Mostly-Destructive Consequences of Non- and Nowhere-Analyticity of the Spacetime Metric; and new "2-sided Mirror Paradigm" hypothetical grandiose universe picture

Warren D. Smith, warren.wds@gmail.com. Online at http://vixra.org/abs/2607.0063. v1, 15 July 2026. v2 (25 July) adds update & new slick mathematical description of mirror paradigm section including new grandiose universe picture after discovered Tzanavaris, Boyle, Turok 2025. v3 (30 Aug.) adds appendix destroying "ER=EPR" with no-go theorem. The first of a series of 3 papers; a short introductory summary of them is http://viXra.org/abs/2610.0071.

Abstract: We explore consequences of my previous discovery (http://vixra.org/abs/2605.0004 Smith 2025) that Einstein general relativity vacuum solutions are *not* necessarily real-analytic, and indeed can be (hence presumably generically are, since the analytically-smooth spacetime metrics form an infinitesimally tiny subset of the much wider class of permitted functions) *nowhere-analytic*. These consequences are hugely destructive for many former physics ideas that implicitly or explicitly assumed analyticity – such as "ER=EPR," "AdS/CFT," or "maximal analytic extensions" – and I will duly obliterate them! A Monte Carlo estimate is that 1-20 *thousand* general-relativity papers are invalidated. However, in an unexpected brief creative interlude during that apocalyptic demolition, I invented the promising new "mirror paradigm," which might be crazy, but if valid offers a simple nice way to resolve most or all famous "paradoxes" associated with black holes, and also provides a whole new mental picture of the entire universe. Amazingly, I found out soon after posting v1 that almost exactly the same idea (under the name "black mirror") was invented independently 1.5 years before by Tzanavaris, Boyle, and Turok (TBT) although mostly for different reasons. At the end I attempt to reconcile all Smith-versus-TBT discrepancies while showing all apparent obstacles to this picture are only apparent, to produce a mirroring hypothesis we hopefully all can agree is the right proposal and makes sense. I also, surprisingly, make new observations even about the hugely-studied Schwarzschild/Kruskal black hole, including proposing arguably-local metrical tests for event-horizons (a feat often claimed impossible by prior authors).

Table of contents

1. Background
2. Destruction commences
3. Interlude: New possibly-revolutionary (crazy?) self-consistent black hole "mirror paradigm"
4. Destruction resumes
5. Conclusion
6. Updates (added for v2 after version v1 of this paper finished & posted)
7. New slick mathematical descriptions of "mirror paradigm" metrics.
    i. New grandiose mental picture of universe.
    ii. Arguably-local tests for horizons.
8. Some good next questions
9. References
10. Appendix: Analytic dirt in one universe abolishes the other (barrier theorem for linearized

Kruskal-Szekeres extension)

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1. Background

Concerning analyticity and analytic continuation, one good reference is Henrici 1974-1986. Note that adding a non-analytic function to an analytic function – even if the non-analytic one is multiplied by an arbitrarily tiny number such as 10^{-100} – yields a non-analytic function. Similarly, adding even the slightest amount of nowhere-analytic-function "pollution" to an analytic function yields a nowhere-analytic function.

Counting and measuring functions. The Cantorian count of the number of *analytic* maps of [-1,1] to $\mathbb{R}$ is the same as the cardinality $2^{\mathbb{N}}$ of the continuum $\mathbb{R}$, because analytic functions are specifiable by a countable sequence of reals: their Maclaurin series coefficients; and a matching lower bound arises from the fact that any real constant is such a function. [Cardinality=$(2^{\mathbb{N}})^{\mathbb{N}}=2^{\mathbb{N}\cdot\mathbb{N}}=2^{\mathbb{N}}$.] The number of *continuous* maps of [-1,1] to $\mathbb{R}$ also is $2^{\mathbb{N}}$ because a continuous function is specified by its values at all rational arguments, which again is a countable set of reals (listable by concatenating all Farey sequences), and all analytic functions are continuous. $2^{\mathbb{N}}$ also counts the number of maps of [-1,1] to itself with continuous k^{th} derivatives, for any particular k≥0 (proof: integrate). However, if a "random" such k-differentiable function is chosen, for any particular k≥0 (even if we insist that its k^{th} derivative be monotonic), then the *probability* that any open subinterval will exist on which it is analytic, equals zero. For example, if we let $F^{(k)}(-1)=-1$ and $F^{(k)}(1)=1$ and then for n=1,2,3,... we at step n we assign the value of $F^{(k)}(x)$ at the n^{th} rational number in (-1,1) to be a random real chosen uniformly from the reals within the real interval whose endpoints are the previously chosen values of $F^{(k)}(x)$ at the two nearest previously-used rationals to x's left and right... then the resulting function F(x) will have a continuous k^{th} derivative but probability=0 of being analytic.

One nice way to think about this is to define functions F(x) for real x∈[-1,1] via their **Chebyshev series $F(x)=\sum_{j\geq 0}a_jT_j(x)$** where $T_n(x)=\cos(n\cdot\arccos x)$ is the degree-n Chebyshev polynomial $T_n(x)$. If expressed as a sum of powers of x times all-integer coefficients, these begin $T_n(x)=2^{n-1}x^n-n2^{n-3}x^{n-2}+...$ where of course the second term occurs only when n≥2. Because the Chebyshev polynomials $T_n(x)$ are orthogonal on [-1,1] with respect to the weighted measure $d\mu(x)=(1-x^2)^{-1/2}dx$ and $\int_{[-1,1]}T_n(x)^2d\mu(x)=\pi/2$ for each n≥1 (for n=0 the answer instead is π), any such function with $\sum_{j\geq 0}|a_j|^2<\infty$ will be L_2 on the thus-weighted [-1,1], whereupon by Carleson's theorem the series will converge for all x in [-1,1] *except* perhaps for a set with measure=0. Also $\int_{[-1,1]}T_n(x)^2dx=(4n^2-2)/(4n^2-1)$. Because $|T_n(x)|\leq 1$ whenever x∈[-1,1], if $\sum_{j\geq 0}|a_j|<\infty$ it follows that the series for F(x) will converge absolutely (and uniformly) for every x∈[-1,1], and the resulting F(x) will be bounded between two constants for all x∈[-1,1]. Because $2\int_{0\leq x\leq X}T_n(x)dx=(n+1)^{-1}T_{n+1}(X)-(n-1)^{-1}T_{n-1}(X)+C_n$ where $C_n=0$ if 2≤n=even and $C_n=2n/(n^2-1)$ if 3≤n=odd, it follows that if $\sum_{j\geq 0}|a_j|(j+1)^m<\infty$ then the Chebyshev series for $F^{(k)}(x)$ will converge

absolutely (and uniformly) for each k=0,1,2,...,m, as well as for every negative-integer k [i.e. k-time integrals of F(x)]; and that all these $F^{(k)}(x)$ will be bounded between two (k-dependent) constants for all $x\in[-1,1]$; and hence that F(x) will have a *continuous* m^{th} derivative for all $x\in[-1,1]$.

So if, for example, **we choose each a_j independently uniformly at random** from the real interval $[-j^{-m-2}, j^{-m-2}]$, then the Chebyshev series F(x) always will represent a function of x that has (at least) m continuous derivatives, with it and all those derivatives bounded between two constants, for all $x\in[-1,1]$. The Cantorian count of such functions as usual is $2^{\mathbb{N}}$. *However*, **with probability=1 this random F(x) will be analytic *nowhere*** in the complex x-plane, and its Chebyshev series will diverge (at least) exponentially for every complex $x\notin[-1,1]$, for example due to the identity $2T_n(x)=L^{-n}+L^n$ where $L=x+(x^2-1)^{1/2}$ where note that $\max(|L|,|L^{-1}|)>1$, causing exponential(n) growth of $L^{-n}+L^n$ to occur, exactly when $x\notin[-1,1]$.

F(x) that are analytic for all $x\in[-1,1]$ necessarily have Chebyshev coefficients a_j whose absolute values are bounded below an **exponentially decreasing** function of j. But with probability=1, our random process produces *sub*exponentially decreasing $|a_j|$, meaning $\limsup_{j\to\infty} j^{-1}|\log|a_j||=0$. For example, the following two Chebyshev series converge for all $x\in[-1,1]$ but are analytic *nowhere*: $\sum_{j\geq 0}\exp(-\sqrt{j})T_j(x)$ and $\sum_{j\geq 0}j^{-53}T_j(x)$. The first is infinitely differentiable for all $x\in[-1,1]$, while the second has at least 51 continuous derivatives. Both these Chebyshev series converge (indeed, quite rapidly) *only* on the real interval [-1,1] but nowhere else in the complex plane.

UPDATE for v3: Other formalizations of the notion that "almost all" functions are nowhere-analytic were by Dart 1973, Shi 2001, and Bastin, Esser, Nicolay 2012.

My **previous paper** (Smith 2025) constructed general relativity (GR) scenarios, which could even have initial conditions analytic everywhere except at a single point, which then would time-evolve to solutions analytic *nowhere* within a positive-measure open set of spacetime locations spreading outward from that initial point at lightspeed. Since nowhere-analytic behavior is *possible*, presumably it should be *generically expected* given the just-mentioned probability=1 results. So in the "dirty" real physical world, expect nowhere-analytic spacetime metrics.

Health and safety warning: Since general relativity is not the same thing as "the true laws of physics" (e.g. there also are Maxwell's equations and presumably also "quantum gravity"), I do not claim to have *proven* that the spacetime metric of *our universe* is nowhere-analytic. I am just saying that, a priori, this seems (especially if we *were* considering GR alone) *overwhelmingly likely*. The rest of this paper will explore the consequences of assuming it.

Although some physics-related partial differential equations, such as the so-called 1D heat equation $\partial U/\partial t=\partial^2 U/\partial x^2$ started at time t=0 have the property that *all* their solutions U are analytic functions of both x and t (when t>0), the EVFEs (Einstein vacuum field equations) evidently do *not* enjoy that property.

Some further examples that generically are nowhere-analytic (beyond those in Smith 2025). One well known class of exact EVFE solutions, discovered by Hans Brinkmann in 1925, are the "pp-wave" spacetimes

$$ds^2 = 2\,du\,dv + dx^2 + dy^2 + H(u,x,y)\,du^2$$

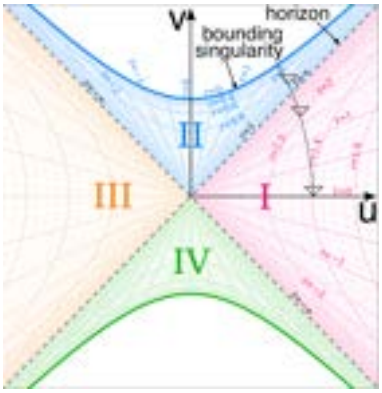
where H(u,x,y) is an *arbitrary* twice-differentiable function satisfying $\partial^2H/\partial x^2+\partial^2H/\partial y^2=0$. (Pp-waves are a subclass of the wider "Kundt class," discussed in ch.31 of Stephani et al. 2003.) An important particular special case is the "exact vacuum plane waves"

$$H(u,x,y) = A(u)\,(x^2-y^2) + 2B(u)\,xy$$

for arbitrary twice-differentiable functions A(u) and B(u) called the "+ and × polarization profiles." Notice that it is perfectly ok with the EVFEs if the dependence of H(u,x,y) on u, e.g. the functions A(u) and B(u), are nowhere-analytic, e.g. $A(u)=\sum_{j\geq 0} 10^{-j}\sin(\pi^j u)$ which for real u is nowhere-thrice-differentiable but everywhere second-differentiable. [This series converges for all real u, but diverges for all nonreal complex u.] Another class of exact solutions depending on arbitrary functions capable of exhibiting nowhere-analyticity are the "Einstein-Rosen cylindrical waves."

Natural-seeming A(u) that are nowhere-analytic since differentiable only finitely many times (with probability=1) are the K^{th} integrals (for any finite integer K≥0) of "Brownian motion."

There has been a large body of work by many authors (but not me) on "**maximal analytic extensions**" of important GR vacuum (or in some cases nonvacuum) exact solutions. However, there usually is *no such thing* as a "maximal analytic extension" of a *non*analytic function. Nowhere-analytic functions have no analytic extensions. Therefore we now see that all work, by all general-relativists, on the topic of "analytic extensions," presumably has zero physical relevance. Let us now review some of that work.



First came Martin D. **Kruskal**'s 1960 "Maximal extension of the Schwarzschild" eternal nonrotating uncharged black hole metric, which involves four regions wikipedia calls

I. exterior region (u>|v|): "our universe"
II. interior black hole (v>|u|)
III. parallel exterior region (u<-|v|): "other universe"
IV. interior white hole (v<-|u|).

(Picture adapted from wikipedia article, in turn based on fig.2 in Kruskal 1960. All 45° inclined lines are null geodesics.) The initial investigators, including K.Schwarzschild and J.Droste, had only been aware of regions I and II. Kruskal's other two regions III and IV are not deducible from parts I and II of the spacetime by using the EVFEs, in the sense if any point in I or II emits a flash of light, that light classically can never reach anyplace in III and IV. No signal traveling at speeds≤c (where c=299792458 meter/sec is the speed of light) can ever reach anyplace in III and IV from anyplace in I and II. However parts III and IV of Kruskal's metric nevertheless are deducible by *assuming* – a better word might be "hoping" – that the spacetime metric is analytically smooth, so we can use "analytic continuation" to make the deduction.

