Pi-Greek

Rolando Zucchini
Independent math researcher Italy

ABSTRACT

This paper reports the most significant studies on the knowledge of Pi-Greek. The number that has stimulated mathematicians throughout the centuries, from Archimedes to Descartes to Euler. It was the numerous attempts to prove the squaring of the circle that led to increasingly precise approximations of **Pi**. The proofs continued even after the Royal Academy of Sciences in Paris, in 1775, abandoned examining the many alleged "solutions" to this problem. The irrationality of π was demonstrated by Ferdinand von Lindemann in 1882. This article is mainly a historical research on the number Pi-Greek.

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Remark
This article is mainly a historical research on the number Pi-Greek, supported by some new mathematical considerations.

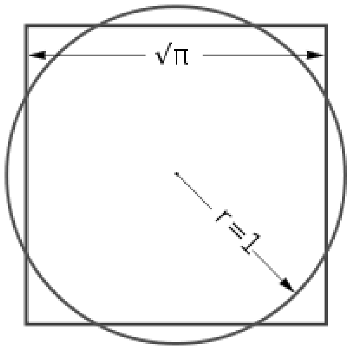
1. Preface

Given a circle, the problem of its quadrature it consists in drawing a square with the same area, requiring: 1) using an ungraduated straightedge and a compass, 2) using a finite number of intermediate steps. Since the area of a square is l^2, and that of a circle is πr^2, from the equality :

$$l^2 = \pi r^2$$

Letting $r = 1$, we obtain: $l = \sqrt{\pi}$.

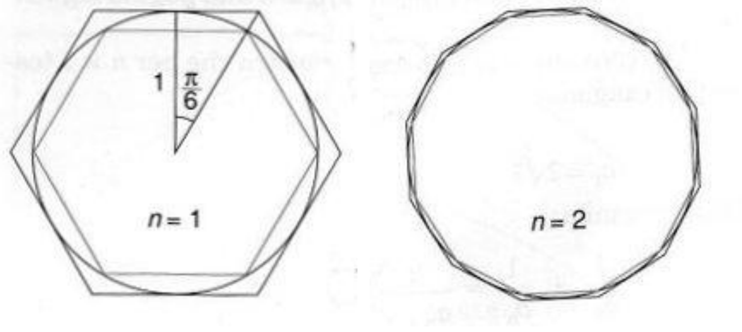
In other words, it is a matter of constructing $\sqrt{\pi}$ using the ruler and compass.



The problem of *squaring the circle* is perfectly equivalent to that of *rectifying the circumference*, that is, drawing a segment whose length is equal to the circumference of the circle being examined. The problem of *squaring the circle* persisted from antiquity until the 19th century. In 1775, the Paris Academy of Sciences was forced to no longer accept the alleged *solutions to squaring the circle* because so many were arriving that their revision required the full-time work of many of the Academy's geometers. Attempts continued even after F. von Lindemann (Carl Louis Ferdinand von Lindemann; Hannover 1852 – Munich 1939) proved the transcendence of π in 1882 (a number is said to be transcendental if it cannot be a solution to any algebraic equation). It should be emphasized that proving π to be a transcendental number implies the impossibility of solving the problem of squaring the circle. But, as happened with Euclid's fifth postulate, for more than two thousand years, mathematicians tried in vain to solve an unsolvable problem. Their attempts, however, led to increasingly precise approximations of π.

