

Title: How to derive the Fine Structure Constant using probability and Compton equations.

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*----- Abstract | start.

I use the relativistic Compton equations to calculate the momentum response of a 'target' electron to vacuum photon collisions redirected from a 'source' electron. The equations give the ratio of the momentum of the 'target' electron after the collision divided by the momentum of the incoming photon over the range of all angles of the 'target' electron's deflection.

By summing evenly (over the maximum cross-section) over all angles we obtain the ratio of 1/2.1412 when using photon frequencies of 2 and 4 times the mass equivalence of an electron in the ratio of 5.6269 to 1. 4 times the mass equivalence may be 2 x (2 times electron mass equivalence photons acting simultaneously) or a double photon composite.

The fraction 1/8 is a probability based on spherical geometry – the ratio of the maximum cross-sectional area of a sphere to its surface area (1/4) and a further probability of (1/2) due to the even chance of the electromagnetic properties of a photon matching those of an electron. This gives a total probability of (1/4) x (1/2) = 1/8.

There are 2 such 1/8 probability collisions – one in which a photon is deflected by a 'source' electron towards a 'target' electron and a second 1/8 probability collision at the 'target' electron.

The Fine Structure Constant is therefore divided into 3 parts.

$1/8 \times 1/8 \times 1/2.1412 = 1/137.0360$ based on probability, spherical geometry and the use of the Compton scattering equations.

1/2.1412 {0.4670} is calculated using the Compton scattering equations.

1/2.1412 x 1/8 (probability) gives 1/17.1295 {0.05838} Coulomb's Law.

1/17.1295 x 1/8 (probability) gives 1/137.0360 {0.007297} the Fine Structure Constant.

A 'free' electron may be pictured/averaged as a speed of light particle contained in an approximately circular orbit with fixed angular momentum by collisions with vacuum momenta.

*----- Abstract | end.

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Page 5. Data points. Strategy and formulae to calculate $p_e \cos \phi / (h v_i / c)$ at specific points.

Page 15. Electron - kinetic picture. Electron 'radius'. Compton wavelength.

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Page 16. The factor 1/8.

Page 24. Reference 1. Compton equations.

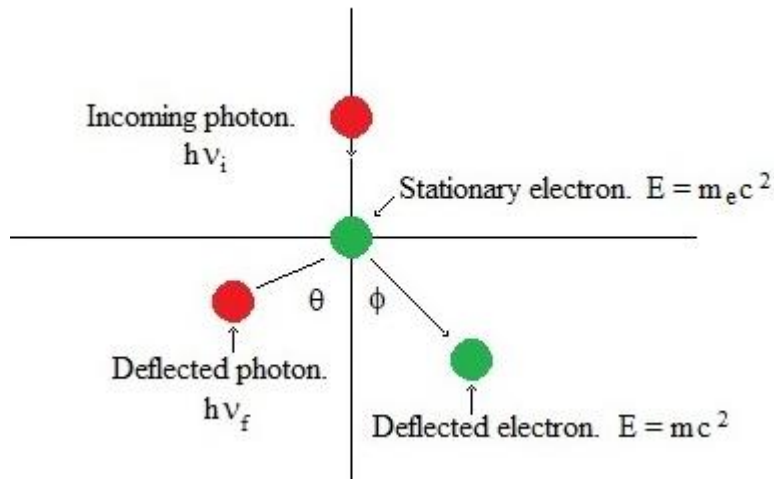
Page 25. Reference 2. Explaining items to do with Coulomb's Law.

Page 30. Reference 3. To do with section: Electron - kinetic picture. Electron 'radius'.
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Page 31. Reference 4. Examples of calculations.

Index | end.

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*----- Equations, Constants and Formulae | start.

*----- Equations | start.

(a) Relativistic mass increase of an electron: $m = m_e / (1 - v^2/c^2)^{1/2}$

(b) Electron rest energy $E = m_e c^2$

(c) Total electron energy $E = m c^2$

(d) Total electron energy with electron momentum p_e : $E^2 = m_e^2 c^4 + p_e^2 c^2$

*----- Equations | end.

*----- Constants | start.