"**Einstein-Rosen (ER)** bridges" aka "wormholes," named after Einstein & Rosen 1935, are connections between "two different universes" (or perhaps between faraway places in the same

universe?). These, according to wikipedia, "now are understood to be intrinsic parts of the maximally extended version of the Schwarzschild metric." Wikipedia's claim, however, was *false*, because the metric proposed by Einstein & Rosen is *not* equivalent to Kruskal's metric. Einstein & Rosen 1935 had simply employed the most-usual mass-M Schwarzschild (t,r,θφ)-coordinatization

$$ds^2 = -c^2[1-H/r]dt^2 + [1-H/r]^{-1}dr^2 + r^2[d\theta^2 + \sin(\theta)^2 d\varphi^2]$$

with circumferential radial coordinate r (where $H=2GMc^{-2}$ is the value of r at the *h*orizon, i.e. Schwarzschild radius, ds^2 is the squared element of pseudolength, and $G\approx 6.674\times 10^{-11}$ $meter^3 kg^{-1} sec^{-2}$ is Newton's gravitational constant; this is E&R's EQ 5), then changed variables from r to w with $w^2=r-H$ and therefore 2wdw=dr, thereby obtaining two exterior-Schwarzschild metrics (one with w>0, the other with w<0) *glued* together on their common horizon-spheres w=0. E&R claim on their p.75 that their metric is analytic ("regular") everywhere, unlike Kruskal's which has two boundary-curve singularities (the hyperbolas at the top and bottom of the picture, corresponding to r=0 in the Schwarzschild coordinatization). However, $g_{tt}=0$ on E&R's horizon w=0, causing $\det(g_{ab})=0$ there [although $g_{tt}>0$ and $\det(g_{ab})<0$ everywhere else], which as far as I am concerned causes the whole horizon surface to *be* a singularity in E&R's metric, which therefore is *non*analytic when w=0. Einstein & Rosen then tried to claim that the Einstein vacuum field equations $R_{k\ell}=0$ (here $R_{k\ell}$ is the Ricci curvature tensor while g_{ab} is the metric tensor) could equally validly, as far as they knew based on experimental evidence, be replaced by $\det(g_{ab})^2 R_{k\ell}=0$ [or for that matter $\det(g_{ab})^p R_{k\ell}=0$ for an arbitrary even integer $p\geq 0$]; and E&R's metric is an exact solution of these alternative field equations; so therefore they do not care about Warren D. Smith's judgment that the w=0 surface is singular. Warren D. Smith, however, thinks this is not legitimate: Einstein & Rosen **cheated**. Almost the whole point of Einstein's original derivation of general relativity had been his demand that everything be *invariant* if you change your coordinatization of the metric. Their alternative field equations $\det(g_{ab})^p R_{k\ell}=0$ disobey that demand if $p\neq 0$. E&R's metric and w-using coordinatization are bad in the sense that $\det(g_{ab})=0$ when w=0. If E&R changed coordinates to a better coordinate system (e.g. resembling Kruskal's), with $\det(g_{ab})\neq 0$ there, then their spacetime would *not* satisfy $\det(g_{ab})^p R_{k\ell}=0$ on its horizon! Meanwhile, Kruskal's re-coordinatization of the Schwarzschild metric *does* satisfy $R_{k\ell}=0$ on its horizon, and indeed everywhere between the two hyperbolas.

A lower-dimensional analogy: if you have two flat sheets of paper, and cut a 1-inch diameter circular hole in each, then glue the two sheets together along the perimeter of that circle ("throat"), then are the gluing points "regular points" on the resulting 2-manifold? I contend they are not, i.e. are a metrical singularity (albeit a rather weak kind of singularity). If we had cut and glued *square* rather than circular holes, everybody would agree that the square-corners were metrical singularities, since they are surrounded by 540°, not 360°, of angle. But all other points on the perimeter of the square are metrically-regular. If we replace the "square" hole by a regular N-gon, then again points on the N-gon edges are metrically regular, but the corners are N metrical singularities with angle excess 720°/N each. If we make N large, then each of these singularities becomes weaker in the sense that their angle-excesses each approach zero. In the limit $N\rightarrow\infty$ we get a circular hole, whose entire perimeter consists of singular points, albeit each of them is "infinitely weakly" singular. [A (3+1)-dimensional situation extremely-resembling those two glued

sheets of paper is analysed by Visser 1989.] Analogously, I claim E&R's metric actually is *not* a solution of the Einstein *vacuum* field equations. E&R hid that fact with their "multiply by $\det(g_{ab})^p$" sophistry. In fact, E&R's metric solves Einstein's *non*vacuum field equations with an infinitesimally-thin spherical shell of "flattened exotic matter," with negative areal mass-density proportional to 1/M, located on the wormhole-throat w=0 surface while everyplace *else* is vacuum. Later I shall determine the density and pressures of that matter. In contrast, Kruskal's is a genuine, no-cheating, solution of the *vacuum* field equations.

E&R's metric has *no* positive-measure intermediate region lying inside all horizons, while Kruskal's "maximal analytic extension" does have such regions, namely II and IV in the spacetime diagram. So clearly, the E&R and Kruskal metrics are *inequivalent*. Alternatively, E&R could have differently-defined w via $(H^2+\tilde{w}^2)^{-1/2}\tilde{w}^2=r-H$ (but didn't) which still would have caused the left hand side to be proportional to $\tilde{w}^2$ for small $|\tilde{w}|$, but for large $|\tilde{w}|$ would, more intuitively and simply, cause $\tilde{w}$ to asymptotically equal r. That would have been a different coordinatization of E&R's same metric. Another, which makes it clearer that E&R's metric is non-analytic at w=0 (now p=0), arises from changing coordinates from r to p where r=|p|+H. Then

$$ds^2 = -c^2[1-H/(|p|+H)]dt^2 + [1-H/(|p|+H)]^{-1}dp^2 + (|p|+H)^2[d\theta^2 + \sin(\theta)^2 d\varphi^2].$$

By their same kind of transformation applied to the r-coordinate of the Reissner-Nordström instead of Schwarzschild metric, Einstein & Rosen also could and did obtain electrically-*charged* wormhole metrics (automatically positively charged on one side, negatively on the other). They also could have done this, even more generally, for the full Kerr-Newman spinning/charged black hole metric family, but did not since Kerr & Newman were ≤6 years old at the time. Einstein & Rosen's main objective was not to create "wormholes," but rather to alter the Schwarzschild black hole metric to *remove* its singularity at the centerpoint r=0 of the original metric, while keeping it unaltered everywhere outside its horizon-sphere. (It has been claimed that Ludwig Flamm in 1916 already realized wormholes existed, but Matt Visser contends "you definitely need hindsight" to detect that realization inside Flamm's paper.) But E&R *failed* in their singularity-removal goal if you agree with me that E&R's wormhole "throat" sphere-surface *is* a metrical singularity: E&R's removal of the centerpoint singularity came at the cost of creating a new metrical singularity on the (previously metrically-regular) horizon-sphere.

Unfortunately, apparently in most modern (ER=EPR and post-1970?) usage, most people talking about "Einstein-Rosen" wormholes *really* have in mind *Kruskal 1960's* maximal-analytic-extension metric, although they very rarely actually say so, and never mention the fact E&R cheated. For example, this is revealed around 52:41 in Susskind's video-recorded 4 Nov. 2014 Stanford lecture on ER=EPR that he put on the internet; and also in the first line of the "conclusion" of the Maldacena-Susskind 2013 paper – albeit unfortunately *without* explicitly saying so, *without* citing Kruskal 1960 (although M&S, in order to maximally confuse their readers, did cite E&R 1935) and Maldacena-Susskind 2013 do not actually *state* a metric. That unstated renaming convention is infuriating, obnoxious, disgusting and insults *everybody*.

At about one hour and 26 minutes into this same 4 Nov. 2014 lecture on ER=EPR at Stanford, Susskind correctly says that a very large ("Schwinger limit") electric field can create (and then separate) e^+e^- pairs from vacuum. Each such pair has quantum-entangled (since opposite) spins.

At 1:30 Susskind then says an "extreme version of this same Schwinger process" will create a pair of + and - charged *black holes* from vacuum, completely quantum-entangled – everything about one may be deduced by measuring the other – and with an ER bridge between them. Susskind assures his audience that "you can calculate the probability for [this creation process] and you can calculate how it behaves afterwards." But Susskind doesn't actually *do* or cite any such "calculation"; and after searching the literature up to 4 Nov. 2014 I do not believe any such calculation exists. Au contraire: I claim that there is no, and never will be, and never can be, any solution of the Einstein-Maxwell vacuum equations featuring *topology change* such as this (allegedly induced by an electric field in vacuum), for the simple reason that it is mathematically *impossible*. So presumably Susskind has in mind a solution, *not* of the Einstein-Maxwell equations, but rather of some completely reliable more advanced theory of "quantum gravity" combined with electromagnetism, which apparently Susskind possesses but kept secret from everybody else.

Later, at 1:33:30 in the same lecture, Susskind claims that two black holes can and normally will have *many* Einstein-Rosen bridges, note the plural, connecting them. (Where again, I think Susskind really meant "Kruskal.") This multibridge notion has never been justified, by anybody, by any metric solving Einstein's equations. Can such metrics exist and persist? Susskind simply *asserted* so, with no justification whatsoever. Indeed, in this entire (nearly 2 hours long) lecture, Susskind never wrote down a single formula, instead contenting himself with drawing on the board, in the manner of a comic book, rather sloppy pictures of fantasies such as this, apparently under the impression it might convince somebody of something.

The point that Kruskal wormholes are "non-traversible," i.e. it is not classically possible for a signal to travel from region I to III (or from III to I), was made by Fuller & Wheeler 1962, although F&W's paper was entirely unnecessary because that already was obvious from my spacetime diagram copied from fig.2 of Kruskal 1960. F&W claim on p.920 with no cite, and no metric justifying it, that "wormholes" can exist as in their fig.1 joining two locations in the *same* near-Euclidean universe via a possibly-much-shorter route. F&W's actual mathematical analysis, however, concerns the *Kruskal* metric, which does *not* join two locations in the same near-Euclidean universe. It joins two *different* universes. F&W blissfully ignore that contradiction, falsely pretending it is so minor that it does not deserve even a single sentence of discussion.

> **More about traversibility.** The Kruskal "wormhole" is "non-traversible" meaning it lacks timelike curves that enter one hole and emerge from the other. Such curves exist, *but* only if they include acausal, aka spacelike, aka "velocity>c" segments. Inequivalent metrics representing *traversible* wormholes were later found mathematically (Morris & Thorne 1988), *but* all known examples are *non*vacuum solutions involving "exotic" matter with "negative mass." Indeed on p.405 Morris & Thorne sketch an argument by Don Page that negative-mass matter is *necessary* for any traversible wormhole. (They do not say so, but Page's argument is essentially the same as the proof of the Hawking-Penrose "singularity theorem," albeit used for a different purpose; see Hawking & Ellis 1973 for introduction to that.) Exotic matter presumably cannot exist since if it could, then the vacuum could – and thermodynamically would be expected to – decay into it, contradicting all observations indicating the vacuum is stable. Also, the Morris-Thorne-Yurtsever followup paper argued that any traversible wormhole could be used to construct a "time machine" enabling you, paradoxically, to kill your newborn grandmother.

Meanwhile, Einstein & Rosen 1935's eternal-wormhole metric (also joining two *different*, not the same, universes) *is* traversible, i.e. contains timelike geodesics connecting their two universes. (Yet another confirmation that E&R's and Kruskal's metric are inequivalent.) However, if you fall into E&R's black hole from one universe, then when you emerge in the other, the "**direction of time reverses**." (This point was not made by E&R.) That is, E&R's metric's t-coordinate *in*creases in our universe as we go into the "future." But in their other universe (with w<0), we need to regard t as *de*creasing into the future. The reason I say this is Smith 2003, where I pointed out that general relativity, despite its field equations naively appearing to be time-reversal-symmetric, asymmetrically self-generates a **definition of "future"** thanks to irreversible phenomena such as the theorem that (under "energy conditions" assuring "negative mass" cannot exist) black holes can merge but not bifurcate. Falling *into* black holes always corresponds to the "future" time-direction. Then, if we adopt Feynman's view that an electron traveling backward in time is equivalent to a positron moving forwards in time: An electron that falls into E&Rs hole, then emerges on the other side, on both sides with continually increasing t-coordinate, really should be regarded as an electron falling in from our side *and* a positron falling in from the other side, both annihilating at w=0. Note: the gamma rays produced by that annihilation classically could never escape the w=0 horizon surface; and from the point of view of an external observer, the electron "never" reaches w=0, but rather has w>0 forever, only asymptotically approaching $w \to 0^+$ when $t \to \infty$. But from the point of view of a different observer riding on the electron, w=0 (and w<0, if it kept going past w=0 without being annihilated) both are reached in finite time as measured by his own clock. (That's discussed in Misner, Thorne, Wheeler 1973.) Susskind about 57:30 in his lecture claims the "ER" wormholes he has in mind are non-traversible, providing more confirmation that what Susskind falsely called "Einstein-Rosen" really meant "Kruskal."

Important new observation about Einstein-Rosen (and Kruskal). It is today well known (see discussion in Smith 2003) that the Schwarzschild metric is *stable* to small-enough (in suitable explicitly defined norm-senses) perturbations. All such perturbations die out exponentially ("quasimodes"). However, if small perturbations of the horizon surface die out like exp(-t) on our side, then on the other side of the Einstein-Rosen version of Schwarzschild, presumably they also must behave like exp(-t). If so, they actually are *increasing* exponentially as we go into the "future" (for that other-side's reversed notion of "future"). I conclude from this that the Einstein-Rosen metric must be **unstable** and/or that small perturbations to it are generically **illegal** (i.e. cannot exist without destroying the property of being an exact EVFE solution). Either way, we conclude that the ER metric necessarily is **unphysical**. However, note that this kind of time-reversal does *not* need to happen with Kruskal's spacetime (yet another reason it is inequivalent to Einstein-Rosen), and therefore this argument does not exclude Kruskal's metric from being physical. (It still might well be unphysical, but not because of *this* argument.) Nevertheless, it *could* happen with Kruskal, i.e. we *could*, if we wanted, insist that in regions I and II of the Kruskal spacetime diagram, the "future" is the *in*creasing-u direction, while in regions III and IV, it is the *de*creasing-u direction. (Although I've never seen any author before 2020 consider that "**bi-future**" option.) Version 1 of this paper stated that my ER unphysicality argument would also rule out bi-future Kruskal. That statement likely was wrong because "odd symmetry" perturbations F(u,v)=-F(-u,-v) presumably still would be permitted that die out exponentially in both our universe and the other as they go into their respective "futures," so bi-future Kruskal could still be stable, at least within this class of perturbations. The single- and bi-future Kruskal metrics differ in the following sense: With bi, no particle from our universe (I) can ever interact with another coming from the other universe (III). With single, both particles can enter region II and interact there before unavoidably hitting the upper singularity.

"ER=EPR." Mark Van Raamsdonk in 2010 supposedly claimed that an AdS-Schwarzschild black hole metric if "maximally analytically extended" to feature an "*E*instein-*R*osen bridge" aka "non-traversable wormhole" to another faraway such hole (again I believe his "ER" really meant "Kruskal"), via Maldacena's "AdS/CFT-correspondence" corresponds to a pair of "maximally quantum-entangled" thermal conformal field theories. (Actually: I did not see such a claim when I looked in Van Raamsdonk's paper. More likely, this claim instead traces to Maldacena 2003.) Maldacena & Susskind 2013 then proclaimed that "ER=EPR," meaning that all "quantum-entangled" qubit-pairs were somehow secretly connected by an enormous number of invisible Einstein-Rosen (actually Kruskal) "wormholes" that were really responsible for the whole notion of quantum entanglement, and this brilliant insight was going to resolve both the EPR paradox (Einstein Podolsky Rosen 1935) that measurements of quantum-entangled far-distant particles naively apparently could "transmit information faster than light" – as well as later black hole "Hawking information-loss" and ""firewall" quantum-entanglement paradoxes. (More about ER=EPR: Gharibyan & Penna 2013, Jensen & Karch 2013, Sonner 2013.)

