\varphi = 2\pi n$. In the continuum limit, the field equation is

$$\nabla^2\varphi = -4\pi n\,\delta^{(3)}(r), \qquad \varphi(r) = \frac{n}{r}. \tag{3}$$

The interaction potential $V = Jn_1n_2/r$ follows from the overlap of the two defect phase fields. The force magnitude is

$$|F_{\text{Newton}}| = \left|\frac{dV}{dr}\right| = \hbar c\,\frac{n_1 n_2}{r^2}. \tag{4}$$

Matching to $F = Gm_1m_2/r^2$ fixes

$$\boxed{n = \frac{m}{m_P}}, \qquad m_P \equiv \sqrt{\frac{\hbar c}{G}}. \tag{5}$$

III. DEFINITION OF R

Define the Planck force $F_P \equiv J/\ell_P = c^4/G$. The normalized gravitational state parameter is

$$\boxed{R \equiv \frac{F\,r^2}{F_P}}. \tag{6}$$

This definition removes the geometric $1/r^2$ attenuation common to all 3D point-source forces, exposing the local scale-dependent physics encoded in R. The force is recovered as $F = F_P R/r^2$.

IV. DECOMPOSITION OF R

A. Effective decomposition

The parameter R decomposes into a defect rigidity term and a vacuum response term:

$$\boxed{R = R_m - R_0}, \tag{7}$$

where R_m and R_0 are specified below. The additive structure is the simplest ansatz yielding a well-defined $R = 0$ crossover.

B. Defect term R_m

A defect of winding n produces $|\nabla\varphi| \sim n/r$. The elastic energy $E \sim (J/2)\int(\nabla\varphi)^2 d^3x \sim Jn^2 r/\ell_P$. Dividing by $F_P\ell_P = J$ gives the dimensionless contribution

$$\boxed{R_m(r) = n^2\frac{r}{\ell_P} = \frac{m^2}{m_P^2}\frac{r}{\ell_P}}. \tag{8}$$

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C. Vacuum term R_0

The vacuum's coarse-grained phase response at scale r is governed by the dominant lattice mode $k \sim 1/r$, frequency $\omega = ck \sim c/r$. Over one Planck time $t_P = \ell_P/c$, the accumulated phase advance is $\omega t_P = \ell_P/r$. The lowest-order compact $U(1)$ contribution consistent with the lattice scale is

$$\boxed{R_0(r) = 2\pi \frac{\ell_P}{r}}. \tag{9}$$

The 2π normalization follows from choosing the standard $U(1)$ phase period 2π.

D. Explicit form

$$\boxed{R(m,r) = \frac{m^2}{m_P^2}\frac{r}{\ell_P} - 2\pi\frac{\ell_P}{r}}. \tag{10}$$

V. PROPERTIES

A. Sign regimes

$R > 0$: defect rigidity dominates; the net interaction is attractive. $R < 0$: vacuum response dominates; the interaction is repulsive.

B. Critical condition

$$\boxed{R = 0 \iff mr = \sqrt{2\pi}\,\frac{\hbar}{c} \approx 8.82 \times 10^{-43}\ \mathrm{kg \cdot m}}. \tag{11}$$

For $m \gtrsim 10^{-8}$ kg, the critical radius $r = \sqrt{2\pi}\hbar/(mc)$ falls below ℓ_P; such masses are in the $R > 0$ regime at all physically accessible scales.

C. Asymptotics

Large r: $R \approx (m/m_P)^2(r/\ell_P) \gg 0$, recovering the Newtonian regime. Planck scale: $R \sim (m/m_P)^2 - 2\pi$, where the vacuum term is significant.

VI. EFFECTIVE FORCE

From $F = F_P R/r^2$ and Eq. (10):

$$\boxed{F(r) = G\frac{m_1 m_2}{r^2} - 2\pi\frac{c^4}{G}\left(\frac{\ell_P}{r}\right)^4}. \tag{12}$$

The second term is mass-independent, interpreted as an effective vacuum response contribution. At laboratory scales the correction is $\sim 10^{-130}$ N, beyond current sensitivity.

VII. DISCUSSION

We have derived a dimensionless gravitational state parameter R from a discrete $U(1)$ lattice. R decomposes into defect rigidity and compact phase-cycle responses, characterizes the net gravitational state via its sign, and defines a critical line $mr = \sqrt{2\pi}\,\hbar/c$. The decomposition $R = R_m - R_0$ is an effective-model assumption; the coarse-grained vacuum parametrization $R_0 = 2\pi\ell_P/r$ identifies the lowest-order compact $U(1)$ contribution. The parameter R and the critical condition Eq. (11) provide falsifiable structure for discrete-spacetime phenomenology.

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Appendix A: Additivity and scaling

The definition $R \equiv Fr^2/F_P$ separates the geometric $1/r^2$ from scale-dependent physics. Additivity $R = R_m - R_0$ is the minimal structure producing a well-defined crossover. $R_0 \propto \ell_P/r$ follows from the dominant $k \sim 1/r$ mode; subleading terms $\propto (\ell_P/r)^2$ are Planck-suppressed. The 2π coefficient follows from the chosen compact $U(1)$ phase convention.

[1] J. F. Donoghue, Phys. Rev. D **50**, 3874 (1994).
[2] D. J. Kapner *et al.*, Phys. Rev. Lett. **98**, 021101 (2007).
[3] G. 't Hooft, *The Cellular Automaton Interpretation of Quantum Mechanics* (Springer, 2016).
[4] J. B. Kogut, Rev. Mod. Phys. **51**, 659 (1979).
[5] E. G. Adelberger *et al.*, Prog. Part. Nucl. Phys. **62**, 102 (2009).
[6] W. H. Zurek, Rev. Mod. Phys. **75**, 715 (2003).
[7] Y. Y. Fein *et al.*, Nat. Phys. **15**, 1242 (2019).