Nuclear Shell Model Composed of Spin-coupled Dinucleon

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Abstract

This paper constructs a Nuclear Shell Model composed of spin-coupled dinucleon nuclei. Using pn (proton-neutron) and nn (neutron-neutron) pairs formed by antiparallel spin coupling as the basic structural units of the atomic nucleus, a shell model isomorphic to the outer electron orbitals is established. By comparing the microscopic orbital configurations of various typical magic number nuclei, the formation of magic numbers is explained from the perspective of nuclear structure. This paper distinguishes between orbital shell configurations and the actual steady state of the atomic nucleus, thus resolving paradoxes that magic number models cannot explain, such as the instability of certain double magic number nuclei and the stability of isotopes in non-magic number nuclei. Research shows that a complete inner neutron cores is a necessary condition for maintaining the stable state of the atomic nucleus. Magic numbers merely represent the geometric characteristics of fully occupied orbitals and cannot be used as a standard for judging nuclear stability. This model can construct a unified nuclear shell system applicable to nuclei within the magic number range without the need for artificial fitting of empirical parameters, providing a new theoretical framework for studying nuclear structure and isotope stability.

Keywords: nuclear structure, nuclear Shell model, spin-coupled dinucleon, magic number, shell filling, isotope stability

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1. Introduction

The nuclear magic numbers 2, 8, 20, 28, 50, 82 and 126 are empirical laws summarized from extensive nuclear physics experiments. Existing theories including the liquid drop model, collective motion model and relativistic mean field theory each have limited scope of application and struggle to uniformly explain the microscopic formation mechanism of magic numbers [1, 2, 11, 12].

The liquid drop model can only macroscopically describe the overall energy trend of atomic nuclei and fails to interpret internal layered arrangement and the origin of magic numbers [1]; the collective model focuses on collective excitation motion of nuclei without addressing the underlying mechanism of ground-state orbital filling [2]; relativistic mean field theory and nuclear many-body theories rely on numerous adjustable parameters and can only perform numerical calculations for individual nuclides, lacking general applicability [6, 11, 12].

The single-nucleon spin-orbit coupling shell model was awarded the Nobel Prize in Physics in 1963 and has long served as the core mainstream theory for interpreting magic numbers, offering stage-specific reference value for qualitative analysis of light nuclei [5]. Nevertheless, this theory suffers from an inherent defect of inverted underlying logic: the threshold values for segmented orbital filling (2, 6, 12, 8, 32, 44) cannot be strictly analytically derived from angular momentum and spin-coupling Hamiltonian equations. These values are artificially fitted parameters set in reverse after the experimental magic number results are known, lacking physical support at the level of physical nucleon pairing structures within atomic nuclei [7, 15]. Years of relevant reviews and revision works have only optimized numerical performance without eradicating fundamental logical flaws [8, 10, 14]. Studies on exotic nuclei and superheavy nuclear stability islands still adopt the traditional single-particle framework and cannot eliminate magic number paradoxes [13, 16, 19]. International research on magic numbers of exotic nuclei and nuclear tensor forces does not introduce dinucleon pairing structures [17, 18]. Existing analyses of dinucleon correlations only discuss low-lying excitation spectra and fail to establish a complete orbital filling system to explain the magic number sequence [20].

Remarkable contradictions persist in traditional theories: doubly magic nuclei ^{56}Ni and ^{100}Sn are prone to decay, while the non-magic nucleus ^{56}Fe possesses the highest binding energy and the most stable structure. No unified microscopic explanation exists for stability differences among different isotopes [9, 14]. Conventional theories generally detach from physical nucleon pairing structures and require separate corrections for different nuclear regions, lacking unified structural rules applicable to all nuclear regions.

To address the aforementioned shortcomings, this paper uses pn and nn spin pairing as the basic unit, follows the neutron core priority filling rule, and derives the magic number origin through standardized orbital arrangement, providing a new theoretical framework for the study of nuclear structure and isotope stability.

2. Fundamental Composition of the Model and Filling Rules

2.1 Fundamental Particle Units: Proton-Neutron pn Pairs and Neutron-Neutron nn Pairs

Both form scalar bosons with spin s=0 via spin-antiparallel coupling, acting as the filling units that constitute nuclear orbitals.