> "Two black holes which are [quantum] entangled will necessarily have [an Einstein-Rosen, by which he apparently really means Kruskal] bridge between them, two black holes which are unentangled will not. This was a major discovery. It seemed ludicrous at first, but very quickly caught on and is now standard lore."
> – Leonard Susskind, 11 May 2022 Oppenheimer Lecture at UC Berkeley. [For similar hype see Grant 2015, Lin, Ouelette & Cole 2015, and ch.6 of Cowen 2019.] Two other quotes, now from the 4 Nov. 2014 video lecture at Stanford by Susskind:
> "The two [ER 1935 and EPR 1935] papers apparently had nothing to do with each other [but] it has turned out that they are so deeply related that one can almost put an equal sign between them, they are so deeply connected, joined at the hip."
> "Lots of other people have different thoughts than I do [but] they're all wrong. They are."
> To be clear, as of 2026 I am *not* ready to regard this as "standard lore," and not willing to accept the second or third quotes either – and based on Susskind's perpetual use of ER when he means Kruskal, it appears to me that Susskind never actually read the Einstein-Rosen 1935 paper he cited. That's a pity because both ER and EPR 1935 papers are excellent and well-written.

2. Destruction commences

Once we agree that "analytic extensions" are physically-irrelevant nonsense, then **all that about "ER=EPR," wormholes, Kruskal regions III and IV, and probably the "AdS/CFT correspondence" too** (because I think it also implicitly depended upon analytic continuation) are **utter hogwash**. Indeed, even if

1. AdS/CFT were set in a correct-dimensional universe, *and* even if
2. our universe were anti-de-Sitter (AdS) instead of plain de Sitter, *and* even if
3. conformal field theories (CFT) had any physical validity, *and* even if
4. all experimental evidence were not against supersymmetry (SUSY is used/assumed by Maldacena 1998 in his "derivation"), *and* even if
5. Maldacena's "derivation" of the AdS/CFT correspondence came anywhere near achieving mathematical rigor

(none of which look true, but even if they had been) – AdS/CFT & etc. *still* would be **bunk**. That is because Maldacena 2003 and Maldacena & Susskind 2013 certainly both depended upon analytic continuation, e.g. see M&S's appendix A's sentence "These become Rindler coordinates after we analytically continue τ to Lorentzian time." Maldacena 1998 is so completely packed with speculation to such an unbelievable degree that it is hard to say what it depends on – if the term "depends" even has any meaning in such a logic-free environment – but its followup paper by Witten definitely did involve analytic continuation.

Berezin, Dokuchaev, Eroshenko 2016 found the "maximal analytical extension of the **Vaidya** metric," which is a nonvacuum exact GR solution that can be regarded as modelling a *Hawking-evaporating* Schwarzschild black hole. This work too must be thrown in the garbage as "physically irrelevant."

A number of authors, including Carter 1966 & 1968, Boyer & Lindquist 1967, Kim 1996, and Bakun et al 2017, attempted to find/understand the maximal analytic extension of the Kerr (or Kerr-Newman) metric representing a rotating (and in Newman's generalization also electrically *charged*) black hole. The results were astonishing, indicating **Kerr** holes really are doorways connecting an infinitude of other universes! And inside the inner horizon is a "time machine" region containing closed timelike curves! But again: with even the slightest nowhere-analytic pollution, all this is utterly unjustifiable; and presumably no extrapolation whatsoever into the region inside the Kerr hole's infinitely-blueshifting "inner horizon" can be expected to have any physical validity. (Morris & Thorne 1988 had already made similar critical remarks about Kerr's inner horizon in their §IA, but their attack now is hugely bolstered by the non-analyticity juggernaut.)

Also Schwarzschild can be redone with a general nonzero electric **charge** (Reissner-Nordström) and cosmical constant Λ (Schwarzschild had assumed charge=0 and Λ=0), see Mokdad 2017 who again finds a doorway connecting an infinitude of other universes (and Mokdad's metric is *not* equivalent to Einstein-Rosen 1935's, which has no such infinitude of universes) – but again I presume that, with the slightest amount of pollution of the spacetime metric by a nowhere-analytic function, no extrapolation into the region inside the infinitely-blueshifting inner horizon can be expected to enjoy any physical validity. More big-analytic-continuation results again with no physical validity: Hartle & Hawking 1972.

More criticism of "ER=EPR." An **objection by Nikolic 2013**, which as far as I can tell remains unanswered as of year 2026: "Convincing justification of [the ER=EPR] interpretation would require quantitative evidence that *correlations* between two ends of the wormhole are equal to those between the members of the EPR pair. As long as [there is no] such evidence, interpretation of wormholes as EPR pairs does not seem justified." Let me now elaborate on Nikolic's objection, plus invent my own objections.

Imagine we begin with two faraway (lightyears apart) un-entangled Schwarzschild black holes A and B. Midway between them we generate an entangled pair of two photons, e.g. one has helicity +1, the other -1 (but we do not know which) flying in opposite directions toward A and B. (Another version: a neutrino-antineutrino pair with helicities ±1/2.) After the two photons fall into the two black holes, those two holes acquire quantum-entangled spins: One hole has spin +1, other -1, but we do not know which. Therefore according to Susskind, the holes now are joined (but were not formerly) by an Einstein-Rosen bridge. How did this spacetime topology-change suddenly spring into being wormhole-linking two lightyears-apart black holes? It did not. If "physics is local" ("Mach's

principle"?), then the only way this could have happened is that the two photons, were *generated* with an ER bridge connection, which lengthened as the photons moved apart, ultimately becoming the ER-bridge connecting the two holes A and B. So we see that if ER=EPR is valid for entangled pairs of black holes, it *must* be the explanation for entangled photon-pairs, and indeed entangled pairs of *any* two qubits that would, if dumped into black holes, result in quantum-entangled measurable properties (mass, spin, and/or charge) of those holes.

First ER=EPR problem. Einstein-Rosen, and Kruskal, bridges only work for two *equal-mass* Schwarzschild black holes, since the maximally-extended Schwarzschild exact-GR-solution is unique (by "Birkhoff's theorem") and has only *one*, not two, mass-parameters. So if A and B originally had different masses – always true in practice – that whole story simply does not work. This trivially simple objection, incredibly, is nowhere discussed in Maldacena & Susskind's 31-page paper!

Now consider a fancier thought-experiment. Perhaps with the aid of one of Susskind's favorite devices, a hypothetical "quantum computer," generate N spin-based qubits (N arbitrarily large), entangled such that the sum of those 01-qubits is *even*. I.e. all 2^{N-1} classical N-bit-states with even-sum are present in quantum superposition, all equally likely; but the 2^{N-1} odd-sum states have zero probability. With this fancier kind of entanglement, if you measure one of the qubits, then the remaining N-1 qubits are reduced to a fixed (known) sum-parity state; if you measure a second qubit the remaining N-2 then are in a known-sum-parity state, and so on. All qubit measurements act, as far as any measurer can tell, like perfect "fair coin tosses," *except* for the last; once N-1 qubits are measured, the final N^{th} qubit's state is determined. If we drop all *those* N qubits into N mutually-far-separated black holes, we get a state in which all N black holes must somehow be connected by wormholes!

Second ER=EPR problem. Einstein-Rosen and Kruskal bridges inherently only work for *two* Schwarzschild black holes, *not* an arbitrary number N>2 of them. That is another trivially simple objection, again incredibly nowhere discussed in Maldacena & Susskind 2013.

Now at this point an imaginary Susskind appeared on my shoulder and said "the maximal analytic extension of the Kerr metric (Boyer & Lindquist 1967) involves an *infinite* number of Kerr black holes in 'different universes' (or one perhaps could imagine them, or some of them, as being in the same universe, but far separated) metrically linked together inside." And Kerr holes, of course, are really what one should be considering if spin is involved. "So therefore," (continued the imaginary Susskind) "your N-fancily-entangled spins thought-experiment could be handled."

Third ER=EPR problem of the same ilk. The trouble with that is: the maximal analytic extension of the Kerr metric requires that all the Kerr holes have the ***same* mass, |spin|, and |charge|.** Also, "infinity" is *not* the same thing as "an arbitrary integer N>2." Also, if you look in Boyer & Lindquist's paper about the specific pattern of how the infinitude of Kerr holes are metrically linked (which is much *less* symmetric than the group of all N! permutations of N things), then you'll see that if we *measure* the spin of one arbitraily-selected hole, causing it instantly to "divorce" from the N-way "marriage," leaving an (N-1)-way marriage... and so on for a succession of more spin measurements done in any of the N! possible time-orderings... well, this is something that as far as I can see, just cannot happen.

Fourth foundational ER=EPR problem. Kruskal 1960's analytic extension of Schwarzschild metric is an *exact* solution of the GR vacuum field equations. "Exact" means it is not necessarily ok to perturb it. (And you'd better take this worry seriously in view of our refutation of the physicality of Einstein-Rosen 1935 despite 91 years of unqualified praise for it, and the Morris-Thorne-Yurtsever argument that it would yield paradoxes.) The reader may find it helpful to consider the function $F(z)=10^{-100}\exp(-z)$ for complex z with $\mathrm{re}(z)\geq 0$. This function is very close to the function $G(z)=0$. Indeed $|F(z)-G(z)|\leq 10^{-100}$ for all z with $\mathrm{re}(z)\geq 0$. However, they are not exactly equal. If we now consider the analytic continuations of these two functions into the complex z-plane, in particular into the region $\mathrm{re}(z)<0$, then F(z) and G(z) no longer are close approximations. Indeed |F(z)-G(z)| grows *unboundedly exponentially large*. This is not because I chose an atypical nasty F(z). Instead, the well known Liouville's theorem from complex analysis states that every function both analytic and bounded on the entire complex plane, must be constant. In other words, *every* nonconstant analytic perturbation you can make to *any* analytic function in the right halfplane (even if its absolute value is bounded below 10^{-100} everywhere!), will yield *unboundedly huge* perturbations when we use analytic continuation to extend the domain to the left halfplane.

> More strongly, it is known that any function F(z) both analytic and with absolute value bounded *below a polynomial(|z|)* on the entire complex z-plane, must *be* a polynomial(z). Therefore, *every* nonpolynomial analytic perturbation you can make to an analytic function in the right halfplane (even if its absolute value is bounded below 10^{-100} everywhere!), will yield, when we use analytic continuation to extend the domain to the left halfplane, perturbations so huge that they cannot be bounded by any polynomial. An even stronger result arises by combining that with this Paley-Wiener theorem; "The F(z) analytic and bounded in the closed upper halfplane are the functions F(z) that, when restricted to real z, have Fourier spectrum supported on a nonstrict subset of $[0,\infty)$." We deduce from those that any nonconstant analytic F(z) bounded within one halfplane must grow at least *exponentially* along rays into the other halfplane. And matters often are even worse than that: consider the function $H(z)=10^{-100}\sum_{j\geq 0}3^{-j}\exp(-2^j z)$. This function is tiny everywhere in the right halfplane $\mathrm{re}(z)\geq 0$, and analytic wherever $\mathrm{re}(z)>0$. But all along the imaginary axis, H(z) is everywhere differentiable but nowhere second-differentiable; and no analytic continuation into the left halfplane $\mathrm{re}(z)<0$ exists *anywhere*.

Analogously, if a slight perturbation (e.g. caused by "dirt," or some faraway gravitating mass, perhaps another black hole) is made to the Schwarzschild metric on one side of an Einstein-Rosen or Kruskal bridge, it is entirely possible that it will often (usually? always?) cause *unboundedly enormous* perturbations on the other side. For that reason, it was **utterly unjustified** for Maldacena & Susskind (or anybody) to simply *assert* that an Einstein-Rosen bridge could join two holes in the *same* universe. The Kruskal exact solution is *only* valid for a bridge joining *two* – not one, and not three – universes, each asymptotically flat, and each an *exact* Schwarzschild metric. If there is any faraway hole somewhere else, then we do *not* have an exact Schwarzschild metric anymore. No matter how tiny this perturbation is, it might unavoidably destroy validity of the metric as an EVFE solution. So as far as I can tell *no* justification – whether theoretical or via a computer numerical calculation – has ever been given (and certainly not by Maldacena & Susskind in their 2013 ER=EPR paper) of the claim that such "vacuum wormholes" linking faraway holes in the *same* asymptotically-flat universe, could mathematically exist.

Do not be fooled by Misner 1960, which claims to provide "initial conditions" for a wormhole linking two faraway regions in the same universe. *Obviously*, 3-dimensional (*timeless*) metrics mathematically exist (with no physical-reality constraints, e.g. no demand they obey Einstein equations) exhibiting that topology. So what? What matters for physics is whether (3+*1*)-dimensional wormhole space*times* exist, which are long-lasting and nearly time-invariant solutions of the Einstein field equations. (If the two holes circularly orbit, they would be *exactly* time-invariant if it were not for the very gradual decay of that orbit due to energy-loss into gravitational waves, which by separating the holes far enough could be made arbitrarily slow. Also, in *de Sitter* space, there presumably would be an unstable but *exactly-time-invariant* two-holes solution held apart at just the right fixed separation by the repulsion caused by the Einstein cosmical constant Λ.) *That* question – the question that matters – was nowhere addressed by Misner.

Fifth: Probable circular-logic problem: Examining Van Raamsdonk 2010's

> ABSTRACT: In this essay based on 0907.2939, we argue that the emergence of classically connected spacetimes is intimately related to the quantum entanglement of degrees of freedom in a non-perturbative description of quantum gravity. Disentangling the degrees of freedom associated with two regions of spacetime results in these regions pulling apart and pinching off from each other in a way that can be quantified by standard measures of entanglement.

even if VR's arguments were acceptable mathematical proofs (which they are not), I think there still would be a severe risk that they are "circular." I.e. VR's arguments that you need quantum entanglement to get "classically connected spacetimes" are based on physics that *assumed*, as step 1, that we have a classically connected spacetime! For example, the very fact AdS is an exact solution of the Einstein vacuum field equations with attractive-signed cosmical constant Λ rests on an assumption we *have* a classical spacetime manifold. AdS underlay Maldacena's attempts to show "holography," which underpinned VR's arguments. And the very existence of quantum mechanics itself (e.g. Schrödinger equation) rested on assumptions we have a smooth spacetime manifold. If VR wants to convince me his arguments are not circular, then he'd presumably need to redo all of general relativity and quantum field theory, starting from sticks and stones, in a way that did not depend on any assumptions we have a smooth spacetime manifold (or anyhow made it clear there was no circularity involved – probably not easy because there historically were *many* logical steps to worry about) before he could even get started. No noncircularity demonstration anything like that ever happened, and VR's approach was to blissfully ignore this entire worry.