2. Archimedes

Archimedes (of Syracuse; 287 (?) – 212 BC) significantly advanced the knowledge of π. In his book *Sulla misura del cerchio*, after establishing that the ratio of the surface area of a circle to the square of its radius is equal to the ratio of its circumference to its diameter, he considered polygons with 6, 12, 24, 48, and 96 sides, and carefully calculated approximations of π that led him to evaluate it : $3 + 10/71 < \pi < 3 + 1/7$ or $223/71 < \pi < 22/7$, obtaining $3.1408 < \pi < 3.1429$. He considered a circle of radius 1 that he circumscribes and inscribes with polygons of $3 \cdot 2^n$ sides. Indicating with a_n the semi-perimeter of the circumscribed polygons and with b_n that of the inscribed polygons, for n = 1 we have $a_1 = 2\sqrt{3}$ and $b_n = 3$ with $3 < \pi < 3.46$; for n = 2 we have the dodecagons with $a_2 = 12/(2 + \sqrt{3})$ and $b_2 = 6/\sqrt{(2+\sqrt{3})}$ and $3.10 < \pi < 3.21$.



For every n → $1/a_n + 1/b_n = 2/a_{n+1}$ and $b_n \cdot a_{n+1} = (b_{n+1})^2$.
Using these recurrence formulas, it is possible to approximate π with the desired precision. Using this method, Archimedes approximated π up to n = 5, that is, 96 sides of the inscribed and circumscribed polygons, calculating the square roots by successive approximations, downward and upward, which led him to the approximation: $3.1408 < \pi < 3.1429$.

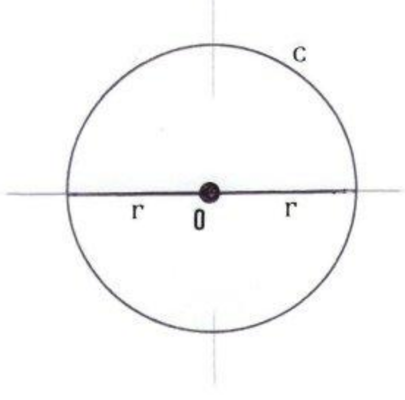
If we indicate with $l(p)$ the length of the side of the regular polygon of p sides inscribed in the circle of radius 1, with purely geometric considerations, we arrive at the conclusion that the side of the polygon with twice the number of sides is $l(2p) = \sqrt{(2-\sqrt{(4-l^2(p))})}$. From this formula we obtain:

$$l(6) = 1 \rightarrow l(12) = \sqrt{2-\sqrt{4-l^2}(6)} = \sqrt{2-\sqrt{3}}$$

From the previous relation, another expression for the sequence b_n of the semi-perimeters of the inscribed polygons can be deduced. From $b_1 = 3$, we obtain $b_n = 3 \cdot 2^{n-1}/l(3 \cdot 2^n)$, for n = 2, in fact, we obtain $b_2 = \frac{6}{\sqrt{2-\sqrt{3}}}$.

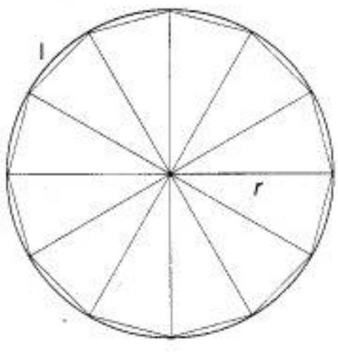
2.1. *π* incommensurable

Since ancient times, **pi** has been the ratio between the circumference and the diameter of the circle, which are two incommensurable quantities: $\pi = c/2r$



This ratio is an *invariant*, that is, it does not vary with the radius of the circle in a plane in which the Pythagorean theorem and the theorem of Thales (of Miletus ; 624 – 548 BC (?)) are valid. That is to say, π is invariant in the *Euclidean plane*. But even in the *non-Euclidean hyperbolic plane* of N. I. Lobaçevskij (Nikolaj Ivanovic Lobaçevskij ; 1792 - 1856), π is used to calculate the circumference of a circle: $c = \pi k(e^{r/k} - 1/e^{r/k})$, where k is the constant of the *hyperbolic plane* and *e* is the irrational exponential. When r, the radius of the circle, varies, the formula remains unchanged and π is invariant in the *hyperbolic plane*.