2.2 Shell Orbital System Isomorphic to Extranuclear Electron Orbitals

The principal quantum number N takes values of 1, 2, 3, 4, 5, 6…, Subshell orbitals are denoted as s^2, p^6, d^{10}, f^{14}, g^{18}…, The superscript number represents the maximum number of pn pairs or nn pairs that the corresponding subshell orbital can accommodate

2.3 Filling Priority of nn Neutron Pairs

The nn pairs fill inner orbitals first to form a neutron core, after which pn pairs sequentially fill outer orbitals.

2.4 Closed-Shell Filling Principle

Once pn pairs or nn pairs fill the maximum capacity of a subshell orbital or the total upper limit of particles accommodated by a principal quantum shell, the current subshell or principal shell is fully filled with priority.

3. Microscopic Orbital Structures and Steady-State Comparison of Magic Nuclei

3.1 Magic Number 2: ^{4}He, Z=2, N=2 (Doubly Magic Nucleus)

The orbital configuration: pn $1s^2$.

The N=1 principal shell is fully occupied by two groups of pn pairs, forming a closed shell with high spatial symmetry and weak overall Coulomb repulsion. It serves as the fundamental steady-state unit for light nuclear systems.

3.2 Magic Number 8: ^{16}O, Z=8, N=8 (Doubly Magic Nucleus)

The orbital configuration: pn $1s^0$, $2s^2$, $2p^6$.

The N=2 principal shell is completely filled with pn pairs. Vacant orbitals reserved in inner layers form buffer zones, dispersing proton charge distribution and yielding an overall stable nuclear structure.

3.3 Magic Number 20: ^{40}Ca, Z=20, N=20 (Doubly Magic Nucleus)

The orbital configuration: pn $1s^2$, $2s^2$, $2p^6$, $3s^0$, $3p^0$, $3d^{10}$.

The N=1 and N=2 principal shells are filled in order, while the 3d suborbital within the N=3 principal shell is fully occupied. Large vacant regions exist in 3s and 3p orbitals, diluting neutron accumulation effects and keeping the overall system energy at a low value. This nucleus represents a standard doubly magic nucleus.

3.4 Comparative Analysis of ^{56}Ni and ^{56}Fe

3.4.1 ^{56}Ni, Z=28, N=28 (Doubly Magic Nucleus)

The orbital configuration: pn $1s^2$, $2s^2$, $2p^6$, $3s^2$, $3p^6$, $3d^{10}$.

All principal shells with N=1, N=2 and N=3 are fully filled, forming a geometrically perfect closed doubly magic shell. Traditional magic number theories predict extremely high stability for this nucleus, yet it features an extremely short half-life and undergoes rapid decay, constituting a typical magic number paradox. No independent inner neutron core buffer structure exists inside the nucleus; dense pn pairs accumulate throughout the volume, and concentrated proton distribution drastically amplifies Coulomb repulsion. Despite geometrically fully closed shells, massive internal stress prevents long-term steady state.

3.4.2 ^{56}Fe, Z=26, N=30 (Non-Magic Nucleus)

^{56}Fe is not a magic nucleus, yet it fully satisfies the full-shell stability criteria of the traditional shell model, with principal shells N=1, N=2 and N=3 perfectly and completely filled.

The orbital configuration: nn $1s^2$ || pn $2s^2$, $2p^6$, $3s^2$, $3p^6$, $3d^{10}$.

Its inner 1s orbital consists of nn pairs, forming a compact neutron core without proton accumulation in the central region, which drastically weakens Coulomb repulsion and minimizes various potential energies of the system. It is the nucleus with the highest binding energy and the most stable structure across the universe.

The stark contrast between the two nuclei directly proves that pure geometric closed-shell or fully occupied magic-number configurations cannot serve as criteria for judging nuclear steady states, and an intact inner nn neutron core is an essential prerequisite for the stability of medium-mass nuclei.

3.5 Magic Numbers 50: Comparative Analysis of ^{100}Sn and ^{120}Sn

3.5.1 ^{100}Sn, Z=50, N=50 (Doubly Magic Nucleus)

The orbital configuration: nn $1s^0$, $2s^0$, $2p^0$ || pn $3s^2$, $3p^6$, $3d^{10}$, $4s^2$, $4p^6$, $4d^{10}$, $4f^{14}$.

The outer pn pairs form fully closed high-order shells, but all inner orbitals N=1 and N=2 are empty. There is no neutron core structure at all, lacking inner neutron buffer and neutron core support. Protons accumulate densely, keeping the system in a high-energy excited state accompanied by intense radioactive decay.