Planck's constant: $h = 6.6261 \times 10^{-34} \text{ kgm}^2\text{s}^{-1}$

Rest mass of the electron: $m_e = 9.1094 \times 10^{-31} \text{ kg}$

In some texts the rest mass of the electron is shown as m_0

Electron self-orbital radius $r_e \sim 1.9308 \times 10^{-13} \text{ m}$ {page 17}

Speed of light (in a vacuum): $c = 2.9979 \times 10^8 \text{ ms}^{-1}$

$h/c = 2.2102 \times 10^{-42} \text{ kgm}$

$h/(m_e c^2) = 0.080933 \times 10^{-19} \text{ s}$

$(m_e c^2)/h = 12.3559 \times 10^{19} \text{ s}^{-1}$

$(m_e c) = 27.3092 \times 10^{-23} \text{ kgms}^{-1}$

$(m_e c^2) = 81.8711 \times 10^{-15} \text{ kgm}^2\text{s}^{-2}$

*----- Constants | end.

Photon/electron collision.

Input:

(a) ν_i (base value for the frequency of the incoming photon)

Example: $12.3559 \times 10^{19} \text{ s}^{-1}$

(b) multiplier k on ν_i e.g. $k = 2$ for double the frequency $2\nu_i$ compared to the base value ν_i equivalent to the mass of an electron as in $h\nu_i = m_e c^2$

Examples: 1, 2 or a fraction or a decimal value

(c) ϕ /degrees

Examples: 30 degrees, 60 degrees

Intermediate (derived):

(a) θ = angle between the final direction of the photon and the initial direction of the photon.
Degrees/radians

Example: 81.7868 degrees or 1.4274 radians

(b) ν_f = photon final frequency (after collision). Unit s^{-1}

Example: $6.6532 \times 10^{19} \text{ s}^{-1}$

(c) p_e = electron momentum (after collision). Units kgms^{-1}

Example: $29.1083 \times 10^{-23} \text{ kgms}^{-1}$

Output:

(a) $p_e \cos\phi / (h\nu_i/c)$ No units. Value between 0 and 1

Example: 0.9231

h and c are constants. ϕ and ν_i are given

we need to derive p_e from p_e^2 but for p_e^2 we first need to derive ν_f and for ν_f we need to derive θ .

So working backwards we need to find: θ , v_f , p_e^2 and finally p_e

Formulae:

cot = cotangent

tan = tangent

atan = arctangent

$$\gamma = hv_i/m_e c^2$$

$$\theta = 2 \times \text{atan}[(\cot\phi)/(1 + hv_i/m_e c^2)]$$

*===== θ | start.

Source (2). [Compton scattering - Wikipedia](https://en.wikipedia.org/wiki/Compton_scattering)

{https://en.wikipedia.org/wiki/Compton_scattering (28th May 2026.)

$$\text{Equation 4: } \cot\phi = [1 + (hv_i)/m_e c^2] \times \tan(\theta/2)$$

Rearrangement of $\cot\phi = [1 + (hv_i)/m_e c^2] \times \tan(\theta/2)$ with:

$$\tan(\theta/2) = \cot\phi / (1 + [(hv_i)/(m_e c^2)])$$

$$\theta/2 = \text{atan}\{\cot\phi / (1 + [(hv_i)/(m_e c^2)])\}$$

$$\theta = 2 \times \text{atan}\{\cot\phi / (1 + [(hv_i)/(m_e c^2)])\}$$

*===== θ | end.

*===== v_f | start.

The formula for v_f comes from Reference 1. Compton equations.

Page 231 (j) Equation 8.16: $v_f/v_i = 1/[1 + \gamma(1 - \cos\theta)]$

$$v_f = v_i / \{1 + [(hv_i/m_e c^2) \times (1 - \cos\theta)]\}$$

Divide top and bottom by v_i :

$$v_f = 1 / \{(1/v_i) + [(h/m_e c^2) \times (1 - \cos\theta)]\}$$

Multiply top and bottom by c :

$$v_f = c / \{(c/v_i) + [(h/m_e c) \times (1 - \cos\theta)]\}$$

$$v_f = c / \{[(h/m_e c) \times (1 - \cos\theta)] + (c/v_i)\}$$

*===== v_f | end.

*===== p_e^2 and p_e – (need v_i and v_f) | start.

