

# Arbitrary approximation of the shifted-LRC by the equi-split-LRC

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**Abstract:** - Consider an instance of the shifted Lonely Runner Conjecture (shifted-LRC) where  $n$  runners (except the stationary runner 0) have integer speeds and start from real values in  $[0,1]$  at time  $t=0$ . We show that one can derive an alternative vector of starting points that can be made to be arbitrarily close to the initial vector of starting points. The alternative starting point of each runner  $i$  is a rational in  $[0,1]$  and is expressible as  $(q_i / P)$  where  $P$  is a large prime and  $q_i$  is an integer in  $[0, P-1]$ . We then introduce a new LRC variant called the equi-split-LRC. The LRC instance allows a minimal loneliness gap of  $f$  for runner 0 from the remaining  $n-1$  runners, if and only if, the equi-split-LRC instance with the same vector of speeds and alternative vector of starting points simultaneously allows a minimal loneliness gap of  $f/P$  for the arc-center of each of  $P$  sectors into which the circle is divided from the remaining  $n-1$  runners. Here,  $f$  is a desired fraction in  $[0,1]$ . This finding is important in the light of recent counter-examples to the shifted-LRC.

## 1. Introduction

The reader is referred to papers [1][2] for a literature survey on the shifted-LRC (where the  $n-1$  moving runners are allowed to have arbitrary starting points on the circumference of the circle at time  $t=0$ ) and LRC (where the  $n-1$  moving runners have the same starting point as the stationary runner 0 at time  $t=0$ ). Recently, counter-examples [1] have been found to the shifted-LRC (with as low as 6 runners), in which certain vectors of rational starting points of the runners prevent the minimal  $1/n$  gap. The aim of this paper is to derive a new LRC variant from the shifted-LRC called the equi-split-LRC with the same vector of speeds, but with a different loneliness criterion. The equi-split-LRC is interesting, because the “danger-zone” for all three LRCs - the non-shifted LRC (referred to simply as the LRC), the shifted-LRC, and the equi-split-LRC, is the same and equal to  $2f$ .

## 2. The approach

Theorem 1 below talks about approximating the given vector of real starting points with a vector of rational starting points.

**Theorem 1:** Denote the initial given vector of  $n-1$  real starting points in the shifted-LRC instance as  $A = \langle a_1, a_2, \dots, a_{n-1} \rangle$ . There exists an alternative vector of  $n-1$  rational starting points denoted as  $B = \langle (q_1 / P), (q_2 / P), \dots, (q_{n-1} / P) \rangle$ , where  $P$  is an arbitrarily large prime number, and where each  $q_i$  is an integer in  $[0, P-1]$  such that  $\text{MAX}(\text{absolute}((q_i / P) - a_i))$ , over integers  $i$  in  $[1, n-1]$  is arbitrarily small.

**Proof:** We know from well-known theories on prime numbers that if  $\{P_1, P_2, \dots, P_M\}$  is the set of prime numbers below  $(P_M + 1)$ , then the number  $P = ((P_1 P_2 \dots P_M) + 1)$  is also prime. We are thus able to choose a larger prime number  $P$ , for which we can keep obtaining an optimum value of integer  $q_i$  in  $[0, P-1]$  such that  $\text{absolute}((q_i / P) - a_i)$  will tend to decrease from the earlier value of  $\text{absolute}((q_i / P) - a_i)$  calculated from the earlier and smaller prime  $P$ . There will be exceptions to the statement that  $\text{absolute}((q_i / P) - a_i)$  will keep decreasing if we choose larger values of  $P$ , for example if  $a_i$  is a rational whose denominator is a prime constant in which case  $\text{absolute}((q_i / P) - a_i)$  will be 0 when  $P$  is that prime constant and will increase momentarily for a higher value of  $P$ . But it is true that there exists an optimum value of integer  $q_i$  in  $[0, P-1]$  such that  $\text{absolute}((q_i / P) - a_i)$  will tend to 0 as  $P$  tends to infinity.

**Hence Proved**

### Definitions: -

For all the 3 three LRC variants, we are given  $n-1$  moving runners with non-zero integer speeds. The LRC variants differ mainly in the starting points of the runners, and in their aims.

1. **LRC:** The  $n-1$  moving runners start from 0 (the stationary runner 0) at time  $t=0$ . The aim is to find the time  $t=T_{LRC}$  at which every runner is separated by atleast  $f$  from 0.
2. **Shifted-LRC (SLRC):** The  $n-1$  moving runners start from points given by vector  $A$  at time  $t=0$ . The aim is to find

the time  $t=T_{SLRC}$  at which every runner is separated by atleast  $f$  from 0.

