Emergent Gravity from fundamental Triangles with inner Torsion-Twist Degrees of Freedom – an Analogon to Structure of Penrose-Triangles

Abstract:

Investigated is a model of spacetime in which fundamental triangles possess not only geometric properties such as area and edge lengths but also internal degrees of freedom representing twist, which encodes their relative orientation with respect to neighboring triangles. While local triangles are flat and consistent, global misclosures can accumulate, acting analogously to holonomies in curved spacetimes. This approach enables the construction of a spin-network-like model in which gravity can be interpreted as a statistical emergent phenomenon arising from these accumulated twists. The concept is inspired by Penrose-triangles: local consistency combined with global twist. Therefore this description is called the Penrose-triangle-ansatz (PTA).

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1. Introduction/Motivation:

Modern approaches to quantum gravity — including Regge calculus, spin networks, and loop quantum gravity — describe curvature not by smooth fields but by discrete structures [1.],[2.]. A central problem is how local, nearly flat building blocks can give rise to global curvature.
The Penrose triangle [3.] illustrates a phenomenon relevant in this context: locally consistent geometry can accumulate a form of global torsion "twist." It is proposed to utilize this concept as a physical degree of freedom in a model of fundamental triangles.

2. Mathematical forms of fundamental model:

2.1. Bricks of building:

1.Knots: Triangles (T_i) with geometric parameters (A_i) (Area, lengths of edges, angles).

2. Edges: Edges (e_{ij}) between neighbored triangles carry torsion-twist:

$((\tau_{ij}) \in SO(3)) \vee ((\tau_{ij}) \in SU(2))$.

2.2. Holonomy and curvature:

For a closed loop

$$\left(\gamma = \left(T_1 \rightarrow T_2 \rightarrow \dots T_n \rightarrow T_1\right)\right) \quad (1.)$$

the holonomy is defined as:

$$\left[\Phi_\gamma = \prod \tau_{ij}\right] \quad (2.)$$

1. $\left(\Phi_\gamma = I\right) \rightarrow$ flat loop (3a./3b.)

2. $\left(\Phi\gamma \neq I\right) \rightarrow$ effective curvature

2.3 Energy-function / action:

A basic formulation:

$$\left[S = \sum_\gamma f\left(\Phi_\gamma\right),\right] \quad (4a.)$$

where $\left(f(I) = 0\right) \wedge \left(f\left(\Phi_\gamma \neq I\right) > 0\right)$. (4b.)

Thus, nontrivial holonomies act as energy contributions that control macroscopic curvature.

2.4. Quantisation from pure rotation to operator:

Instead of a fix rotation:

$$\left(\tau_{ij}\right) \quad (5a.)$$

there now an operator is introduced:

$$\left(\tau^{ij}\right) \in SU(2) \quad (5b.)$$

It acts on a local Hilbert-space of edge:

$$H_{ij} = span| j, m\rangle \quad (6a.)$$

with:

1. $(j) \in \frac{1}{2}(N), (spin)$ (6b./6c.)

2. $(m) = (-j, \dots, j)$

2.5. Physical Interpretation:

Each edge now carries not only an orientation, but also a quantized geometric moment:

$$\left(\tau^{ij}\right) \sim \exp\left(iJ^{ij} \cdot \theta\right) \quad , \quad (7.)$$

where are :

$\left(J^{ij}\right)$ = generator of rotation (spin operator),
(θ) = discrete or continuous rotation parameters.

2.6. Discrete geometry becomes spectral:

In this picture, geometry is no longer continuous, but rather has a state of form of a quantized area:

$$A \sim \ell_{PL}^2 \sqrt{j(j+1)} \quad \text{(8a.)}$$

Twist becomes discrete:
The "residual rotation" around a loop:

$$\hat{\Phi}_\gamma = \prod_\gamma \hat{\tau}_{ij}$$

(8b.)

has a discrete spectrum of eigenvalues:

$$\hat{\Phi}_\gamma |\psi\rangle = e^{i\varphi n} |\psi\rangle \quad . \quad \text{(8c.)}$$

2.7. Interpretation of the twist as a quantum degree of freedom:

The central idea of the model becomes precise here by interpreting, that the "Penrose-like twist" is not a geometric error or fault, but a quantized state of the connection. This means classical inconsistency → disappears and is replaced by:

1. superposition of twists,
2. discrete holonomy states,
3. measurable phases.

2.8. Loop-quantisation of holonomy:

For a closed loop:

$$\gamma = (T_1 \rightarrow T_2 \rightarrow \cdots \rightarrow T_n \rightarrow T_1) \quad \text{(9a.)}$$

there is defined:

$$\Phi^\gamma = P \cdot \exp\left(i\gamma \sum A^{ij}\right) \quad \text{(9b.)}$$

with:

$A^{ij} = SU(2)$ -connection-operator.