3.5.2 ^{120}Sn, Z=50, N=70 (Single-Magic Nucleus)

The orbital configuration: nn $1s^2$, $2s^2$, $2p^6$ || pn $3s^2$, $3p^6$, $3d^{10}$, $4s^2$, $4p^6$, $4d^{10}$, $4f^{14}$.

The inner N=1 and N=2 principal orbitals are fully filled to form a complete neutron core; outer pn pairs completely occupy all subshells of n=3 and n=4 without extra high-order subshell filling. The intact inner neutron core fully screens Coulomb repulsion between protons, yielding a defect-free structural configuration. This nuclide possesses the highest natural abundance among tin isotopes with a well-completed structure.

3.6 Magic Numbers 82 and 126: Comparative Analysis of ^{208}Pb and ^{204}Pb

3.6.1 ^{208}Pb, Z=82, N=126 (Doubly Magic Nucleus)

The inner orbital configuration: nn $1s^2$, $2s^2$, $2p^6$, $3s^2$, $3p^0$, $3d^{10}$.

The entirely vacant 3p orbital forms an extensive buffer zone. Surplus 44 neutrons construct layered neutron cores that effectively alleviate strong Coulomb repulsion in heavy nuclei.

The outer orbital configuration: pn $4s^2$, $4p^6$, $4d^{10}$, $4f^{14}$, $5s^2$, $5p^6$, $5d^{10}$, $5f^{14}$, $5g^{18}$.

The outer pn pairs fully occupy all subshells of N=4 and N=5, featuring saturated and symmetric orbital spin coupling, forming an ultimate steady-state structure within the heavy nuclear mass region.

3.6.2 ^{204}Pb, Z=82, N=124 (Single-Magic Nucleus)

The inner orbital configuration: nn $1s^2$, $2s^2$, $2p^6$, $3s^0$, $3p^0$, $3d^{10}$ Large vacant regions exist in the low-order 3s and 3p orbitals of the N=3 shell, providing greater surplus buffer capacity for the neutron core.

The filling pattern of the outer pn pairs is consistent with that of ^{208}Pb.. This nuclide has an extremely long half-life with no decay channels at the nuclear structural level, classified as an ultra-long-lived stable heavy nucleus.

4. Unified Analysis of the Formation Mechanism of Magic Numbers

Traditional theories directly equate magic numbers with stability criteria for atomic nuclei, claiming that doubly magic full-shell nuclei must be stable while non-magic nuclei exhibit poor stability.

Core comparative examples including ^{56}Ni and ^{56}Fe as well as orbital configurations of all magic nuclei demonstrated in this paper prove that magic numbers merely represent geometric quantitative features arising from layered filling of shells by dinucleon pairs. As mathematical characteristics of orbital arrangement, they cannot independently determine the physical steady state of atomic nuclei.

The actual stability of atomic nuclei is governed by core structural conditions: complete filling of the inner nn neutron core enables effective buffering of proton Coulomb repulsion and elimination of internal structural stress within nuclei.

This perfectly resolves the paradoxes that magic number models cannot explain: the doubly magic ^{56}Ni lacks a neutron core and is unstable at high energy, whereas the non-magic ^{56}Fe with a complete neutron core is the most stable nucleus in the universe. This model can self-consistently unify and explain all laws governing magic nuclei and isotope stability without artificially fitted or adjusted parameters.

5. Conclusion

1. This paper successfully establishes the dinucleon spin-coupling shell model of atomic nuclei, taking spin-singlet pn and nn pairs as fundamental units and constructing a complete, self-consistent system of nuclear orbital filling rules and microscopic criteria for nuclear steady states.

2. This paper thoroughly resolves the classical paradox inherent in the 1963 single-nucleon spin-orbit coupling shell model: the doubly magic full-shell nucleus ^{56}Ni identified by traditional theories is structurally unstable with intense radioactivity, while the non-magic nucleus ^{56}Fe has the highest binding energy and the most stable structure in the universe. This contradiction is fully and self-consistently interpreted via the neutron core buffering and screening mechanism proposed herein.

3. Nuclear magic numbers only represent geometric mathematical characteristics of fully occupied orbitals rather than physical criteria for nuclear stability. The integrity of the inner nn neutron core structure serves as the core determinant governing the steady state of medium and heavy atomic nuclei.

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