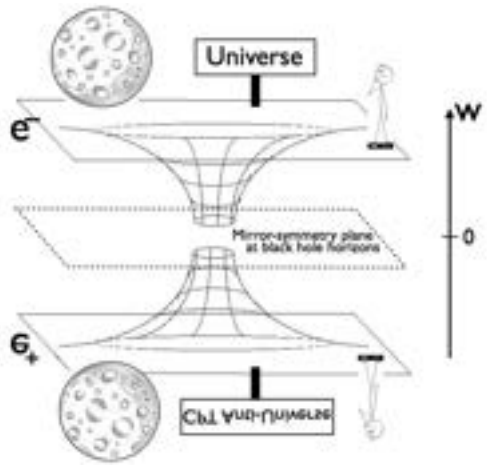
"Evil conspiracy theory" Historical note. Kruskal 1960 was actually written and submitted (and its figures drawn) by J.A.Wheeler, *not* Kruskal, but only Kruskal appears as an author! That is because Kruskal showed his mathematical work underlying the paper to Wheeler, requesting his feedback as a GR expert, whereupon Wheeler said "not important." [Both Kruskal and Wheeler were Princeton professors – Kruskal in applied maths, Wheeler in physics.] Years later, Wheeler decided it actually was very important hence wrote & submitted the paper with Kruskal's name on it, as repentance. (Kruskal, in Europe at the time, was astonished to receive *Physical Review*'s acceptance letter!) I believe that if Kruskal had written it, then it would have included some of the internals of his calculations, rather than merely their results; and some discussion of his motivations. More importantly, I suspect that **"Kruskal's" unjustified claim in his fig.1 that the two Schwarzschild holes could alternatively be regarded as residing, *not* in two different universes, but rather only one (in the limit of far separation) was *invented solely by Wheeler*.**

Why? First, I doubt Kruskal would have made such a nonrigorous claim sans critical examination. Second, Wheeler was an excellent artist and draftsman and drew that figure. Kruskal was not and did not. (Incidentally, I knew Kruskal personally fairly well.) Third, Wheeler had an evil ulterior motive (in which Kruskal was uninvolved) for introducing same-universe wormholes: they set the stage for his elaborate "geometrodynamics" physics-fantasy. Fourth, Wheeler repeated his error on p.920 of Fuller & Wheeler 1962 whereupon Morris & Thorne 1988 repeated it again in their fig.1b. My theory of the evil history of Kruskal's paper thus explains how and why Wheeler snuck a completely mathematically-unjustified (but unfortunately self-propagating) idea into the physics literature in a paper by another author who never wanted to, and did not, propose that idea!

If we consider the diagonal line u+v=0 in Kruskal 1960's fig.2 spacetime diagram separating the traditional Schwarzschild half u+v>0 from the "new universe" u+v<0 (null geodesics correspond to 45°-inclined lines in this picture), then hypothetical analytic complex-valued perturbations of forms $10^{-100}\exp(-K[1\pm i][u\mp iv])$ with K>0 would be tiny throughout the old universe, but assume unboundedly enormous values in the new one.

The logical **error** repeatedly commited by Wheeler was that he took the limit of far-separation *first*, then considered a Kruskal wormhole metric linking the two faraway locations *second*. Really, you need to have Kruskal-like wormhole metrics linking two (*not* necessarily far apart) locations first, then consider the far-distance limit second. That way, your limit can hope to exist! Wheeler's wrong way, no wormhole solution need exist, and therefore no limit need exist. Mathematicians call this "unjustified reordering of limits." It is not allowed. No valid argument has ever been produced by Wheeler or any later wormholer that any wormhole EVFE solution (whether traversible or not) can mathematically exist linking two locations in the same asymptotically-flat universe.

W.Rindler in 1965 (and others later, e.g. G.'t Hooft 2018) suggested *not* interpreting Kruskal's metric as a wormhole joining two universes (or two exterior-anythings), but rather *identifying* Kruskal's "two" spacetime points ±(u,v). This option would (to quote Rindler) "destroy the arrow of time within the [black hole] interior." But note that the sufficient analytic extensions of the Kerr metric include closed timelike curves inside the inner horizons, so it would be surprising if the Schwarzschild limiting case lacked them. Also, for Kerr, Kerr-Newman, and Reissner-Nordström, the central singularity is *timelike*, meaning timelike geodesics generically permanently avoid hitting it ("repulsion"), unlike the spacelike singularity in Kruskal/Schwarzschild, which every infalling geodesic is doomed to hit after a bounded amount of time measured by a clock carried by the infaller. With Rindler's re-interpretation, that doom can be avoided, or at least postponed, by an infaller equipped with good-enough rocket engines. Furthermore (**'t Hooft**) within that region quantum field theories would need to be CPT-invariant, causing antimatter and matter to be the same thing. These both suggest the possibility that the laws of physics inside and outside black holes radically *differ*! There is no classical way for external experimental physicists to decide that. But neither Rindler or 't Hooft said *what* could possibly cause the *laws of physics* to thus drastically change when you pass through a not-metrically-special horizon surface. And 't Hooft overlooked my bi-future suggestion, which, if combined with the Rinder two-point-identification trick, would *not* yield closed timelike curves whereupon there would *not* be any need for antimatter and matter to be the same inside the horizon, and infallers would resume all being doomed.



3. Interlude: New possibly-revolutionary (or crazy) self-consistent "mirror paradigm" for black holes.

I now point out that an analogue of Rindler's same point-identification trick could be applied *not* to Kruskal 1960's, but rather to the Einstein-Rosen 1935 metric: identify points with w and -w coordinate (but otherwise same), while reversing our notions of charge, parity, and time-direction for each particle with negative w. (This time-direction switch is in *addition* to the one I already argued was necessary with Einstein-Rosen, and causes increasing-t to again point into the "future" on both sides of w=0.) The w<0 side then, quite literally, would be a CPT "mirror anti-universe" with black hole horizons playing the role of the "mirror." Masses falling into the horizon then could *not* "pass through to the other side" but rather would instantly annihilate into pure energy with their antimatter mirrored versions (also falling in, from the anti-universe side). That energy then would *stay* on the w=0 horizon *surface*. There would be ***no* black hole "interior,"** only a surface that effectively would be an impassable but absorbing *boundary* for our universe.

The mirror paradigm offers the potential to explain, very simply, both Bekenstein-Hawking black hole "surface entropy" and "Hawking radiation" much more naturally, perhaps now without "paradoxes," while also getting rid of the singularities that resided inside (former) black hole notions, and *not* using (utterly unjustifiably) analytic continuation and related ridiculous "AdS/CFT holography" fantasies.

List of features of "2-sided-mirror" paradigm:

1. E&R's full spacetime metric (both universes combined) does *not* rest on any need for analytic continuation, so pollution of the metric by nowhere-analytic functions is not a devastating objection (in fact no objection at all) to the mirror paradigm.
2. For example: Hawking's "information loss paradox" had *been* paradoxical because an encyclopedia tossed into the black hole would fall *deep inside* the horizon, where it could no longer communicate its information up to that horizon and then into Hawking radiation. If there *is no* black hole interior, that paradox is avoided.
3. The question of how black hole entropy possibly could be proportional to *surface area* while the information constituting that entropy instead presumably sat far inside, while the horizon was metrically not special? Again, no longer a problem.
4. "Firewall" proposal? Kind of happens automatically due to antimatter-annihilations (plus anyhow is not needed since no interior).
5. The horizon surface *is* made special: as I pointed out, with Einstein & Rosen 1935's metric, the horizon is a metrical singularity while everyplace else is regular. And as the mirror-symmetry-plane between the universe and mirrored anti-universe, it plays a special role as a surface of sure-annihilation, at which energy is trapped; and all observers agree on its location.
6. Hawking radiation? Regarding it as "quantum tunneling" now actually makes some intuitive *sense*.

7. "Time stopping" singularities such as the Schwarzschild r=0 point demanded to exist inside "trapped surfaces" by Penrose theorems? Banished.
8. The objection that both ER black holes need to have exactly the same mass, |charge|, and |spin|? Not a problem since the "mirror universe" is assumed to be an exact, albeit CPT-reversed, copy of our universe.
9. "Wormholes" to faraway places in the same universe (never-justified fantasy which might be mathematically impossible)? No longer needed nor present.
10. Morris-Thorne-Yurtsever "time machine" paradoxes? Since the mirror paradigm version of E&R's wormholes effectively are *not* traversible – any traverser would get annihilated, and then the resulting gamma rays would get trapped – those paradoxes cannot occur.
11. My instability or unperturbability objections about Einstein-Rosen? Gone due to the extra time-reversal causing perfect mirror-symmetry. Indeed black holes of *non*-Kerr-Newman metrical form (e.g. swiftly dying perturbations thereof) still could be "mirrorified" by the same sort of Einstein-Rosen trick involving defining, via squaring, a coordinate 0 on the horizon, positive above it, and there is nothing below.
12. CPT-invariance of quantum field theories? Actually now has a useful *purpose*.
13. The paradoxical fact that a black hole formed from a (>99.9% baryonic) star ultimately yields, after Hawking-evaporation, >99.9% *non*baryons (contradicting heavily experimentally confirmed laws of baryon-number and lepton-number conservation), and indeed should output antimatter and matter equally? No longer contradictory, because the mass-energy content of the black hole is pure-energy, convertible in the "standard model" into any particle type.

In contrast, the faraway "cosmological horizon" in de Sitter spacetime (which also has a positive temperature and quantum-radiates, according to Gibbons & Hawking 1977) is *not* observer-independent; but since it never was troubled by "paradoxes" we presumably do not need any analogous special treatment for it.

Is the mirror paradigm a **brilliant** answer, or **crazy**? The main reason to suspect it brilliant is the above 13 pieces of evidence. Want more than 13? See the final section, which extends this list to 21. The main reason to suspect it crazy is the fact Einstein & Rosen 1935 "cheated" and were wrong, i.e. their metric is *not* an EVFE solution. To overcome that objection, I need to hypothesize as a **new law of physics** that the "flattened" energy trapped on the E&R horizon surface can "*act* exotic" i.e. act for the purposes of its effect on the metric via the Einstein field equations, as though it had negative-mass areal-density. In addition to negative effective areal mass-density, there could also be effective negative pressure in the radial direction (Morris & Thorne 1988 call this "radial tension") as well as large |pressure| in angular (perpendicular to radial) directions. All this should be computable by writing the Einstein-Rosen wormhole as a special case of the general Morris-Thorne 1988 wormhole family (their EQ 1), then using their helpful formulae 17, 18, and 19; and I'll do that in the final section.

Whyever should that effective negative density and pressure happen? Well, the E&R horizon is a special place. It is a metrically-*singular* surface. Therefore, Einstein's ordinary laws (which are partial differential equations) cannot be mindlessly applied. Mass-energy located on it, is "flattened" to be 2- not 3-dimensional. There is no time, in the senses that the time-time entry g_{tt} of E&R's metric tensor equals zero on the horizon; and any particle staying on a black hole horizon (at least, in the old Kruskal 1960 view of what a "horizon" was) needs to be traveling at lightpseed and

therefore cannot experience time. I had said any particle entering the horizon instantly annihilates with its antiparticle (entering from the other side) into pure energy, but given that there is no time, perhaps such particles are permanently trapped in a state of being "half-annihilated, half not-yet-annihilated." All this specialness suggests to me that some *new* law of physics *must* be hypothesized to handle this. The law that I am suggesting – i.e. that we get Einstein-Rosen-style metrical and CPT "mirroring" at horizons, arguably is the **simplest** possible such law, at least from the macroscopic point of view.

What happens to a star collapsing into a black hole? I would guess that, at the moment each of its particles got cut off from the external universe, it would annihilate with its mirrored anti-particle and "flatten" onto the expanding horizon-surface. An explicitly calculable such collapse (exact GR solution) is "Oppenheimer-Snyder dust-ball collapse": Describe the interior of the dust-filled-ball region by a homogeneous FLRW "contracting universe" metric, while its exterior is a Schwarzschild vacuum metric. The centerpoint of the dust ball, at some moment, becomes an event-horizon sphere, initially of zero radius. That sphere's radius then expands at the speed of light until it reaches its final value $2GMc^{-2}$, then stays at that value forever after. This switchover occurs at the exact moment the dust-ball radius and horizon-sphere radius are equal. I.e. horizon-radius-expansion occurs exactly while the horizon-sphere lies inside the dust-ball, but fixed-radius occurs exactly while the horizon-sphere encloses the dust-ball. The horizon-sphere (according to old-style classical GR) is never metrically special. But according to the mirror paradigm, there is no horizon-interior and as the horizon-sphere expands through the dust-cloud, each dust particle gets annihilated and flattened the moment the sphere hits it, then stays on that sphere surface forever after. I would like to know if there is any purely-*local* way the first particle could "know" it is on a horizon in order to *start* that process. The final section will examine this local-test question.

Is my whole mirror paradigm fantasy/proposal "science" at all, i.e. susceptible to experiment? Classically, there is no way an external experimenter can tell what happens inside, or on the horizon of, a black hole. So the classical answer is "no." However, "quantum tunneling" effects such as "Hawking radiation" might in principle, at least partly, convert that into a "yes." Also, even with no experiments at all, we still are allowed to dismiss physical theories containing logical contradictions. As Hawking pointed out, the combination of Einstein general relativistic gravity with semiclassical quantum analyses yielding "Hawking radiation," "Hawking temperature," and "Bekenstein-Hawking entropy," appear to yield contradictions ("paradoxes") among previously-accepted laws of physics. *Therefore, those laws of physics must change.* The mirror paradigm is such a change, and a quite minor and simple one, which seems to get rid of all those contradictions. And if correct, then it would be part of future "theories of everything" – some of which hopefully would have experimentally checkable consequences.

Bottom line: I think the mirror paradigm *is* pretty crazy, but certainly far less than "ER=EPR" and "AdS/CFT"; and I presently see no way to attack its putative validity.

4. Destruction resumes

The two key innovative concepts in **Misner & Wheeler's "geometrodynamics" vision** had been "mass without mass" and "charge without charge." That is: all "mass" was postulated to really be wormholes which look *externally* like two Schwarzschild black holes ("localized masses") but actually there never was any "mass" (in the old sense of the word) there – the wormhole is hollow

and there is "nothing inside" – the stress-energy tensor shows pure-vacuum everywhere. (Actually this is not really true, since a curvature singularity exists forming a boundary for the Kruskal metric. On this boundary, null and timelike geodesics *end*. However, "mass without mass" really would be true using Einstein-Rosen wormholes, which E&R claimed singularity-free; but Wheeler did not use the latter and I criticized E&R's solution as not really being a *vacuum* solution of the field equations, and not really singularity-free, at all.) Similarly, all "charges" were postulated to really be wormholes with electric field lines heading into one black hole, and emerging from the other, thus giving the *illusion* of two faraway point charges (one -, other +) but without any actual source of charge – the $\vec{\mathrm{E}}$-field is divergence-free everywhere. There remained the minor matter of somehow explaining why all electrons have the *same* charge and mass, presumably via some sort of quantum-addition to their vision – and explaining why there are far more electrons than positrons – small tasks left to future authors.