The formula for p_e^2 is found in Reference 1. Compton equations (j).

Source (1). Electromagnetic Radiation F. H. Read, ISBN 0 471 27714 2

$$p_e^2 = [(hv_i - hv_f + m_e c^2)^2 - m_e^2 c^4]/c^2$$

$$p_e = +\sqrt{[(hv_i - hv_f + m_e c^2)^2 - m_e^2 c^4]/c^2} \text{ \{+ implies take the positive square root\}}$$

*===== p_e^2 and p_e – (need v_i and v_f) | end.

*----- Equations, Constants and Formulae | end.

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Data points | start.

*----- Contents | start.

Calculate an approximation to the Fine Structure Constant with 2 data points then increase the accuracy with more data points up to 80 to get more exact calculations.

Strategy. Formulae required. Calculate (momentum out)/(momentum in) [$p_e \cos \phi / (h v_i / c)$] at specific points. Derive the Fine structure constant.

*----- Contents | end.

*----- Calculations to derive figures for the 2 data points at: $\phi = 30$ degrees and $\phi = 60$ degrees at 2 average points (over each double part of the cross-sectional areas) in 4 equal cross-sectional areas | start.

Summary | start.

Strategy, input, output, formulae, graphics, points, angles, calculations.

Summary | end.

Strategy.

Input: v_i | multiplier k on v_i | ϕ /degrees (at each point)

Derive: θ | v_f | p_e (at each point)

Output: (averaged) $p_e \cos \phi / (h v_i / c)$

Input:

(a) v_i (base value for the frequency of the incoming photon)

Example: $12.3559 \times 10^{19} \text{ s}^{-1}$ ($> 3 \times 10^{19} \text{ s}^{-1}$ are high energy [gamma rays])

(b) multiplier k on v_i e.g. $k = 2$ for double the frequency $2v_i$ compared to the base value v_i equivalent to the mass of an electron (m_e) as in $h v_i = m_e c^2$

Examples: 1, 2 or a fraction or a decimal value.

(c) ϕ /degrees

Examples: 30 degrees, 60 degrees

Intermediate (derived):

(a) θ = angle between the final direction of the photon and the initial direction of the photon.

Degrees/radians

Example: 81.7868 degrees or 1.4274 radians

(b) ν_f = photon final (after collision) frequency. Unit s^{-1}

Example: $6.6532 \times 10^{19} s^{-1}$

(c) p_e = electron momentum (after collision). Units $kgms^{-1}$

Example: $29.1083 \times 10^{-23} kgms^{-1}$

Output:

(a) $p_e \cos \phi / (h\nu_i / c)$ No units. Value between 0 and 1

Example: 0.9231

h and c are constants. ϕ and ν_i are given

we need to derive p_e from p_e^2 but for p_e^2 we first need to derive ν_f and for ν_f we need to derive θ .

Formulae:

*===== θ | start.

Source (2). [Compton scattering - Wikipedia](https://en.wikipedia.org/wiki/Compton_scattering)

{https://en.wikipedia.org/wiki/Compton_scattering (28th May 2026.)

Equation 4: $\cot \phi = [1 + (h\nu_i / m_e c^2)] \times \tan(\theta/2)$

Rearrangement of $\cot \phi = [1 + (h\nu_i / m_e c^2)] \times \tan(\theta/2)$ with:

$\cot = \cotangent$

$\tan = tangent$

$\text{atan} = arctangent$

$\tan(\theta/2) = \cot \phi / (1 + [(h\nu_i) / (m_e c^2)])$

$\theta/2 = \text{atan}\{\cot \phi / (1 + [(h\nu_i) / (m_e c^2)])\}$

$\theta = 2 \times \text{atan}\{\cot \phi / (1 + [(h\nu_i) / (m_e c^2)])\}$

*===== θ | end.

*===== ν_f | start.

The formula for ν_f comes from Reference 1. Compton equations.