Pi-Greek can be defined as the ratio between the surface area of a circle and the square of its radius: $\pi = S/r^2$. This definition is equivalent to the previous one. In fact, by decomposing a regular n-sided polygon inscribed in a circle into a sequence of n equal isosceles triangles (similar to circular sectors as n increases) with their vertex at the center of the circle we have: $S = r \cdot n \cdot l/2 = r/2 \cdot n \cdot l$. When $n \to +\infty$, $n \cdot l \to c$ and therefore $S = c \cdot r/2$, but $c = 2\pi r$ and therefore $S = \pi r^2$ (see figure below).



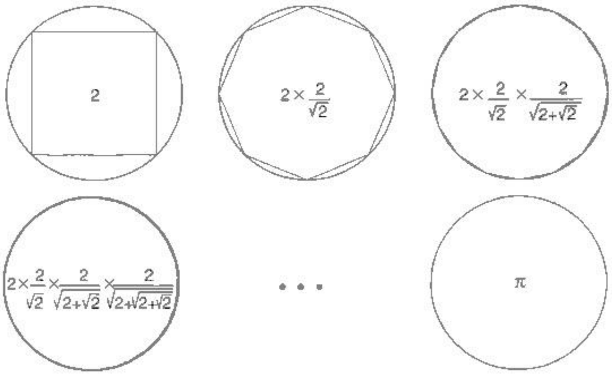
2.2. π approximations after Archimedes

After Archimedes, π became the *pure* mathematical number and posed a challenge to mathematicians for two millennia. However, nothing new came after him, except increasingly precise approximations.

Claudius Ptolemy (100-170) used $\pi = 3 + 8/60 + 30/60^2 = 3 + 17/120 = 377/120 = 3.141\overline{6}$ in his astronomical calculations.

In 1596, Ludolph van Ceulen (or Keulen; Hildesheim 1539 – Leiden 1610(?)), a professor at the University of Leiden, Holland, using Archimedes' method, calculated 20 decimal places of π with a polygon of 60×2^{33} sides. In 1609, he arrived at 35 decimal places of π.

François Viète (Latinized Franciscus Vieta; Fontenay-le-Comte 1540 – Paris 1603), using elementary geometric considerations on a 2^n-sided polygon, wrote the first infinite formula of π (see figure).



The number 2 corresponds to the area of a square inscribed in a circle of radius 1. The number $2x2/\sqrt{2} = 2x1/\cos(\pi/4)$ corresponds to the area of the octagon. Multiply by $1/\cos(\pi/8)$ to convert to a 32-sided polygon, and so on. These steps are permitted by the formula :

$$\cos\alpha = \frac{\sqrt{2cos\,2\alpha+2}}{2}$$

$$\cos\alpha = \frac{\sqrt{2cos\,2\alpha+2}}{2} = \frac{\sqrt{2(cos\,2\alpha+1)}}{2} = \frac{\sqrt{2}}{2}\cdot\sqrt{cos2\alpha+1} = \frac{\sqrt{2}}{2}\cdot\sqrt{2cos^2\alpha} = \frac{\sqrt{2}}{2}\cdot\sqrt{2}\cdot\cos\alpha = \cos\alpha$$

In 1621, Willebrord Snellius (Leiden 1580 – 1626) used trigonometric functions to approximate π. He approximated a circular arc by excess and defect with straight line segments, using the formula:

$$\frac{3sin\alpha}{2+cos\alpha} < \alpha < \frac{2sin\alpha+tg\alpha}{3}$$

which offers a method for calculating π. In fact, for certain angle values, for example of the form $\pi/(3x2^{n-1})$ in regular $3x2^n$-sided polygons, sinα, cosα, tgα, are expressed by irrational values. Considering a circle divided into six arcs (n = 2), the Snellius formula gives the same precision as *Archimedes' exhaustion method* for a 96-sided polygon. The above formula can be demonstrated if we take into account that for very small values the angle coincides with the corresponding arc. With reference to figure a :

$$\text{arc}(BF) = 3\alpha r < BG_1$$
$$BG_1/E_1B = tg\alpha \rightarrow BG_1 = E_1B\,tg\alpha$$
$$E_1B = r(2\cos\alpha + 1)$$
$$BG_1 = r(2\cos\alpha + 1)tg\alpha = r(2\sin\alpha + tg\alpha)$$