But: I suspect that **no GR vacuum solution exists** that describes a wormhole connecting two far-separated black holes in the same (de Sitter or asymptotically-flat) universe. More generally I suspect that no solution of the Einstein-Maxwell equations exists describing a wormhole connecting two far-separated oppositely-electrically-charged black holes in the same universe. I.e: **There is no "mass without mass"! There is no "charge without charge"!** Anybody who wants to shove the "geometrodynamics" vision down our throats (such as Wheeler 1963), needs, as a first step, to refute my suspicions by **proving existence** of such GR-solutions. As a second step, you'd want to argue that they can somehow survive the onslaught from nowhere-analytic metrical pollution. Neither underhanded stunts *pretending* to provide such an existence proof "trivially," and with another author's name conveniently on it; nor 3D-only papers like Misner 1960 attempting to fool the reader into thinking wormhole existence was "proven," are acceptable substitutes.

Sixth objection to ER=EPR: How could we have two *electrons* with entangled spins, when geometrodynamics would instead demand electrons be wormhole-linked to *positrons*? One way to generate two spin-entangled electrons is a double beta-decay of, say, $^{100}\mathrm{Mo}\rightarrow{}^{100}\mathrm{Ru}$ (halflife=7.1×10^{18} years; both these isotopes have nuclear spin=0 and parity=+). And under the as-yet-unproven hypothesis that some nuclei can experience "*neutrinoless* double beta decay," the two emitted electrons necessarily would be *maximally* entangled.

Seventh objection to ER=EPR: Electrons, protons, and atomic nuclei have too great charge, and also in all spin≠0 cases too great spin (the latter also is true about neutrons), in comparison to their masses, to *be* Kerr-Newman black holes. Instead, any Kerr-Newman-like attempt to model them as point particles via classical Einstein-Maxwell necessarily would conclude they are naked singularities with *no* event horizon anywhere for anybody to attempt to "extend" the metric across. So: question: how can two of these particles be entangled via ER=EPR, if no ER or Kruskal (nor any other metrical kind of) "bridge" is classically possible?

5. Conclusion

As far as I can tell, the whole ER=EPR and geometrodynamics ideas are completely unjustified smoking rubble, with "AdS/CFT" and everything depending on "analytic continuation" dead meat. The fact that the Maldacena 1998 paper introducing "AdS/CFT" was claimed in Cowen 2019 to be the *most-cited theoretical physics paper of all time* (although I consider it "10 out of 10 on the

bullshit meter"!) is an embarrassment.

And when I emailed a draft of this paper to both Maldacena and Susskind, requesting their response (e.g. did they have any defense of ER=EPR?), after 7 weeks none came.

6. Updates (added for v2 after version v1 of this paper finished & posted)

1. **Javed & Wilson-Ewing 2026** argue that the ER=EPR hypothesis can be, or perhaps even already is, experimentally ruled out by precision measurements of hydrogen atoms.
2. A few days after I posted this paper to the vixra science-paper-server, google brought the paper **Tzanavaris, Boyle, Turok 2025** to my attention. Astonishingly, TB&T, entirely independently, discovered almost the exact same "mirror paradigm" idea that I did in §3, under the name "black mirror"! They had different motivations and arguments than mine, but their mirroring idea is almost identical. In my opinion the optimal-for-science course of action would be to produce a joint paper, iron out whatever discrepancies exist between us, and decide what we jointly believe/not based on our full combined set of arguments. I sent all three emails saying so, but after 2 weeks none responded.

Later update: Since unfortunately it is now pathetically obvious that Tzanavaris, Boyle, & Turok intend to ignore my emails (just like Maldacena & Susskind...), I am left with no alternative but to try to briefly summarize their work here. Furthermore, since their behavior precludes it, I withdraw my offer to co-author a joint paper. I instead will just say in the next section what *I* believe would be the best joint formulation – which is *not* equivalent to TB&T's because mine is better.

TB&T, and/or co-authors, actually wrote a flock of papers related to this. But unfortunately that flock is flawed since it all assumes analyticity and/or uses analytic continuation, hence has zero physical relevance. Their analyticity assumptions/uses of course totally oppose the main point of the present paper. But strangely enough, they nevertheless led TB&T et al to a picture greatly resembling my "mirror paradigm," with that resemblance *especially* great for Tzanavaris, Boyle, Turok 2025. "**Wick rotation**" (another analytic continuation trick) is used in TBT 2025, but since that part of their paper does not affect their version of my "mirror paradigm," that is no basis for rejecting the latter. TBT 2025's fig.3c depicting the "black mirror" concept naively seems to disagree with me. It shows the intersection of their two universes as a *single point* in a radius-time diagram, i.e. a codimension-2 set. However, in my mirror paradigm the horizon is the surface w=0, with codimension=1. However, since they glued the two lines they labeled "BH" together, that disagreement is illusory.

I now go through the papers in the "flock." For each paper, I give some quotes from it that invalidate it for either our main reason – they use analyticity assumptions about the spacetime metric which doom them to physical irrelevance – and/or because of miscellaneous glaring errors:

1. L.Boyle & N.Turok: Two-Sheeted Universe, Analyticity and the Arrow of Time, (2021). "Our universe seems to be radiation dominated at early times, and vacuum energy dominated at late times. When we consider the maximal analytic extension of this spacetime, its symmetries and complex analytic properties suggest a picture in which spacetime has two sheets, exchanged by an isometry..." (This is like my "mirror paradigm" picture.) "The fact that basic considerations of symmetry **and analyticity** force all fields to satisfy a restricted boundary condition at one end of spacetime (the bang) but not at the other end (dS boundary)

is striking. It seems to suggest a fundamental new explanation for why the thermodynamic arrow of time (i.e. the direction in which entropy increases) points away from the bang." My response: Because their quote assumed analyticity, their "explanation" for the "arrow of time" is simply not justified. Also, de Sitter space ("dS") is a constant-curvature spacetime, all of whose geodesics (whether spacelike, timelike, or null) extend bi-infinitely, i.e. it *has no* "boundary."

2. L.Boyle, Martin Teuscher, N.Turok: The Big Bang as a Mirror: a Solution of the Strong CP Problem (2022). Since it uses **analyticity**, this paper has zero physical relevance. They claim the "time=0" *big-bang* singularity also (in addition to the black hole horizons I suggested in this paper) is a "mirror" boundary between the universe and antiuniverse. They then claim this realization solves the "strong CP problem." But I do not understand that claim.

   We could, however, as a **new law of physics**, hypothesize that this particular mirror has the *asymmetric* property of having a slight bias, of order 10^{-8}, for shoving matter to our side, and antimatter to the antiuniverse side. (Actually, my word "asymmetric" was misleading: both sides symmetrically think this "asymmetry" favors them, so it really isn't asymmetric.)

   An analogy: A famous experiment by Wu, Ambler et al 1957 found that Co-60 atoms β-decaying in a strong magnetic field preferentially emit electrons toward the "north pole" of the external magnet; meanwhile, by CPT symmetry, anti-Co-60 atoms inside the same magnet would preferentially emit positrons in the opposite direction. So this device "asymmetrically" favors pushing antimatter to one side and matter to the other.

   To explain my CPT reasoning: consider a coil (wire wound round an iron horsehoe) creating a magnetic field along the vertical z-axis. Midway between the two poles of this electromagnet sit Co-60 atoms. The Co-60 atoms emit electrons preferentially toward the *north* pole of the external magnet. This creates a steady-state situation featuring an electron-excess located above the Co-60 atoms. In a CPT-reflected universe (using the yz plane as the mirror plane), the coil conducts positrons not electrons (C), and both time T (hence lepton flow direction in the wire) and coil clockwise/anticlockwiseness (P) are reversed, the net effect of all three being a magnetic field *reversal*; and we have anti-Co-60 atoms, which therefore emit positrons not electrons. By CPT symmetry, the steady-state situation must feature a positron excess above the anti-Co-60 atoms, i.e. in the direction from those anti-atoms toward the *south* pole of their external magnet.

   Such hypothesized new laws, analogous to the Wu, Ambler, et al 1957 experiment but now concerning universe-sheet-boundaries instead of atoms β-decaying inside magnets, could resolve questions like "why is matter prevalent in our universe today?" – basically simply by proclaiming them resolved!

3. L.Boyle, Kieran Finn, N.Turok: The big bang, CPT, and neutrino dark matter, Annals Physics 438 (2022) #168767. "In this paper, we **analytically extend** the metric across the bang..." Zero physical relevance.
4. N.Turok & L.Boyle: A Minimal Explanation of the Primordial Cosmological Perturbations (2023) "We impose CPT symmetry by **analytically extending** the spacetime to a 'mirror' universe on the other side of the bang." Zero physical relevance.
5. L.Boyle & N.Turok: Thermodynamic solution of the homogeneity, isotropy and flatness puzzles (and a clue to the cosmological constant), Physics Letters B 849 (Feb.2024) #138442. Their elliptic-function solution of FRW cosmological model is *not* new and was already available in at least one textbook over a decade before they did anything. However, I never saw anybody before using analytic continuation starting from that to deduce various claims. So *that* part is new, at least to me. Is their analytic continuation starting from it, and

their consequent deductions, legitimate? I am unsure. As they point out, some deductions about Hawking radiation may be elegantly found via analytic continuation tricks. However, since Hawking radiation can be derived *without* recourse to those tricks, its presumed validity is not hurt. And also, if we were analysing an *exact* (non-dirty) Schwarzschild metric, which *is* analytic, then *for the purpose of that mathematical analysis alone* continuation is justified. I could continue in this vein. For example it is possible to derive (well, "derive" probably is the wrong word; better is "obtain") the Kerr family of black hole metrics from Schwarzschild via the "Newman-Janis algorithm." The Kerr-Newman more-general family is obtainable in the same way but starting from Reissner-Nordström instead of Schwarzschild (which in fact historically *was* the way the Kerr-Newman solution was first found). These can be regarded as "a sophisticated combination of hope-based heuristic symbolic transformations and analytic continuation tricks." In these cases it all luckily works, although Newman fully admitted that he had little clue when and why his techniques worked versus when they did not. In particular, if you started from a generic *dirty* Schwarzschild metric, then tried to use the Newman-Janis algorithm to obtain a "dirty" Kerr metric, that would *not* work at all. It would be total garbage. But anyhow, the non-rigorous hope-based nature of these tricks and the fact that they generically are unusable garbage,*does not matter* because at the end, Newman et al could (and with the assistance of 5 others did) fully-rigorously check that the Kerr-Newman metric and field exactly solves the Einstein-Maxwell vacuum equations. So analytic continuation certainly has valid uses, and can be a very powerful tool *when* applicable. Similar remarks perhaps could justify B&T's analogous work in this Feb.2024 paper. B&T provided no such justification, and acted blissfully unaware of the whole analyticity issue, but *maybe* such a rescue is possible.

6. L.Boyle, Kieran Finn, N.Turok: CPT-Symmetric Universe, Physical Review Letters 121 (2018) #251301. "We propose that the state of the universe does *not* spontaneously violate CPT. Instead, the universe after the big bang is the CPT image of the universe before it, both classically and quantum mechanically. The pre- and post-bang epochs comprise a universe/anti-universe pair, *emerging from nothing* directly into a hot, radiation-dominated era." "[Three] testable predictions follow:
    i. the three light neutrinos are Majorana and allow neutrinoless double β decay;
    ii. the lightest neutrino is massless;
    iii. there are no primordial long-wavelength gravitational waves."
   I'm guessing their prediction ii eventually will be experimentally falsified, but year-2026 experimental evidence is unable to answer this question. (And as far as I know iii *already* has been experimentally falsified by cosmic microwave background sky maps.) Also, BF&T fail to mention that their predictions i and ii *logically conflict*, see Smith 2023. And as usual their paper should have zero physical relevance because based on "the **analytically extended** FRW background..."

I think Boyle, Turok et al did produce the interesting idea that it **looks natural** for exactly one "anti-universe" partner to exist for our universe, in the sense that, if you were going to create a universe, presumably you would simultaneously create an anti-universe in order not to violate laws such as baryon- and lepton-number conservation. However, I do not see any reason our universe necessarily ever got "created" – with some future better understanding of physics (e.g. future "quantum gravity" theory, which everybody agrees would be required to describe early-universe conditions) the universe perhaps could persist forever without there being any creation-time.

This flock-review has actually been a good **demonstration of the destructive power** of the present work: the physical relevance of the six Boyle, Turok et al flock-papers is instantly mostly seen to be **zero**. It might be possible to find alternate logic-paths avoiding analytic continuation and analyticity assumptions – plus appropriately repair their other errors that I spotted en passant – and thereby rescue some of their work. I don't know which parts are rescuable and which are not, and in view of their non-cooperation have not attempted to find out. For the present (July 2026) all six must be considered busted/illegitimate.

Moral: Ignore generic spacetime nowhere-analyticity at your peril!

Worldwide damage assessment: More generally, it is interesting to ask ***how many* science papers the present work invalidates**. I can crudely approximately answer that question by Monte Carlo sampling. The total number of science papers published so far (July 2026) is ≈100 million, which is 40 times the number on arXiv. On the other hand, of the papers I cited in my "references" section in a slightly older version of *this* paper, I found 16 on arXiv with 30 not; and the ratio (30+16)/16≈2.9 is well short of 40/1. This discrepancy is due to

a. My reference-list is biased in favor of arXiv papers because those are more convenient for me to access and find,
b. The arXiv's composition is biased to contain a larger fraction of GR-related papers, but a smaller fraction of papers in certain other fields, than the scientific literature generally.

Consequently the truth should lie *between* 2.9 and 40. Using arXiv automated search tools and cursory examinations, I estimate that 500 arXiv papers partly or wholy about general relativity use analytic continuation or assume analyticity. Therefore (by multiplying by 40) as a crude estimate, 20000 published science papers are invalidated "at level 1." But 20000 would drop to only 1450 if the multiplier "40" were replaced by 2.9. But papers also can be invalidated "at level 2" if, say, they depend on a cited level-1 paper; then more papers could be invalidated "at level 3" if they cite a level-2 paper, etc. So 1450-20000 should merely be lower bounds.