3. **Equi-Split-LRC (ESLRC):** The  $n-1$  moving runners start from points given by vector  $B$  at time  $t=0$ . The aim is to find the time  $t=T_{ESLRC}$  at which every runner is separated by atleast  $f/P$  from the centers of arcs of  $P$  sectors on the circle numbered from 0 to  $P-1$ , where the arc of sector  $j$  is in  $[(j-0.5)/P, (j+0.5)/P]$ .

It is evident that in all three LRC variants, the total unoccupied length of the circle should be  $2f$ , by all  $n-1$  runners.

We now state the main result of this paper in Theorem 2, where the symbol  $\leftrightarrow$  denotes “if and only if”.

**Theorem 2:**  $(T_{LRC} / P) = T_{ESLRC}$

**Proof:** This result comes from the joint application of Theorem 1 and Theorem 5 of our previous paper [2]. We remark that since the non-zero speeds of the runners are integers, the positions of the runners in the LRC, SLRC and ESLRC, are periodic, irrespective of the real values in  $A$ . This remark is crucial since it establishes a finite amount of time in which the minimal loneliness gap needs to be checked. We observe the following situations.

**Situation at  $t=0$ :** From Theorem 1, it is clear that each alternative starting point in  $B$  is actually the center of the arc of some sector, so the position of each runner  $i$  for the ESLRC is given by  $q_i/P$  where  $q_i$  is an integer. The equivalent situation for the LRC is that the position of each runner  $i$  is 0, which is also the center of Sector 0. For the SLRC, of course, the starting point is given by  $a_i$  that is arbitrarily close to  $q_i/P$ .

**Situation at  $t=T_{LRC}$ :** In the LRC instance, each runner  $i$  is separated by atleast  $f$  from the stationary runner  $R_0$ , so it would mean that its position is given by  $I_i+y_i$ , where  $-f > y_i > f$ , where  $absolute(y_i) < 1$ , where  $I_i$  is some integer.

**Situation at time  $t=T_{LRC}/P$  for the LRC:** In the LRC instance, the position of runner  $i$  would be  $(I_i/P + y_i/P)$ , where  $I_i/P$  is the center of some sector and  $-f/P > y_i/P > f/P$ , and where  $absolute(y_i/P) < 1/P$ .

**Situation at  $t=T_{LRC}/P$  for the ESLRC:** In the ESLRC instance, the position of runner  $i$  would be  $((I_i+q_i)/P + y_i/P)$ , where  $(I_i+q_i)/P$  is again the center of some sector and  $-f/P > y_i/P > f/P$ , and where  $absolute(y_i/P) < 1/P$ . It is clear that this is the time defined as  $T_{ESLRC}$ .

**Hence Proved**

## References

- [1] Monica Blanco, Francisco Criado, Francisco Santos, *Co-loopless zonotopes and counterexamples to the Shifted Lonely Runner Conjecture*, Mar 2026, <https://arxiv.org/abs/2603.24784>.
- [2] Deepak Ponvel Chermakani, *An Average Loneliness Gap of  $1/n$  Can Allow a Minimum Loneliness Gap of  $1/(2n)$* , May 2026, <https://www.vixra.org/abs/2605.0011>.

## About the author

I, Deepak Ponvel Chermakani, am a citizen of India and I wrote this paper, which is original to the best of my knowledge, out of my own interest and initiative during my spare time. I completed a fulltime two-year Master of Science Degree in Electrical Engineering from University of Hawaii at Manoa USA ([www.hawaii.edu](http://www.hawaii.edu)) in Aug 2015, a fulltime one-year Master of Science Degree in Operations Research with Computational Optimization from University of Edinburgh UK ([www.ed.ac.uk](http://www.ed.ac.uk)) in Sep 2010, a fulltime four-year Bachelor of Engineering Degree in Electrical and Electronic Engineering, from Nanyang Technological University Singapore ([www.ntu.edu.sg](http://www.ntu.edu.sg)) in Jul 2003, fulltime high schooling from National Public School Indiranagar in Bangalore India in Jul 1999, and fulltime middle schooling from Bishop Cotton Boys School in Bangalore India in Jul 1997. I am most grateful to my parents (my mother Mrs. Kanaga Rathinam Chermakani and my father Mr. T. Chermakani) for their sacrifices in educating me and bringing me up.