From which : $3\alpha r < r(2\sin\alpha + tg\alpha) \rightarrow \alpha < (2\sin\alpha + tg\alpha)/3$

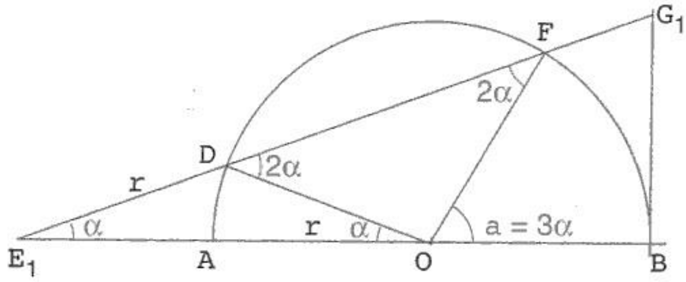


Figure a

With reference to the figure b :

$$\text{arc}(BF) = ar > BG_2$$
$$BG_2/E_2B = HF/E_2H = r\sin\alpha/(2r + r\cos\alpha)$$
$$BG_2 = 3r(\sin\alpha/(2 + \cos\alpha))$$

From which : $ar > 3r(\sin\alpha/(2 + \cos\alpha)) \;\rightarrow\; a > 3\sin\alpha/(2 + \cos\alpha)$

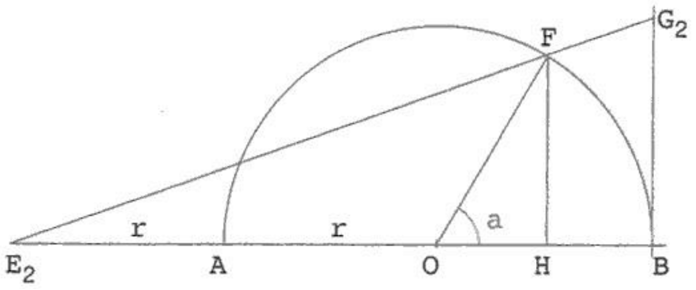


Figure b

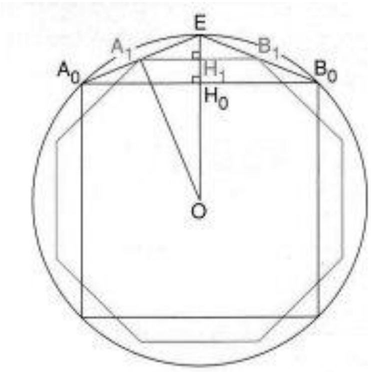
Place α = a it turns out : $\frac{3sin\alpha}{2+cos\alpha} < a < \frac{2sin\alpha + tg\alpha}{3}$

3. **Renè Descartes (Cartesio)**

René Descartes (La Haye en Touraine; 1596 – Stockholm 1650) studied the squaring of the circle using the method of *isoperimeters*. This consists of fixing the length of a perimeter and constructing a sequence of polygons having the same perimeter but with an increasingly larger number of sides. Consider a sequence of regular polygons P_0, P_1, P_2, …, P_n all having the same perimeter p. Suppose P_n has 2^{n+2} sides, let A_nB_n denote one of its sides, O the center of the circle circumscribed to the polygon and H_n the midpoint of A_nB_n. Set $OH_n = r_n$, let E be the midpoint of the arc A_nB_n, A_{n+1} the midpoint of the chord A_nE and B_{n+1} the midpoint of the chord B_nE. $A_{n+1}B_{n+1}$ is the side of the polygon P_{n+1} and is equal to $A_nB_n/2$. Since the triangles $OH_{n+1}A_{n+1}$ and $A_{n+1}H_{n+1}E$ are similar, the following proof follows:

$$(A_{n+1}H_{n+1})^2 = EH_{n+1} \times H_{n+1}O$$
$$EH_{n+1} = H_{n+1}H_n = r_{n+1} - r_n$$
$$A_{n+1}H_{n+1} = \frac{A_nH_n}{2} = \frac{A_0H_0}{2^{n+1}} = \frac{r_0}{2^{n+1}}$$
$$(r_{n+1})^2 - r \cdot r_{n+1} - \frac{r_0^2}{4^{n+1}} = 0$$
$$r_{n+1} = \frac{r_n + \sqrt{r_n^2 + r_0^2/4^n}}{2}$$

The figure shows the first step of this process : from the square with side A_0B_0 to the octagon with side A_1 B_1.



In each of these steps, the perimeter of the polygons remains the same, while the radius of the circles circumscribed to them decreases. In the limit, the *isoperimetric method* yields a radius of the circle circumscribed to the polygons that is in ratio 2π with the perimeter p fixed at the start, thus allowing π to be plotted with the desired precision. However, achieving the perfect result requires infinite steps.

4. *π* irrational

The irrationality of **pi** was first demonstrated in 1768 by Lambert (Johann Heinrich Lambert; Mulhouse 1728 – Berlin 1777). Lambert's proof involves three steps:

Step 1: Any number that can be written as a continued fraction where the sequence of a_i and b_i is irrational under certain conditions :

$$\cfrac{a_0}{b_0 + \cfrac{a_1}{b_1 + \cfrac{a_2}{b_2 + \cdots \; \cdots b_{n-1} + \cfrac{a_n}{b_n + \cdots}}}}$$

Step 2 : Being

$$\operatorname{tg} x = \cfrac{x}{1 - \cfrac{x^2}{3 - \cfrac{x^2}{5 - \cfrac{x^2}{\cdots}}}}$$

Step 3 : If π is rational, for x = π/4, from a certain point onwards $x^2 = (\pi/4)^2$ is irrational, but tg(π/4) = 1, so this is absurd, therefore π is irrational. → Q.E.D.

5. *π* transcendent

It was the advent of mathematical analysis that prompted the most significant developments in the understanding of π.

5.1. John Wallis

The work of John Wallis (Ashfor 1616 – Cambridge 1703) on infinite products anticipated the infinitesimal calculus of Leibniz and Newton. He found the following infinite product formula for π, published in 1665 in his work *Arithmetica Infinitorum* :

$$\pi = 2 \times \frac{2\times 2}{1\times 3} \times \frac{4\times 4}{3\times 5} \times \frac{6\times 6}{5\times 7} \times \frac{8\times 8}{7\times 9} \times \cdots$$

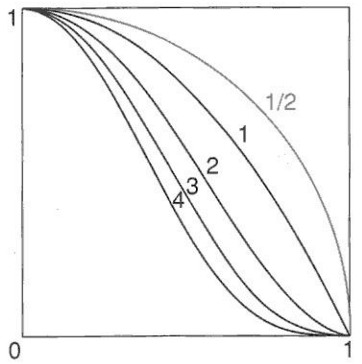
which could also be written :

$$\pi = 2\prod_{p=1}^{\infty} \frac{4p^2}{(2p-1)(2p+1)} = 2\prod_{p=1}^{\infty}\left(1-\frac{1}{4p^2}\right)^{-1} = \lim_{n\to\infty} \frac{2^{4n} \times n!^4}{n\cdot(2n)!^2}$$

Wallis's method consists of studying the area of a quarter of a circle with center at O and radius 1, and therefore of equation $x^2 + y^2 = 1$, from which $y^2 = 1 - x^2$, and therefore: $y = \sqrt{(1 - x^2)}$.
He considers the areas bounded by the curves of equation:

$$y = (1 - x^2)^h$$

for different values of h. For h = 1/2 we have a quarter circle.