7. New slick mathematical descriptions of "mirror paradigm" metrics. Resulting new grandiose mental picture of universe.

Mirror paradigm Schwarzschild black hole metric:

$$ds^2 = -c^2[1-H/|r|]dt^2 + [1-H/|r|]^{-1}dr^2 + r^2[d\theta^2 + \sin(\theta)^2 d\varphi^2]$$

with circumferential radial coordinate r, and where $H=2GMc^{-2}$ is the value of r at the *h*orizon, i.e. Schwarzschild radius. (The reason r is called "circumferential" is that the circumference of each sphere r=R≥H agrees exactly with its Euclidean value 2πR.) The coordinates are (t,r,θ,φ) with t assuming arbitrary real values, r assuming real values with |r|≥H, θ assuming real values with |θ| ≤π/2, and 0≤φ<2π with φ regarded as "wrapped" modulo 2π; and where "increasing |t|" is regarded as "future-directed" and we employ the

(-t,-r)-Identification convention:	(t,r,θ,φ) using particles such as e^-	is *identified*	(-t,-r,θ,φ) changed to antiparticles such as e^+

with

Mirror paradigm Kerr and Kerr-Newman black hole metrics: For Kerr, which generalizes Schwarzschild to permit black holes with nonzero angular momentum J, use the (t,r,θ,φ) coordinates of EQ 2.13 of Boyer & Lindquist 1968, *except* replace r by |r| in all their formulas. For Kerr-Newman, which is the generalization of Kerr to permit the black hole to possess nonzero electric charge Q, again use (what wikipedia calls) the "Boyer-Lindquist" coordinates, except again replacing r by |r| in all their formulas, the point of which is that we now permit negative r. The region with r<0 is called the "anti-universe," as opposed to r>0 which is the "universe." For both Kerr and Kerr-Newman the "spin parameter a" is defined by $a=JcG^{-1}M^{-2}$. *Except*: for both Kerr and Kerr-Newman, perform my above (-t,-r) point-identifications with charge-reversal and reversal-involving definition-of-future, and now with

$$H = GMc^{-2} \pm [G^2M^2c^{-4}-a^2-Q^2Gc^{-4}/(4\pi\varepsilon_0)]^{1/2}$$

being the formula for the value of r at the horizon. The ± sign is + for the outer and - for the inner horizon – choose whichever one you prefer (but only one, and stick with it); both work but are inequivalent. The *outer* horizon (+ sign) is the "better" choice for the purpose of saving us from famous black hole paradoxes.

One. Not two. Provided we agree to use my point-identification convention, the "universe" and "anti-universe" actually are mathematically *the same*. They are *not* two distinct arenas. There is only one unified arena. And each electron, is not really just an electron. It simultaneously is a positron, when you widen your view to also know about the anti-universe part of the unified picture. (This "one arena" view differs from the "two" view stated by Boyle, Turok, et al – and I contend is better.) Also I reiterate that we *refuse to permit* r with |r|<H, so all these black holes *have no "interiors."* This removed-interior serves as (what mathematicians call) a "crosscap."

Nature of black hole horizons as *singular* surfaces. The horizon surface normally has been regarded as a "sphere"; however in view of my point identifications could instead be regarded as nonEuclidean *non*orientable "elliptic plane geometry." It really is not either, since neither descriptor provides the full true space*time* picture. By writing our above mirror-Schwarzschild metric as a member of the family (their EQ 1) of Morris-Thorne 1988 "spherically symmetric wormhole metrics" we can work out its stress-energy tensor. This tensor is *zero* everywhere for both Kerr and Schwarzschild, in the sense that both exactly solve the Einstein *vacuum* field equations, *except* that the horizon *surface* is *non*vacuum since that is where all the mass-energy of the black hole resides (and classically, with our definitions of "future," none of it can ever leave that surface, although more can accumulate if mass-energy infalls). This causes this tensor to be proportional to δ(|r|-H) where δ is the Dirac delta pseudofunction. The only question is what the proportionality constants are. I will use Morris & Thorne's helpful EQs 13, 17, 18, and 19 to work that out for the first time. (Warning: I have not checked M&T's work and am just taking these equations on trust.) I now provide the details. M&T's function b(r) is b(r)=Hsgn(r), so its derivative is b'(r)=2Hδ(r). But because we have *removed* the interval (-H,H) from the r-coordinate real number line and glued r=+H to r=-H, we'll instead regard its derivative as b'(r)=[δ(r-H)+δ(r+H)]H. Meanwhile, M&T's other function $e^{2\Phi(r)}$ is $e^{2\Phi(r)}=1-b(r)/r=1-H/|r|$. Therefore

$$2\Phi(r) = \ln(1-H/|r|), \quad \text{causing} \quad 2\Phi'(r) = (|r|-H)^{-1}\, r^{-1}\, H \quad \text{and} \quad 2\Phi''(r) = (|r|-H)^{-2}\, (H-2|r|)\, r^{-2}\, H.$$

These three functions have infinite |values| when |r|=H, which can be problematic, but they are genuine functions, unlike the "Dirac delta function" involved in the formula for b'(r), which is only a *pseudo*function. I say "problematic" because, e.g, Φ'(r) has opposite limiting values ±∞ in the two one-sided limits $r\to H^{\pm}$ with the same endpoint. Now from M&T's EQ 17 we find that the **effective mass-energy density** (which M&T call "-ρ") equals

$$-(8\pi)^{-1}\, [\delta(r-H)+\delta(r+H)]\, c^2 G^{-1}\, H^{-1}.$$

which corresponds to an infinite negative **effective areal mass-density** uniformly distributed over the surface of the whole horizon sphere, whose surface area is $4\pi H^2$. (Any nonzero *areal* mass density necessarily has infinite absolute value if regarded as an ordinary *volume*-based density, even if finite as an areal density. However, here the situation is worse than that because even as an areal density it still is infinite.) Note that the **distance** L between the concentric spheres r=H and r=R in the Schwarzschild geometry does *not* equal |R-H|, but rather

$$L = \int_{H<r<R} (1-H/r)^{-1/2} dr = (1-H/R)^{-1/2} R + H\tanh^{-1}([1-H/R]^{1/2}) = 2H^{1/2}(R-H)^{1/2} + 3H^{-1/2}(R-H)^{3/2} - \ldots$$

The stated series expansion in powers of $(R/H-1)^{1/2}$ converges when |R-H|<H; the distance L is real if R>H but imaginary if 0<R<H, causing L^2 to increase from $-\pi^2H^2/4$ to +∞ as r increases from 0 to ∞. So in terms of the coordinate L instead of r, this mass-density is not proportional to δ(L), but rather equals $(-4\pi H)^{-1}c^2G^{-1}\delta(L^2/(4H))$ whose integral dL, which should tell us the effective *areal* mass density, is negative infinity. Therefore, we deduce that the **effective areal mass-density** on the horizon is negative-*infinite*. This infinity is caused by the **infinite distance-distortion factor** between L versus r as r→H. Meanwhile in the wormhole scenarios analysed by Visser 1989 (which are not the same as mine, but do have the same "flattened matter" property that all matter occurs on a codimension-1 "throat" spherelike convex surface, compressed to infinite ordinary |density|), he finds *finite* negative areal mass-energy density on his wormhole-sphere "portal between two universes" surface. The reason Visser's areal mass density is finite (indeed on "flat regions" of his horizon/gluing surface, it is zero) but mine infinite, is Visser 1989's (artificially designed) geometry has no distance-distortion. Next using M&T's EQ 18, we find that their "**radial tension τ**" (defined in their EQ 13) involves no Dirac delta and therefore must equal **zero** everywhere. [This naively appears to contradict Kuhfittig 2020. However, it really does not because Kuhfittig near the end of his §2 needed Morris & Thorne's "shape function" b(r) to satisfy three conditions, which are *not* satisfied by my Einstein-Rosen mirror-paradigm b(r) function.] Finally, using M&T's EQ 13 and 19, we find that their "**lateral pressure p**" (where "lateral" means "in angular directions perpendicular to the radial direction dr") equals either ±∞ on the horizon – i.e. **infinite** but with unclear sign! – or perhaps the "average" of these two limiting values should be taken, which would yield zero. At least naively, what seems to matter for this purpose is the integral $\int x^{-1}\delta(x)dx$ (integrated across zero, e.g. from -1 to +1), whose "Cauchy principal value" is zero. For example, $P.V.\int x^{-1}\delta(x)dx=0$ happens if $\delta(x)=(2S)^{-1}$ when |x|<S, but 0 otherwise, assuming you should take the limit S→0+ at the end of calculations involving Dirac delta functions. But less naively, we want to integrate dL *not* dr, which should cause *positive-infinite* effective lateral pressure. I now call your attention to the following quote from Medved, Martin, Visser: Dirty black holes: Symmetries at stationary non-static horizons

2004: "The sum of the energy density and the transverse pressure [must] tend to zero as the horizon [is] approached." Meanwhile in the wormhole scenarios analysed by Visser 1989 (which are not the same as either mine or MMV 2004's) the same is true, albeit there it instead is expressed as "The surface energy density equals the sum of the two principal surface-tensions" with both sides of this equation being negative on the throat of Visser 1989's wormholes. If so, then the transverse pressure should *not* be regarded as zero, but rather equal to the negated mass-density. The effective transverse pressure therefore would be positive and infinite, agreeing with me. A claim equivalent to all this is that the "flattened matter" effectively has "traceless" stress-energy tensor – as would be expected if it consisted of flattened "pure radiation" gas – except with an overall sign change to make the mass-density negative.

To make the mirror paradigm work we need to hypothesize the **new law of physics** that the "flattened mass-energy" on black hole horizons behaves right in terms of its *effect* on the metric, to make our version of the Einstein-Rosen metric actually happen and persist. In particular, we need to hypothesize the above negative-infinite behavior for "effective mass density" etc. – something occuring in no prior laws of physics.

Mirror paradigm FLRW cosmological models: If you also wanted, following Boyle & Turok, to regard the "creation of the universe" (aka "big bang") as yet another "black mirror," then you would proceed as follows. Use the usual (t,r,θ,φ) coordinatization of the FLRW cosmological model metric

$$ds^2 = -c^2dt^2 + A(t)^2\,[(1-kr^2)^{-1}dr^2 + r^2d\theta^2 + r^2\sin(\theta)^2d\varphi^2],$$

where $k\in\{-1,0,+1\}$ is fixed (k=0 for a "flat" universe), the twice-differentiable function A(t) is the time-varying "scale-factor of the universe," and we allow t and r to assume all real values while θ and φ as usual assume real values with $|\theta|\leq\pi/2$ and $0\leq\varphi<2\pi$. This model's metric is an analytically-smooth highly symmetric *approximation* to the universe. The actual universe presumably is everywhere h-fold differentiable for some suitable small number h, but nowhere-analytic and unsymmetric. Whenever A(t)=0 this model's metric is singular because $\det(g_{ab})=0$. Traditionally A(t)=0 when t=0 (the "bang") and then the mass-energy density and pressure both are infinite. *But* we again employ our usual (-t,-r) two-point-identification convention with charge-reversal and definition-of-future-reversal while demanding $A(t)^2=A(-t)^2$. Then the "universe" is the region with t≥0, while the "anti-universe" is the one with t≤0, coinciding on the spacelike surface A(t)=0 representing the "big bang" singularity. But I do not regard these as two distinct objects. Thanks to the two-point-identification convention I regard them as *one* unified object, see below grandiose universe picture.

Mirror paradigm Kruskal: Same coordinatization as Kruskal 1960, but *remove* the interiors of regions II and IV from Kruskal's spacetime diagram, and perform the 2-point-identification protocol

(-u,-v)-Identification convention:	(u,v,θ,φ) using particles such as e^-	is *identified* with	(-u,-v,θ,φ) changed to antiparticles such as e^+

and regard "increasing v·sgn(u)" as "future-directed." The black hole "horizon" consists of the line

u=v in the diagram; and there is mass-energy on it. The "white hole horizon" is the other line u+v=0. There is vacuum everywhere else.

Grandiose hypothetical universe picture trying to combine/improve the independent discoveries by Smith + Tzanavaris-Boyle-Turok: The universe/anti-universe (3+1)-dimensional curved spacetime is like a transparent curved 2-dimensional sheet (*one* object) which has **two sides**. Writing viewed from the underside appears to be mirror-ɒnitiɒw, but it is the same writing. I live on the "top" side of the (3+1)-dimensional universe-sheet and hence consider myself right-handed, made of matter, and part of the universe. The anti-me (who really is the *same* person, *not* a duplicate) living on the "underside" of the same (3+1)D universe-sheet always at the same location as me, thinks he is left-handed, made of antimatter, and part of the anti-universe (even though in his language he *calls* those concepts right-handed, matter, and the "universe"). In this analogy the top side of the sheet is the "universe" and the underside "anti-universe," but these should be regarded as two aspects of the *same* object. The reason you and anti-you are always at the same location is because of the exact CPT symmetry of the laws of physics governing your motions. (If you know about my "cloud QFT" ideas, Smith 2023-2026: the "raindrop" locations are the same on both sides of the sheet, i.e. raindrops impale the sheet like rivets. If you don't know, believe or care about cloud and raindrops, then disregard that.) We may regard all fundamental particles such as electrons as penetrating the sheet like rivets, but with one end of that rivet being an electron, with the other a positron. Note that the "thermodynamic arrow of time" therefore automatically is the same in both the universe and antiuniverse. The universe and anti-universe are *not* "analytic continuations" of each other (and if the universe metric is nowhere-analytic, which a priori is overwhelmingly probable, no analytic continuations can even exist; and such things as the Boyer-Lindquist 1967 "maximal analytic extension of the Kerr metric" are unphysical) but actually *the same object*. Because we are on opposite sides of the (zero-thickness but normally *impenetrable*) sheet, your particles cannot annihilate with anti-you's anti-particles. However, the event-horizon surfaces of black holes, and cosmological big-bang or big-crunch A(t)=0 surfaces if there are any, are unusual (since metrically-singular with what Einstein's field equations would consider "infinite mass-density") codimension≥1 subsets of the sheet where such penetration and annihilations both are permitted. Severe extended metric singularities like those **damage** the sheet, which is why such penetration and annihilation occurs there and why new additional laws of physics apply there. The simplest possible kind of such "damage" is: we postulate that the sheet simply **vanishes** there.