Page 231 (j) Equation 8.16: $\nu_f/\nu_i = 1/[1 + \gamma(1 - \cos\theta)]$

$$\nu_f = \nu_i / \{1 + [(h\nu_i/m_e c^2) \times (1 - \cos\theta)]\}$$

Divide top and bottom by ν_i :

$$\nu_f = 1 / \{(1/\nu_i) + [(h/m_e c^2) \times (1 - \cos\theta)]\}$$

Multiply top and bottom by c :

$$\nu_f = c / \{(c/\nu_i) + [(h/m_e c) \times (1 - \cos\theta)]\}$$

$$\nu_f = c / \{(h/m_e c) \times (1 - \cos\theta) + (c/\nu_i)\}$$

*===== ν_f | end.

*===== p_e^2 and p_e – (need ν_i and ν_f) | start.

The formula for p_e^2 is found in Reference 1. Compton equations (j).

Source (1). Electromagnetic Radiation F. H. Read, ISBN 0 471 27714 2

$$p_e^2 = [(h\nu_i - h\nu_f + m_e c^2)^2 - m_e^2 c^4] / c^2$$

$$p_e = +\sqrt{[(h\nu_i - h\nu_f + m_e c^2)^2 - m_e^2 c^4] / c^2} \quad \{+ \text{ implies take the positive square root}\}$$

*===== p_e^2 and p_e – (need ν_i and ν_f) | end.

Overview of calculating the angle ϕ between the deflected electron and the direction of the incoming vacuum photon | start.

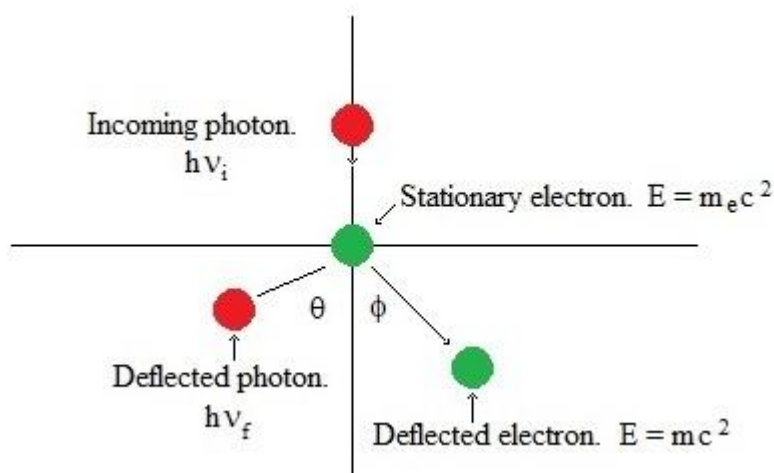


Figure: Data points 1.

The analysis in this section is for the 4 equal areas of the maximum cross-section of the sphere as shown in the 'Plan view' of the next diagram {Figure: Data points 2.}.

We need to calculate ϕ at points where it is midway (by area) in the circle of radius b and midway (by area) in the outer annulus defined by the radii b to d .

The 2 'mid' points (by cross-sectional area) are at ' a ' and ' c ' in the plan and elevation views below.

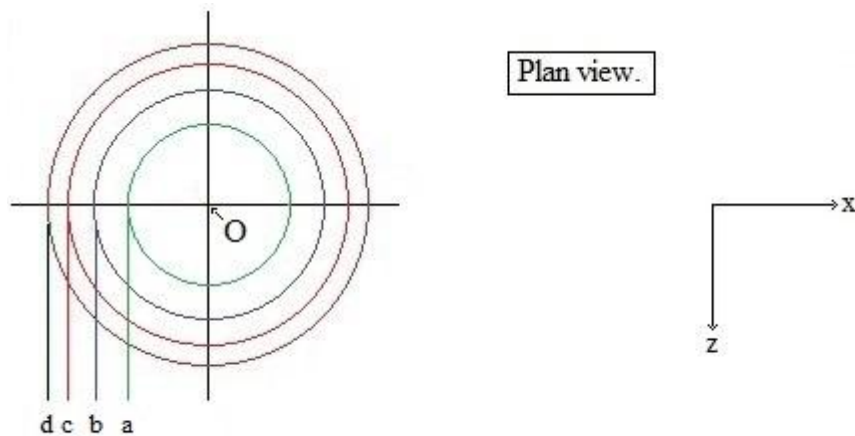


Figure: Data points 2.