He broke these curves into small rectangles, and studied the formula:

$$S_p = 1 + 2^p + 3^p + \cdots + n^p$$

He was able to recognize certain properties of these curves that led him to his famous formula for calculating π. Wallis's formula is truly interesting because it is the first infinite formula for π in which radicals do not appear. However, as can be seen from the diagram below, the first three exact digits of π are obtained after a considerable number of multiplications.

$$2 \times \frac{2 \times 2}{1 \times 3} \times \frac{4 \times 4}{3 \times 5} = 2{,}8444444444$$

$$2 \times \frac{2 \times 2}{1 \times 3} \times \frac{4 \times 4}{3 \times 5} \times \frac{6 \times 6}{5 \times 7} = 2{,}9257142857$$

$$2 \times \frac{2 \times 2}{1 \times 3} \times \frac{4 \times 4}{3 \times 5} \times \frac{6 \times 6}{5 \times 7} \times \frac{8 \times 8}{7 \times 9} = 2{,}9721541950$$

$$2 \times \frac{2 \times 2}{1 \times 3} \times \frac{4 \times 4}{3 \times 5} \times \frac{6 \times 6}{5 \times 7} \times \frac{8 \times 8}{7 \times 9} \times \frac{10 \times 10}{9 \times 11} = 3{,}0021759545$$

$$2 \times \frac{2 \times 2}{1 \times 3} \times \frac{4 \times 4}{3 \times 5} \times \frac{6 \times 6}{5 \times 7} \times \frac{8 \times 8}{7 \times 9} \dots \frac{50 \times 50}{49 \times 51} = 3{,}1260789002$$

$$2 \times \frac{2 \times 2}{1 \times 3} \times \frac{4 \times 4}{3 \times 5} \times \frac{6 \times 6}{5 \times 7} \times \frac{8 \times 8}{7 \times 9} \dots \frac{500 \times 500}{499 \times 501} = 3{,}14002381$$

$$2 \times \frac{2 \times 2}{1 \times 3} \times \frac{4 \times 4}{3 \times 5} \times \frac{6 \times 6}{5 \times 7} \times \frac{8 \times 8}{7 \times 9} \dots \frac{5000 \times 5000}{4999 \times 5001} = 3{,}1414355$$

5.2. William Brouncker

William Brouncker (Castlelyons, Ireland, 1620 – Oxford 1684) was, with John Wallis, a founder of the *Royal Society*. He transformed Wallis's formula for the infinite product into:

$$\frac{4}{\pi} = 1 + \cfrac{1}{2 + \cfrac{3^2}{2 + \cfrac{5^2}{2 + \cfrac{\cdots}{\cdots + \cfrac{(2n+1)^2}{2 + \cdots}}}}}$$

He dealt with continued fractions of the type :

$$a_0 + \frac{b_0}{a_1}\,;\quad a_0 + \cfrac{b_0}{a_1 + \cfrac{b_1}{a_2}}\,;\quad \dots\,;\quad a_0 + \cfrac{b_0}{a_1 + \cfrac{b_1}{a_2 + \cfrac{\cdots}{\cdots + b_n}}}$$

These can also be written with the notation :

$$a_0 + b_0/(a_1 + b_1/(a_2 + b_2/(a_3 + \dots b_n/(a_n + \dots) + \dots)$$

and demonstrated a remarkable property of these fractions : *a number is rational if and only if its continued fraction is regular and finite*. Since π is irrational, it is written as an *infinite continued fraction* with the notation :

$$\pi = 3 + 1/(7 + 1/(15 + 1/(1 + 1/(292 + 1/(1 + \dots) \dots)$$

Considering a finite number of elements of this expansion, we obtain the so-called reduced fraction which give rather precise approximations of π:

3/1   22/7   333/106   355/113 … 103993/33102 …

5.3. James Gregory

James Gregory (Drumoak 1638 – Edinburgh 1675) attempted in vain to prove that the problem of squaring the circle was impossible. Towards the mid-17th century, the idea that the problem was impossible began to gain traction among the more enlightened mathematicians, and, rather than seeking a solution, they attempted to prove its impossibility.
He was the discoverer of the formula :

$$\operatorname{arctg}(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{2k+1}$$

as a primitive of $\frac{1}{1+x^2} = 1 - x^2 + x^4 - x^6 + x^8 - \cdots$

His formula was the basis for the calculation of π. In fact, for x = 1 we obtain :

$$\pi = 4\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots\right) = 4 \cdot \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1}$$

Which, albeit very slowly, converges to π, and for large k gives rather exact values. Convergence is faster for x < 1, because the closer x gets to 0, the faster the series converges.

G.W. Leibniz (Leipzig 1646 – Hannover 1716), starting from Gregory's formula, by transformation, he obtained the formula :

$$\pi = 8\left(\frac{1}{1\times 3} + \frac{1}{5\times 7} + \frac{1}{9\times 11} + \cdots\right) = 8 \cdot \sum_{k=0}^{\infty} \frac{1}{(4k+1)(4k+3)}$$

which converges faster.

5.4. Isaac Newton

Isaac Newton (Colsterworth 1642 – London 1727) found an interesting formula for calculating π. Starting from the formula for the expansion of the binomial :

$$(1+x)^n = 1 + \binom{n}{1}x + \binom{n}{2}x^2 + \binom{n}{3}x^3 + \cdots + x^n$$

$$\binom{n}{i} = \frac{n!}{i!(n-i)!}$$

which can also be written :

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \frac{n(n-1)(n-2)}{2\times 3}x^3 + \dots$$
$$+ \frac{n(n-1)\cdots(n-p+1)}{p!}x^p + \dots x^n$$

It can be generalized by extending it to infinity :

$$(1+x)^\alpha = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2}x^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{2\times 3}x^3 + \dots +$$
$$+ \frac{\alpha(\alpha-1)\cdots(\alpha-p+1)}{p!}x^p + \dots$$

Since :

$$\mathrm{arcsen}(x) = x + \frac{1}{2}\times\frac{x^3}{3} + \frac{1\times 3}{2\times 4}\times\frac{x^5}{5} + \dots +$$
$$+ \frac{1\times 3\times\cdots(2p-1)}{2\times 4\times\cdots(2p)} \times \frac{x^{2p+1}}{2p+1} + \dots$$

We obtain :

$$\pi = 6\left(\frac{1}{2} + \frac{1}{2}\times\frac{1}{3}\times\frac{1}{2^3} + \dots \frac{1\times 3\times\cdots(2p-1)}{2\times 4\times\cdots(2p)} \times \frac{1}{2p+1} \times \frac{1}{2^{2p+1}} + \dots\right)$$

This Newton formula converges rapidly to π.

5.5. Eulero

Euler (Leonhard Euler; Basel 1707 – St. Petersburg 1783) discovered multiple formulas for π, including:

$$\pi^2/6 = 1 + 1/4 + 1/9 + \dots + 1/n^2 + \dots$$

From this we have:

$$\pi = \sqrt{6\left(1 + \frac{1}{4} + \frac{1}{9} + \dots + \frac{1}{n^2} + \dots\right)}$$

to the increases of n it offers increasingly precise approximations of π.
For n = 1000, we have $\pi = 3.14063805$.
Using a similar method, Euler found that :

$$\pi^2/8 = 1 + 1/3^2 + 1/5^2 + \dots 1/(2n+1)^2 + \dots$$

which, for n = 100, gives $\pi = 3.14159265$.

Euler used the symbol π in his treatise on infinite series: *Variae observationes circas series infinitas*. It was his fame as a great mathematician that definitively established the notation π to indicate the ratio between the circumference and its diameter.

