# Deriving The Dot Product From Polar Coordinates

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**Abstract**

Using polar coordinates the dot product is easily derived.

## Polar coordinates

How do you show one form of the dot product is $\mathbf{A} \cdot \mathbf{B} = |A||B| \cos\theta$. If $\mathbf{A} = (x_a, y_a) = (a\cos\theta_a, a\sin\theta_a)$ and $\mathbf{B} = (x_b, y_b) = (b\cos\theta_b, b\sin\theta_b)$ and $\mathbf{A} \cdot \mathbf{B} = x_a x_b + y_a y_b$, we have

$$\begin{aligned} \mathbf{A} \cdot \mathbf{B} &= ab\cos\theta_a\cos\theta_b + ab\sin\theta_a\sin\theta_b && (1) \\ &= ab(\cos\theta_a\cos\theta_b + \sin\theta_a\sin\theta_b) && (2) \\ &= |A||B|\cos(\theta_a - \theta_b) && (3) \\ &= |A||B|\cos\theta && (4) \end{aligned}$$

where $\theta$ is the angle between the vectors.