This description then exactly is the structure of Wilson-loops in gauge-theories.

2.9. Emergence of gravity in quantised Model:

In limit of continuum the condition :

$$\langle \Phi^\gamma \rangle \neq I \quad \text{(10a.)}$$

leads to effective curvature of:

$$\left[R \sim \lim_{A \to 0} \frac{1}{A} \left(1 - Tr\left(\Phi^\gamma\right)\right) \right] \quad \text{(10b.)}$$

This description exactly is an analogon to field-strength in Yang -Mills-theories:

$$F_{\mu\nu} \sim [D_\mu, D_\nu] \tag{11.}$$

Ergo there are two themes described. First the limit of continuum of an operator (Φ^γ) and second effective curvature as an analogon to field-strength of $(F_{\mu\nu})$. Then the interpretation is, that in the limit of continuum the expectation value of (Φ^γ) is not the identity, ergo there is $[\langle\Phi^\gamma\rangle \neq I]$. This explanation means, that the operator (Φ^γ) in average has a deviation from trivial identity.

Now the effective curvature. Define the curvature (R) as limes over small areas $(A \to 0)$, in analogy to Wilson-loops: $\left[R \sim \lim_{A\to 0} \frac{1}{A}(1 - Tr(\Phi^\gamma))\right]$. Here (Φ^γ) is quasi interpreted as operator for parallel transport over the area (A).

Analogy to Yang–Mills-theories :

In Yang–Mills-theories there is the field-strength $(F_{\mu\nu})$ defined by the commutating derivation of $[F_{\mu\nu} \sim [D_\mu, D_\nu]]$, Therefore the analogon comes clear: (Φ^γ) plays the role of exponential parallel-transport $(e^{i\int F})$ and the curvature (R) is like the linearized field-strength.

<u>Summary in one collection of equations:</u>

$$\boxed{\langle\Phi^\gamma\rangle \neq I} \tag{12a.}$$

$$\boxed{(limit\ of\ cont.)\, R \sim \lim_{A\to 0} \frac{1}{A}(1 - Tr(\Phi^\gamma)) \sim (effective\ curv.)\, F_{\mu\nu} \sim [D_\mu, D_\nu] \sim (Yang-Mills-Fieldstrength)} \tag{12b.}$$

More detailed for a description in form of style of Wilson-loops and limit of continuum there is the following writing:

$$\left[\Phi^\gamma = P\exp\left(\oint_\gamma \Gamma\right)\right] \tag{13a.}$$

where (Γ) describes the connection form and (P) the ordering of paths. Then there is for an infinitesimal loop:

$$(\gamma = \partial A) \tag{13b.}$$

$$[\Phi^\gamma = I + R_{\mu\nu}\Sigma^{\mu\nu} + O(A^2)] \quad , \tag{13c.}$$

where $(\Sigma^{\mu\nu})$ is the oriented element of area and $(R_{\mu\nu})$ is the curvature.

From this condition then follows:

$$[\langle\Phi^\gamma\rangle \neq I \;\; \to \;\; R \neq 0] \tag{14a.}$$

and the effective curvature can be identified as:

$$\left[R \sim \lim_{A \to 0} \frac{1 - Tr(\Phi^{\gamma})}{A} \right] \quad (14b.)$$

The analogon to Yang–Mills-theory then is described by:

$$F_{\mu\nu}[D_{\mu}, D_{\nu}] = [\partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}] \quad (15.)$$

2.10. Connection to the fundamental idea:

Now the Penrose-Ansatz is physically explained:

Penrose-Idea	Quantised Theory
Torsion-twist in triangle	SU(2)-Holonomy
global Inconsistence	Structure of phase
Impossible geometry	Superposition of states
visual paradox	Operator-algebra

Table 1: Physical interpretation of this ansatz over Penrose-triangles.

2.11. Short in-between summary:

The twist is no geometrical error but a quantized degree of freedom of the connection – and gravity then generates from the statistical dynamics of these quantized holonomies.