Note

Among the many modern formulas for calculating π, we mention that of the Indian mathematician Srinivasa Ramanujan (1887–1920) :

$$\pi = (102 - 2222/22^2)^{1/4} = 3.14159265358$$

A strange formula, like many others he invented without providing any proof.
In 1985, William Gosper, using a Ramanujan formula entered into a computer, calculated 17 million decimal places of π.

6. Transcendence of the Pi-Greek

Charles Hermite (Dieuze 1822 – Paris 1901) proved the transcendence of the constant *e* in 1873. He attempted, unsuccessfully, to prove the transcendence of the invariant π. His work was taken up by Ferdinand von Lindemann who proved it in 1882. The proofs of the transcendence of *e* and π are rather complicated and require in-depth knowledge of mathematical analysis; however, they do not fall within the predominantly popular and didactic purpose of this short essay. For these, see the texts by A. Baker (Alan Baker, b. 1939): *Transcendental number theory*, Cambridge Mathematical Library 1975, and by D. H. Bailey: *Numerical result of the transcendence of the constant involving π, e and Euler constant in mathematical computation* (1988). In the cited texts, the proofs of Hermite and von Lindemann are the result of simplifications made to them by K. Weirstrass (Karl Weirstrass; Ostenfelde 1815 - Berlin 1897) and D. Hilbert (David Hilbert; Königsberg 1862 – Göttingen 1943).

7. Finale

An ancient problem, dating back to 500 BC, has occupied the minds of eminent mathematicians for over two thousand years. Only thanks to advances and new discoveries in every field of mathematics has it found a full and satisfactory explanation.

8. Historical note

In 1775, the Royal Academy of Sciences in Paris declined to examine the many alleged "solutions" to squaring the circle and other problems. Below is a summary of the text in which it justified its decision.

This year, the Academy has decided not to consider any further solutions to the problems of duplicating the cube, trisecting the angle, or squaring the circle, nor any machine that claims to achieve perpetual motion.
We believe it appropriate to explain the reasons that prompted this decision.
The problem of duplicating the cube was raised by the Grecians. Legend has it that the oracle of Delos, consulted by the Athenians to end the plague, ordered the construction of an altar to the god Delos in the shape of a cube twice the size of the one in his temple. (…)

The problem of squaring the circle is different: the squaring of the parabola discovered by Archimedes and that of the lunulae by Hippocrates of Chios raised the hope of squaring the circle, that is, of finding the measurement of its surface. Archimedes demonstrated that this problem, and that of rectifying the circumference, depended on one another, and have since been confused. (…)

This long experience was enough to convince the Academy of the limited utility to science of examining all these so-called solutions.
Other considerations also convinced the Academy: there is a popular rumor according to which governments have promised considerable rewards to whoever succeeds in solving the problem of squaring the circle; believing this rumor, a multitude of people give up useful occupations to launch themselves into research into this problem, often without knowing it thoroughly, and always without having the necessary knowledge to attempt a successful solution. (…)
Many were unfortunate enough to believe they had succeeded in their attempt, and they refused to give in to the arguments with which the geometers refuted their solutions, which they often did not understand, and ended up accusing them of envy and bad faith. Sometimes their obstinacy degenerated into pure madness. (…)
Humanity therefore demanded that the Academy, convinced of the utter futility of analyzing the solutions to the problem of squaring the circle, seek to destroy, with a public declaration, popular opinions that have been disastrous to many families. (…)
Squaring the circle is the only problem rejected by the Academy that could give rise to useful research, and if a geometer were to succeed in solving it, the Academy's resolution would only increase his glory, demonstrating the geometers' opinion of the difficulties, not to mention the insolubility, of the problem.

Remark
History is repeating itself in our times with the Syracuse Conjecture, better known as the Collatz Conjecture.

Reference / Essential bibliography

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