More features of "2-sided-mirror" paradigm (continuing prior list):

14. Black hole horizons are sheet *boundaries*, where the sheet *ends* – explaining why black holes have no interior. (Similarly for the cosmological big-bang and/or big-crunch surfaces, if any.)
15. It also explains why, when you fall into a black hole, i.e. reach an edge of the universe-sheet, you (if you wanted to keep going along the surface of the sheet) would try to keep going on its other side, i.e. within the the anti-universe. This is as though an ant were walking along one side of a finite sheet of paper. When it reaches the edge of the sheet, to keep walking forward it must do so along the other side of the sheet: ——↩
16. This explains why matter-antimatter annihilation happens at sheet-boundaries and why "penetration" to the other side of the sheet occurs only there.
17. *Why* do black-hole horizons cause sheet-ending? We've postulated that *infinite* matter densities cause metrical singularities, which cause sheet-damage, and hence sheet-

vanishing, and hence sheet-ending. But a black hole horizon is, at least initially, a metrically-regular place with only a finite matter-density nearby! I postulate the answer is **fedback instability**. Suppose along a newborn black hole horizon, a sheet *cut* occurs. Normally such a cut would immediately re-heal and you would not even know it had happened. However, in this case, any further matter falling into the horizon will (since the cut-sheet ends there, *and* since matter cannot ever emerge from a horizon (since it cannot travel at superluminal speeds, so nothing can "bounce off" the cut) necessarily will *pile up* at the cut, "flattening" and immediately causing infinite mass-energy density to appear there. This infinite density in turn will cause sheet-damage, preventing re-healing. Observe: Although this pathology happens at black hole event-horizons, it *cannot* happen at any "normal" location in spacetime. Event-horizons are special in that respect. A highly-related question:

Are there "local tests" for horizons? One idea is: we could demand (3+1)-dimensional spacetime be (in the horribly-bad jargon historically used by the GR community) "globally hyperbolic," i.e. *foliated* by 3-dimensional spacelike surfaces, each of which "extends to infinity" and is regardable as a "surface of constant time"; and **demand** that every two worldlines (of two points, say a red point and a blue point on each such surface, each always moving at sublight speeds) be causally bidirectionally connected, i.e. "at any time the 'red' point (or a point infinitesimally near it) could emit a lightspeed signal, which could travel to eventually hit the blue point's future-worldline." That is a hypothetical demanded connectedness-like property of foliated universe-sheets and the worldlines of pairs of points on their foliation-leaves. Since that property would be disobeyed as soon as one of the points enters a black hole interior through its event-horizon, the sheet *must* end at such horizons, so no "black hole interiors" can exist.

You might worry that two-worldline demand was "nonlocal," whereas you want all laws of physics to be **local**. However, there is a version of that condition which arguably seems local (the question depends on your definition of "local"; you might not agree), and presumably is equally satisfactory. The existence of this condition indicates that **contrary to the widely stated myth that the Schwarzschild and other black hole horizons are not metrically special, arguably-local tests exist which show they *are* special.** The basis for that myth had been: All the classic curvature invariants of, e.g, Carminati, McLenaghan, and Zakhary 1991-2007 stay finite at the horizon; and Kruskal 1960's coordinatization is analytically smooth everywhere, including at the horizon. But my local test (II below) is more sophisticated and subtle than that. It concerns, not merely (3+1)-spacetimes, but rather (3+1)-spacetimes *foliatable* into 3D spacelike leaves. I now **formulate my proposed demands/test** using "timelike*geodesics" instead of "worldlines" and employing the logic symbols "for all" (∀) and "exists" (∃).

Definition: By "timelike*geodesic" we mean a timelike geodesic (which is a continuous curve on the manifold with continuous derivative whose speed<c always), *except* that wherever this curve touches a spacetime sheet-boundary, we permit it to attain lightspeed if that boundary is a null surface, and no longer require it to be geodesic there, although we still demand continuity and speed≤c.

Two demands that (we hypothesize) physical (3+1)-spacetime metrics must satisfy:
I ("global hyperbolicity"):

∃ at least one foliation of the spacetime into 3D spacelike leaves L, such that:
∀ timelike*geodesics W

W∩L ≠ ∅, and indeed W∩L is a single point.

II ("bidirectional causality" – here is an arguably-localized version):

∃ foliation of the spacetime into 3D spacelike leaves demonstrating its global hyperbolicity
∀ leaves L in that foliation
∀ points A∈interior(L)
∃ ε>0
∀ points B∈interior(L) with |PseudoDistance(A,B)^2|≤ε
∀ timelike*geodesics $\tilde{B}$ with B∈$\tilde{B}$
∃ at least one lightspeed trajectory starting from A that reaches $\tilde{B}$.

Demand II is "local" in the sense that we allow ε>0 to be arbitrarily small. It is our first horizon test.

Example: Let us use II to test the Kruskal point A with (u,v)=(1,1). This A lies on the event horizon u=v. Consider points B arbitrarily near A in the spacetime diagram with u>v so B lies outside the horizon in "region I." Give B an initial velocity perpendicular-to-radial with magnitude equal to the (subluminal) "escape velocity." Then clearly B will follow a timelike geodesic $\tilde{B}$ which in both the far future and far past goes arbitrarily far away from the hole. Meanwhile, if A emits a lightspeed signal, that signal can travel inside the hole (into region II in the Kruskal diagram) or stay on the u=v line, but can never reach region I and hence never intersect the future portion of $\tilde{B}$. Therefore, the test fails. But for any alternative A lying slightly *outside* the horizon in region I, the test would succeed: I claim A's signal (if an omnidirected flash of light) would always eventually reach some future point on $\tilde{B}$, no matter what B's subluminal initial velocity was.

Non-Example: Now consider a plane in Minkowski flat spacetime traveling perpendicular to itself at lightspeed. You might worry that matter would "pile up" to infinite density on any such moving cut-plane, so our whole instability-story also would happen on any such cut, *not* just black-hole horizons. However, our test II would *pass* for any two spatially-separated Minkowski points A,B if that plane were not a cut, but would *fail* for A on and B outside a black hole horizon *regardless* of whether said horizon was a cut. That is the difference: demanding bidirectional-causality test II pass, *forces* black hole horizons to get cut.

Was that really a "local" test? That depends on your personal definition of the word "local." But what matters more than your personal preference, is the preference of *physics* itself – specifically what matters is whether this test "has teeth," i.e. whether its failure could plausibly trigger the postulated "sheet-cutting" instability. I suspect the answer is "yes": bidirectional causality plainly is a connectedness-like property, so it plausibly is the *reason* the universe-sheet stays connected, i.e. uncut.

Tanatarov & Zaslavskii 2014 gave a second, quite different, but also arguably purely-local, test for horizons. Generically almost everywhere, real-world "dirty" spacetime metrics (even if artificially resticted to be either "axisymmetric" or "static") have Weyl curvature tensor of "algebraically general" Petrov type I. But *on event-horizons*, T&Z proved they always have "algebraically special" Petrov type II, D, III, N, or O, and generically only the first two. But I warn you that T&Z were unable to prove this in full generality; they only were able to do it in for *axisymmetric* or *static* spacetimes,

where they enjoyed 1-degree-of-freedom worth of symmetry to make their analytical task simpler. I only *conjecture* its validity for *general* spacetimes. (The reason for my conjecture is I think 1 degree of symmetry is not enough to matter – if the conjecture were going to fail for general spacetimes, it should have failed for at least one of those two subclasses.)

"Petrov type" was a brilliant invention by A.Z.Petrov in 1954 employing his deep understanding of linear algebra. The Weyl curvature tensor $C^{ab}{}_{pq}$ may be regarded as performing a map from antisymmetric 2-indexed tensors X_{ab} to themselves via $C^{ab}{}_{pq}X_{ab}\rightarrow X_{pq}$. Since in 4-dimensional spacetime, antisymmetric tensors X_{ab} have six independent components, this map could be regarded as a multiplication by a 6×6 matrix at each point of spacetime. However, known symmetries of the Weyl tensor actually force any eigentensors to belong to a *four*-dimensional subset. Therefore the Weyl tensor (at any given point of spacetime) really has at most four linearly-independent eigentensors. These eigentensors in turn are associated with certain null vectors in the original (3+1)-spacetime, called the "principal null directions" at each point of spacetime. Then the possible "Petrov types" are

- Type I: four different principal null directions (generic case),
- Type II: one double and two single principal null directions,
- Type D: two double principal null directions,
- Type III: one triple and one single principal null direction,
- Type N: one quadruple principal null direction,
- Type O: the Weyl tensor vanishes ("conformally flat" spacetime).

This explanation has been inadequate and for a fuller explanation I refer you to Petrov's 1969 book.

Example: The Kerr-Newman black holes are type D, meaning that at every point of those spacetimes, there are exactly two (each double) principal null directions. But "dirty" Kerr-Newman black holes are type I.

A perhaps **more-intuitive characterization** of the concept that a spacetime has "algebraically special Petrov type" was the Goldberg-Sachs theorem: "A vacuum solution of the Einstein field equations admits a shear-free **null-congruence** if and only if its Weyl tensor is algebraically special." A "congruence" is a set of geodesics such that every point of the spacetime lies on exactly one of the geodesics. A "null congruence" is a congruence all of whose geodesics are null, i.e. lightspeed. So the intuition behind Tanatarov & Zaslavskii's theorem presumably is: *locally*, infinitesimally near event horizons, there is a way – indeed, a shear-free way – to partition a spacetime metric asymptotically resembling our metric arbitrarily well, into disjoint curves, each of which (to some high order asymptotic accuracy) is a null geodesic falling into the black hole. However, this is a special property of spacetime *infinitesimally near horizons*; for general "dirty" spacetimes no such partition into null geodesics exists.

Finally, for a possible third (potentially considerably simpler!) test, see open question C at the end.

Still more features of "2-sided-mirror" paradigm:

18. Why should black-hole horizons, as sheet boundaries, act as though they have negative-infinite mass-energy areal-density? Cutting the sheet along some surface, is associated with

an effective energy proportional to the area of the cut-surface. This resembles the fact that cutting paper with scissors requires energy proportional to the length of the cut.

19. You might worry about this effective energy being *negative* infinite – does this imply the vacuum should be energetically unstable and want to cut itself to pieces (contrary to observation)? Two possible answers to that worry:
    i. *Effective* energy is not actual energy. And the amount/nature of the effective energy always is chosen just right to cause the *actual* mass-energy (as measured externally via its gravity) to be conserved.
    ii. Maybe spacetime *cannot* "cut itself" since the only permitted way to do that is on codimension≥1 subsets on which infinite energy-density resides – or surfaces yielding violation of the "bidirectional causality" test II demand – those are the only permissible kind of "scissors."
20. Similarly, Bekenstein-Hawking *entropy* is, for black-hole horizons, proportional to the area of the horizon, just like there is entropy associated with paper-sheet boundaries (per unit cut-length) differing from the entropy of the same amount of uncut paper.
21. And of course, new laws of physics should be *expected* at sheet-boundaries. So their presence in the Grandiose Hypothesis should not be a surprise.

Smolin "Life of the cosmos" fantasized multiverse: rejected. Lee Smolin in his 1997 book *Life of the cosmos* hypothesized that each time a black hole forms, it spawns a newborn "daughter universe" in which the set of fundamental physical constants defining the laws of physics can *differ* slightly from ours. In this way we get an enormously-branching "multiverse" evolving via "Darwinian evolution" to try to maximize "fitness." And the definition of "fitness" of a universe is roughly "the number of black holes it creates." In unpublished work I tried to investigate this by asking (via both observational data and theoretical modeling) whether our universe's parameters indeed seem to be trying to maximize fitness. The (rather tentative) answer seemed to be "yes, it looks that way." In any case, the "mirror paradigm" picture suggested here is **incompatible** with Smolin's fantasy. There is only *one* CPT-transformed partner of our universe (the "anti-universe"), and only *one* "other side of the sheet," *not* an unboundedly large number of daughters each of which is *not* a CPT-transformed carbon copy of our universe, but rather quite different. Also, Smolin's picture involves mother→daughter *directionality*, while the mirror-paradigm picture has no asymmetric directional preference. And conceivably even with the mirror paradigm the universe might still seem to be "trying" to maximize fitness, e.g. merely to increase Bekenstein-Hawking entropy.

If Kruskal-like "maximal analytic extension" were what really happened inside black holes (which I contend is nonsense) then that would be a third picture, incompatible with both Smolin's "life of cosmos" and my "2-sided mirror" pictures.

8. Some good next questions

A. What is the effective stress-energy tensor (in the mirror-paradigm picture) at the horizon of a Kerr-Newman black hole? Originally I hoped that the "mirror" surfaces would feature some knowable formula for the effective mass areal-density on them. But it later became clear that "areal density" is not the right quantifier because it is infinite. So some other quantitative means of describing the nature and strength of the singularity on such surfaces, is wanted.
B. Seek better understanding of "local tests" for horizons. In particular:
C. Prove or disprove: there always exists a Lanczos tensor $Ł_{abc}$ for any physical (3+1)-

spacetime, with the property that $|Ł_{abc}Ł^{abc}|=\infty$ everywhere on every black hole event-horizon; and try to characterize the set of such Lanczos tensors.

Re question C, for the usual Schwarzschild coordinatization, the simplest known Lanczos tensor is $Ł_{trt}=-Ł_{rtt}=GMr^{-2}$, with all 62 other components zero (written in units with c=1). Because this tensor obeys $Ł_{abc}Ł^{abc}=2G^2M^2r^{-3}(r-H)^{-1}$ we see $|Ł_{abc}Ł^{abc}|$ is infinite on the horizon r=H – it works. (It also is infinite when r=0, but finite everywhere else.) And this is not the only Lanczos tensor that "works" for the Schwarzschild metric; and I also have found analogous suitable Lanczos tensors for the Λ-Reissner-Nordström and Λ-Kerr generalization(s) of Schwarzschild.

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Appendix: Analytic dirt in one universe abolishes the other (barrier theorem for linearized Kruskal-Szekeres extension)

The main text of the paper gave reasons to suspect the Kruskal-Szekeres "maximal analytic extension" of the Schwarzschild black hole was unphysical. This appendix proves a ***theorem*** backing that up. This theorem does not rely on any assumed non-analyticity of our universe's metric. It remains fatal even if we assume our universe's metric is an *arbitrarily tiny perturbation* of Schwarzschild which is analytic everywhere outside and on the horizon. ("Arbitrarily tiny" in either the sup or L^2 norms; r-weighted energy norms work too. Warning: "tiny everywhere" is not proven fully-rigorously, but instead will be shown by solving a PDE numerically and observing the output solution remains bounded everywhere; and this solution may be arbitarily rescaled.)

Consequently, I contend the Maldacena-Susskind "ER=EPR" vision now is a smoking crater.

In the Kruskal-Szekeres extension of Schwarzschild, the "second universe," by which we here mean region III of the diagram below right, is not determined by "our universe" (region I in diagram)

via the Einstein field equations, since it lies outside its domain of dependence, aka causal future. The *only* way it is determined is via *analytic continuation* outside that domain.