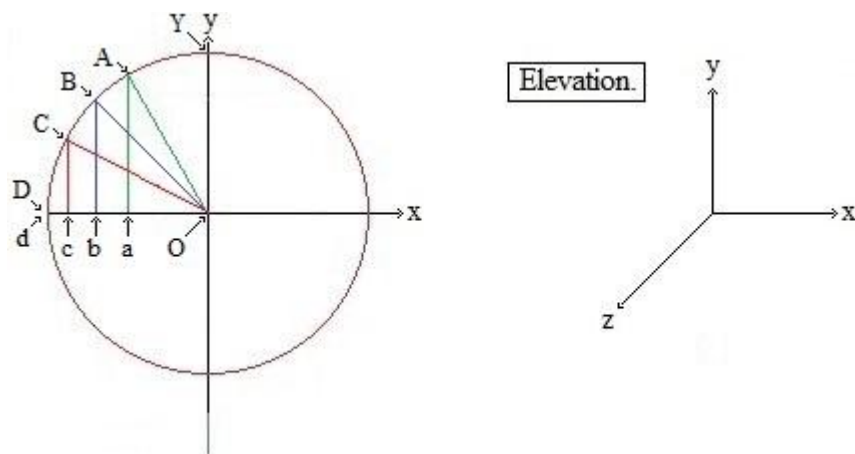


Figure: Data points 3.

Overview of calculating the angle ϕ between the deflected electron and the direction of the incoming vacuum photon | end.

*===== Points | start.

With O at the origin and O to d = 1 we can simplify the calculations because the distances are proportional.

We need 4 equal cross-sectional areas delimited by radii, a,b,c,d about a centre labelled O.

Set d = 1 (maximum radius).

Inner circle radius: a has area πa^2

For equal cross-sectional areas we need each circular annulus to equal the inner circle.

Circular annulus between radii: b and a has area $\pi b^2 - \pi a^2 = \pi a^2$

$b^2 - a^2 = a^2 \Rightarrow b^2 = 2a^2 \Rightarrow b = (\sqrt{2})a$ {assuming a positive root}

Next circular annulus between radii: c and b: Area $\pi c^2 - \pi b^2 = \pi a^2$

$c^2 - b^2 = a^2 \Rightarrow c^2 = b^2 + a^2 \Rightarrow c^2 = 2a^2 + a^2 \Rightarrow c^2 = 3a^2 \Rightarrow$

$c = (\sqrt{3})a$ {assuming a positive root}

Next circular annulus between radii: d and c: Area $\pi d^2 - \pi c^2 = \pi a^2$

$d^2 - c^2 = a^2 \Rightarrow d^2 = c^2 + a^2 \Rightarrow d^2 = 3a^2 + a^2 \Rightarrow d^2 = 4a^2 \Rightarrow$

$d = (\sqrt{4})a$ {assuming a positive root}

If we use d = 1 as the full radius this simplifies the calculations.

{d = $(\sqrt{4})a$ } \Rightarrow {1 = $(\sqrt{4})a$ } \Rightarrow {a = $1/(\sqrt{4})$ }

*----- Summary of the 4 points | start.

$$a = \sqrt{1}/\sqrt{4} = 0.5000$$

$$b = \sqrt{2}/\sqrt{4} = 0.7071$$

$$c = \sqrt{3}/\sqrt{4} = 0.8660$$

$$d = \sqrt{4}/\sqrt{4} = 1.0000$$

*----- Summary of the 4 points | end.

*===== Points | end.

*===== Angles | start.

Using Figure: Data points 3.

$$AO = BO = CO = DO = YO = 1$$

$$aO = a = (\sqrt{1})a = (\sqrt{1}) \times (1/\sqrt{4}) = (\sqrt{1})/2 = 0.5000$$

$$\cos(\text{angle } aOA) = aO/AO = aO = 0.5000$$

angle aOA = $\text{acos}(0.5000) = 1.0472$ radians = 60 degrees
angle YOA = $(90 - 60) = 30$ degrees

$bO = b = (\sqrt{2})a = (\sqrt{2}) \times (1/\sqrt{4}) = (\sqrt{2})/2 = 0.7071$
 $\text{cos}(\text{angle } bOB) = bO/BO = bO = 0.7071$
angle bOB = $\text{acos}(0.7071) = 0.7854$ radians = 45 degrees
angle YOB = $(90 - 45) = 45$ degrees