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3. Differences to classical spin-networks/loop-quantum gravity (LQG):

Feature	Classical Spin-Networks	The Penrose-Triangle-network
Fundamental building stone	Knots: Points/ tetraedral Volumina	Knots: *flat triangles* (2D Building blocks), local consistent
Edges	carry SU(2)-Spins / Holonomies	Carry quantised torsion-twist degrees of freedom interpreted as local Orientation
Geometry	Areas/Volumina quantised, internal mostly defined by spins	Area/Geometry + independent grade of twisting as additional variable
Holonomy	Holonomies determine Areas / Curvature	Holonomies generate from *accumulated torsion-twists*, which originally come from Penrose-Triangle -Ideas
Dimensionality	Spin-Networks often 3D-Tetraeder, 4D-Spinfoams	Fundamental building stone is 2D (Triangle), global Structure 2D/3D, can developed to 3D-Tetraeders
Interpretation of torsion-twist	Operators correspond to geometrical curvature	Operator interpreted as *inner Degree of Freedom* , which is local trivial, but global measurable
Emergent Gravity	Derived from Areas/	Derived primary from *inner Torsion-Twists*,

	Volumina	classical Geometry secundary

Table 2: Comparison of differences between (LQG) and Penrose-Triangle-Ansatz (PTA)

Essential differences:

<u>I. Primary variable is torsion twist-drill, not area:</u>

1. In LQG spin directly is coupled to area.

2.In this model here additionally exists a degree of freedom of orientation, which is independent from area or volume.

<u>II. Local consistence, global Holonomy as emergent curvature:</u>

3. Penrose-inspirated: every triangle is local but global not-closing generates curvature.

4. In LQG: curvature is connected intrinsic with spin.

<u>III. 2D-Basically building stones instead of 3D-Tetraeder:</u>

5. This model begins with flat triangles not with tetraeders.

The elaboration to tetraeders is possible but remains flexible.

<u>IV. Combination from classical idea and quantization:</u>

6. The Penrose-idea delivers a clear concrete motivation.

Quantisation over $SU(2)$ -operators overtakes the log-structure without interpreting the geometric structure in a direct primary form.

<u>3.3. Advantages and new possibilities:</u>

Flexibility of building stones. There could be defined different sorts of twistings on edges, not only SU(2), e,g, inhomogenious inner degrees of freedom, which carry additional physical characteristics.

Explicite visualisation of holonomy; by using the Penrose-analogon it is easy to understand, how global curvature generates from local torsion twists.

New paths of emergence. In LQG spin-networks are strictly geometrically motivated. This ansatz here allows to interprete gravity as a statistic phenomenon of orientation, independent from classical metrics.

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4. Analogy to Penrose-triangle:

Each triangular building block is locally consistent, like the bars of a Penrose triangle. Global loops accumulate twists, analogous to the “impossible” connections of a Penrose triangle. Unlike the Penrose triangle, the global holonomy is consistent and physically interpretable. The internal degree of twist $(\$\tau_{ij}\$)$ can be viewed as a quantum-mechanical degree of freedom, similar to spin variables in spin networks.
3. Schematic interpretation:
A network from some triangles with twists will have following characteristics:
Every edge carries a small rotation/orientation (τ_{ij}) . Holonomy for loops $(e.g.: T_1 \to T_2 \to T_3 \to T_1)$ result in an effective residual-rotation. This residual rotation is the discrete counterpart to curvature in emergent spacetime.

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5. Summary:

The (PTA)- model is a little changed spin-network structure, where the quantized holonomies come from an inner twisting grade.
This is different to classical LQG-models [4.] It conserves the mathematical structure of holonomies [5.], operators and quantized spins [6.]. But it shifts the physical interpretation by a description, where gravity primarily generates by orientation, not by areas. The Penrose-analogon then delivers an intuitive picture for the generation of global curvature from local trivial elements.

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6. Interpretation:

Gravity is interpreted as an emergent phenomenon:
The statistical distribution of twists generates effective curvatures in the continuum. **This approach suggests that the fundamental quantity of spacetime might not be the metric, but rather the internal orientation of the building blocks.** The mechanism unites concepts of holonomy, spin networks, and Penrose figures within a common framework

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7.Conclusion:

Introducing an internal degree of freedom for twisting into fundamental triangles [7.] provides a simple yet profound analogy to Penrose triangles and opens up a new perspective on emergent gravity: gravity appears as the cumulative holonomy of discrete building blocks that are locally flat and consistent.

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8. Discussion:

Extension to 3D triangulations [8.] : embedding of triangles into tetrahedral networks. Quantization of the rotational degrees of freedom $(\$\tau_{ij}\$) \to$ spin operators. Investigation of the continuum limit and derivation of an effective theory of gravity. Simulation of small networks to visualize emergent curvature. Introducing an internal degree of freedom for twisting into fundamental triangles provides a simple yet profound analogy to Penrose triangles and opens up a new perspective on emergent gravity: gravity appears as the accumulated holonomy of discrete building blocks that are locally flat and consistent.

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9. References:

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10. Non-scientific comment:

The laws of nature are written in the language of mathematics … the symbols are triangles …
Galilei, G.

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11. Verification:
This paper definitely is written without support from an AI, LLM or chatbot like Grok or Chat GPT 4 or other artificial tools. It is fully, purely human work in every universe.