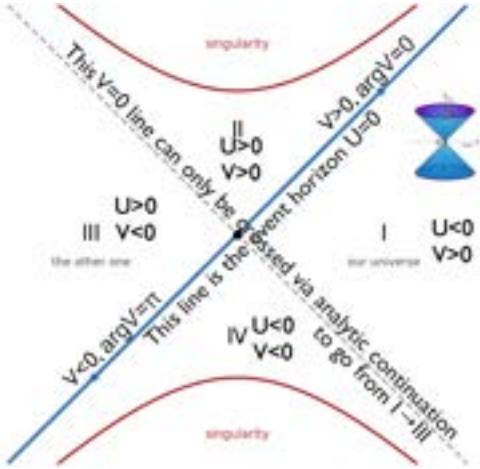
We'll exhibit an ∞-parameter family of infinitesimal perturbations to our universe's metric that are

- rescalable to **any desired sup-norm** (however small) on the horizon;
- **real-analytic at every point** of region I including the horizon;
- **eternal** and also decaying at both temporal ends;
- **obey** the Regge-Wheeler-Zerilli master equation throughout region I, i.e. "physical";
- We have strong numerical **evidence**, but failed to *prove*, that these perturbations have bounded L^2, and L^∞, and r-weighted energy norms on region I.

and for which **no analytic continuation into region III exists**. Therefore (within linearized perturbation theory) the presence of such "dirt" anywhere in universe I *abolishes* universe III.

Setup. The Schwarzschild metric is

$$ds^2 = -(1-2M/r)dt^2 + (1-2M/r)^{-1}dr^2 + r^2(d\theta^2+\sin^2\theta\, d\varphi^2).$$



Introduce the "tortoise coordinate" r*, then the "null coordinates" u,v:

$$r^* = r + 2M\ln((2M)^{-1}r-1), \quad u=t-r^*, \quad v=t+r^*,$$

so-called because radially infalling and outgoing light rays travel along v=constant and u=constant lines in the u,v plane. Let $\kappa=(4M)^{-1}$ denote the hole's "surface gravity." Then define the Kruskal-Szekeres coordinates,

$$\mathbf{U} = -e^{-\kappa u}, \quad \mathbf{V} = e^{\kappa v}.$$

In these coordinates the metric and the radial relation read

$$ds^2 = -32M^3r^{-1}e^{-r/2M}\, d\mathbf{U}\, d\mathbf{V} + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2), \quad \mathbf{UV} = [1-(2M)^{-1}r]e^{r/2M}$$

manifestly regular at the horizon r=2M, which is $\mathbf{UV}=0$. Then Universe I is $\mathbf{U}<0<\mathbf{V}$ and in terms of u and v is the full uv plane. The four regions are defined in the diagram. The border that any continuation between universes must cross is $\{\mathbf{V}=0\}$; the event horizon is $\{\mathbf{U}=0\}$; both are connected real-analytic null hypersurfaces.

Let **D(v)** denote the **characteristic datum**, meaning the perturbation field φ restricted to the event horizon $\{\mathbf{U}=0\}$ as a function of v. Continuation from I to III can be regarded as continuing D from $\arg\mathbf{V}=0$ to $\arg\mathbf{V}=\pi$.

The Regge-Wheeler-Zerilli equations (Regge & Wheeler 1957; Zerilli 1970) are the Einstein (or other) field equations governing small perturbations linearized on top of plain Schwarzschild, and

with $Y_{\ell m}$ angle dependence. They involve certain known "potential functions" which I'll call RWP for "Regge-Wheeler-Potential." For spin s and multipole ℓ, and for even-parity gravitational perturbations, these potential functions are

$$RWP_s(r) = (1-2M/r)\,[(\ell+1)\ell r^{-2} + (1-s^2)2Mr^{-3}], \quad s = 0,1,2 \text{ for scalar, electromagnetic, odd-parity gravitational}$$

$$RWP_Z(r) = 2\,(1-2M/r)\,[\lambda^2(\lambda+1)r^3 + 3\lambda^2 Mr^2 + 9\lambda M^2 r + 9M^3]\,[r^3(\lambda r + 3M)^2]^{-1}, \quad \lambda = \tfrac{1}{2}(\ell-1)(\ell+2)$$

Just write RWP for whichever is in play. In double-null coordinates the **Regge-Wheeler-Zerilli master equation** is $\varphi_{,uv}+(RWP/4)\varphi=0$, and since $\partial_u = -\kappa \mathbf{U}\partial_{\mathbf{U}}$ and $\partial_v = \kappa \mathbf{V}\partial_{\mathbf{V}}$,

$$\varphi_{,\mathbf{UV}} = W(\mathbf{UV})\,\varphi, \quad W = RWP(4\kappa^2\mathbf{UV})^{-1} \quad \text{(commas in subscripts denote partial derivatives)}$$

Note that these **W are analytic at the horizon r=2M.** Hence the Regge-Wheeler-Zerilli equation and Kruskal/Szekeres/Schwarzschild background by themselves do not obstruct analytic continuation. The obstruction we shall exhibit lives entirely in the perturbation.

What reaching universe III requires. On $\{\mathbf{U}=0\}$ the Kruskal coordinate is $\mathbf{V}=e^{\kappa v}$, so $\arg\mathbf{V}=\kappa\,\mathrm{im}(v)$. Universe I is the ray $\arg\mathbf{V}=0$, i.e. all real $\mathbf{V}>0$, corresponding to all pure-real v; but universe III is the ray $\arg\mathbf{V}=\pi$, i.e. all real $\mathbf{V}<0$, corresponding to the complex v on the line $\mathrm{im}(v)=\pi/\kappa$. Therefore, reaching universe III means continuing the datum from the real v axis line up to the parallel line $\mathrm{im}(v)=\pi/\kappa=4\pi M$ in the complex v-plane. So a *barrier* in the complex v-plane at any height b with $0<b<4\pi M$ will prevent every continuation-path from universe I to III. We need functions well behaved on the real axis but stopping dead at that prescribed height. We can get them by starting with lacunary series, which are analytic within the unit disc but inextendible outside it, then conformally mapping disc→strip.

Lemma. For every b>0 there exists a function G(v), bounded and analytic on the strip $\{|\mathrm{im}\ v|<b\}$, whose two edges $\mathrm{im}(v)=\pm b$ are **natural boundaries**: G admits no analytic continuation past either edge at any point of it. G may be scaled to any sup norm, and multiplied by any non-vanishing analytic factor to make it decay along the real axis.

Proof: Step 1: disc. Let h(w) denote Hadamard's lacunary series $h(w)=\sum_{n\geq 1} n!^{-1} w^{(2^n)}$ which converges for all complex w with $|w|\leq 1$. The exponents 2^n obey the Hadamard gap condition with ratio 2>1, and $\limsup|a_n|^{1/\lambda_n}=\limsup|n!|^{(-2^{-n})}=1$ [numerically $(n!)^{-1/2^n}=0.9023,\ 0.9951,\ 0.99996$ at n=6, 12, 20], so the radius of convergence is 1 and by the Ostrowski-Hadamard gap theorem (Titchmarsh 1939) the circle |w|=1 is an impassible "natural boundary" for h. The factorial coefficients make h as regular as could be wished on the closed unit disc: Its Kth derivative is bounded for each K≥0. Nevertheless h(w) cannot be continued across any arc of the disc-boundary |w|=1.

Step 2: strip. The conformal map $w(v)=\tanh([\pi/4][v/b])$ bijects the open strip $\{|\mathrm{im}\ v|<b\}$ conformally

onto the open disc, carrying the two lines im(v)=±b onto |w|=1, with v=±∞ mapped to w=±1. Hence G = h∘w is bounded and analytic exactly on {|im v|<b} with both edges natural boundaries. Note w(v) lies in the *open* disc for every finite real v, so no real point is singular. **Q.E.D.**

Inextensibility Theorem. Fix any b with 0<b<4πM and any B>b, then set

$$w(v) = \tanh((4b)^{-1}\pi v), \quad h(w) = \sum_{n\geq 1} n!^{-1} w^{(2^n)}, \quad D(v) = h(w(v)) / (v^2+B^2)$$

Then D(v) is bounded on the whole real v axis, real-analytic at every point of universe I and of {**U**=0} and decays at both temporal ends. Its domain of analyticity in the complex v-plane is exactly the strip {|im v|<b} with both edges natural boundaries. Hence **no analytic continuation of the resulting solution into universe III exists along any path**.

Long discussion about physicality of D(v). It is not enough merely to say "data D(v) exist" that prevent extension. To contend that dirt in our universe (region I) abolishes the other universe (region III), we want to contend that dirt is *physical*, specifically physical-enough so that there exists a perturbation φ that

1. is analytic
2. satisfies the Regge-Wheeler-Zerilli master equation
3. is suitably "small"

throughout our universe, and agrees on {**U**=0} with D(v).

Proof of global existence and analyticity of the perturbation φ. Fix any $\mathbf{V}_0$>0 and pose the double-null characteristic problem:

$$\varphi = D \text{ on } \{\mathbf{U}=0,\ \mathbf{V}\geq\mathbf{V}_0\}, \quad \varphi = 0 \text{ on } \{\mathbf{V}=\mathbf{V}_0,\ \mathbf{U}\leq 0\}$$

Both surfaces are analytic and both data are analytic, and W is analytic (from its known formula), so the Cauchy-Kovalevskaya theorem provides a solution φ of the Regge-Wheeler-Zerilli master equation analytic throughout the orthant $\{\mathbf{U}\leq 0,\ \mathbf{V}\geq\mathbf{V}_0\}$. That orthant is a subset of the closure of **universe I**, with the prescribed piece of {**U**=0} as part of its boundary. So φ solves the Regge-Wheeler-Zerilli equation, and is analytic, throughout an orthant-subregion of our own universe and has D(v) as its boundary value. And since $\mathbf{V}_0$>0 is arbitrary and the barrier obstructs upward continuation from *every* real v into the complex v-plane at height im(v)=b, nothing depends on its choice. **Q.E.D.**

Proof perturbation bounded within any V-interval (infinite strip $\mathbf{V}_0\leq\mathbf{V}\leq\mathbf{V}_1$) within that orthant. Write the characteristic problem as the integral equation $\varphi(\mathbf{U},\mathbf{V})=D(\mathbf{V})-D(\mathbf{V}_0)+\iint W\varphi$. Then Picard iteration and Grönwall's inequality give $|\varphi|\leq 2\exp(\iint|W|)\sup_{\mathbf{V}}|D(v)|$. Because W depends on **U** and **V** only through their product, that double integral collapses: substituting $p=-\mathbf{UV}$ and then r,

$$\int_0^\infty\int_{\mathbf{V}_0}^{\mathbf{V}} |W| = K \ln(\mathbf{V}\mathbf{V}_0^{-1}), \qquad K = \int_0^\infty |W(-p)|\, dp = \int_{2M}^\infty 2A(r)\, dr = \mathbf{[2(\ell+1)\ell + 1 - s^2](2M)^{-1}}$$

where RWP=(1−2M/r)A(r) and K is **finite**: it is 9/(2M) for odd-parity gravitational ℓ=2, and 13, 12, 21, 37 over 2M for (ℓ,s) = (2,0), (2,1), (3,2), (4,2). Hence

$$|\varphi(\mathbf{U},\mathbf{V})| \le 2\,(\mathbf{V}/\mathbf{V}_0)^K \sup_v |D(v)|.$$

This proves φ **uniformly bounded on every slab** $\mathbf{V}_0 \le \mathbf{V} \le \mathbf{V}_1$, including all the way out to $\mathbf{U}\to-\infty$, and so on every rectangle contained in the orthant. By linearity, replacing D by εD replaces φ by εφ, so on any such slab the field scales down with the datum while the barrier, being a property of the domain of analyticity rather than of the size, is untouched.

Unfortunately, the bound grows like $\mathbf{V}^K$ as $\mathbf{V}\to\infty$, and rescaling by ε shrinks it uniformly without making it bounded. We therefore have *not* proved φ sup-norm bounded over the whole orthant. An objector could complain that a perturbation growing without bound towards large $\mathbf{V}$ would be hard to defend as "physically acceptable dirt" inside our universe.

But crude Picard-Gronwall estimation is enormously pessimistic, because Gronwall discards the sign of W, which is negative throughout region-I (RWP>0 there while $\mathbf{UV}<0$), so the equation is oscillatory rather than growing. Numerical characteristic evolution says the truth is far better:

Empirical numerical fact: $\sup_{\text{orthant}}|\varphi| / \sup_v|D(v)| \le 1$.

Conjecture: $\sup_{\text{orthant}}|\varphi| / \sup_v|D(v)| \le 2$.

Empirical numerical fact: φ has bounded L^2 norm within the whole orthant. (Also: it is possible to prove this *provided* the flux radiated toward ∞ is finite.)

Empirical numerical fact: φ has bounded r-weighted energy norm within the whole orthant.

Corollary (genericity) of those empirical numerical facts (if true). The blocking data are **dense in sup and L^2 norms** among bounded analytic data, i.e. every continuable datum has blocking data arbitrarily near it.

Also, nothing in the lemma relied on the particular exponents 2^n, coefficients $n!^{-1}$, or envelope P(v). Everything works for any Hadamard-gap series $h(w)=\sum_n a_n w^{(\lambda_n)}$ with integer exponents obeying $\lambda_{n+1}/\lambda_n \ge q>1$, of radius of convergence 1, with $\sum_n|a_n|<\infty$; and any envelope-function P(v) analytic and non-vanishing on the closed strip {|im v|≤b} and decaying when re(v)→±∞. Then D(v)=h(w(v)) P(v) has every property asserted in the theorem, and no continuation into universe III exists.

The infinite-dimensional space of free choices:

- **the height b**, anywhere in the interval (0, 4πM) - a continuum;
- **the exponents λ_n**, any integer sequence whose gap ratio is bounded below by some q>1: for

instance 2^n, 3^n, n!...

- **the coefficients a_n**, any absolutely summable sequence with $\limsup|a_n|^{1/\lambda_n}=1$;
- **the envelope-function P(v)**, any analytic non-vanishing decaying factor, for example $(v^2+B^2)^{-1}$ with B>b, or $\exp(-\sigma v^2)$ with σ>0, or $\mathrm{sech}(\alpha[1+v^2]^\beta$ with α,β>0. The latter two envelopes make the data Schwartz class.

Remarks. We suggest taking the perturbation to be **even parity (Zerilli), ℓ=2, m=0** aka "axially symmetric and polarized," i.e. exactly the symmetry class Klainerman & Szeftel 2018 proved Schwarzschild metric globally nonlinearly stable in. It is tempting to invoke such nonlinear stability results to argue the dirt is physically acceptable, but that reasoning is circular: the nonlinear theorems require the data to be small in some norm, and if our perturbation grew without bound it (however rescaled) might fail to be small in that norm: one cannot use stability to prove the very boundedness that stability's hypothesis presupposes. Breaking that loop would need "bootstrapping." Those problems do not arise in the linearized theory.