$cO = c = (\sqrt{3})a = (\sqrt{3}) \times (1/\sqrt{4}) = (\sqrt{3})/2 = 0.8660$
 $\text{cos}(\text{angle } cOC) = cO/CO = cO = 0.8660$
angle cOC = $\text{acos}(0.8660) = 0.5236$ radians = 30 degrees
angle YOC = $(90 - 30) = 60$ degrees

$dO = d = (\sqrt{4})a = (\sqrt{4}) \times (1/\sqrt{4}) = (\sqrt{4})/2 = 1.0000$
 $\text{cos}(\text{angle } dOD) = dO/DO = dO = 1.0000$
angle dOD = $\text{acos}(1.0000) = 0.0000$ radians = 0 degrees
angle YOD = $(90 - 0) = 90$ degrees
*===== Angles | end.

*===== Summary of angles | start.

angle YOA = $(90 - 60) = 30$ degrees
angle YOB = $(90 - 45) = 45$ degrees
angle YOC = $(90 - 30) = 60$ degrees
angle YOD = $(90 - 0) = 90$ degrees

*===== Summary of angles | end.

We need calculations to derive $p_e \cos\phi / (h\nu_i/c)$ as the ratio of momentum input to momentum output for the 2 positions above: a ($\phi = 30$ degrees) and c ($\phi = 60$ degrees) (on the -x axis in the elevation diagram (Figure: Data points 3.) above). The method is shown in the calculations above in this section of the paper.

Above a in the diagram: $\phi =$ the angle YOA = $(90 - 60) = 30$ degrees

Above c in the diagram: $\phi =$ the angle YOC = $(90 - 30) = 60$ degrees

Photon frequency | start.

From $(E = m_e c^2 = h\nu_i)$ we obtain the photon frequency equivalent to the rest mass of an electron as: $\nu_i = m_e c^2 / h = 12.3559 \times 10^{19} \text{ s}^{-1}$

To derive straightforward calculations that approximate to the Fine structure constant we can use the incoming photon 'mass' as double the electron mass with frequency equivalent to

Photon frequency: $2\nu_i = 24.7118 \times 10^{19} \text{ s}^{-1}$

Photon frequency | end.

Calculations with $\phi = 30$ degrees | start.

$$\text{Using } 2v_i = 2 \times 12.3559 \times 10^{19} \text{ s}^{-1} = 24.7118 \times 10^{19} \text{ s}^{-1}$$

Angle YOA = $\phi = 30$ degrees gives the value (calculated just below here) to put at the position [A] that gives the value of $p_e \cos \phi / (h\nu_i/c) = 0.7500$ which is used as the average momentum ratio for the cross-section circling O with radius Ob with 'mid'-point (by circular area) [a] on the negative x axis in Figure: Data points 3.

Here are the calculations:

$$\theta = 2 \times \text{atan}\{\cot \phi / (1 + [(h\nu_i)/(m_e c^2)])\} = 1.0472 \text{ radians}$$

$$v_f = c / \{[(h/m_e c) \times (1 - \cos \theta)] + (c/v_i)\} = 12.3567 \times 10^{19} \text{ s}^{-1}$$

$$p_e^2 = [(h\nu_i - h\nu_f + m_e c^2)^2 - m_e^2 c^4] / c^2 = 2237.1955 \times 10^{-46} \text{ (kg}^2 \text{m}^2 \text{s}^{-2}\text{)}$$

$$p_e = \sqrt{p_e^2} = 47.2990 \times 10^{-23} \text{ kgms}^{-1}$$

$$p_e \cos \phi = 40.9621 \times 10^{-23} \text{ kgms}^{-1}$$

$$h\nu_i/c = 54.6185 \times 10^{-23} \text{ kgms}^{-1}$$

$$p_e \cos \phi / (h\nu_i/c) = 0.7500 \text{ No units}$$

Calculations with $\phi = 30$ degrees | end.

Calculations with $\phi = 60$ degrees | start.

$$\text{Using } 2v_i = 2 \times 12.3559 \times 10^{19} \text{ s}^{-1} = 24.7118 \times 10^{19} \text{ s}^{-1}$$

Angle YOC = $\phi = 60$ degrees gives the value of ϕ to put at the position [C] that gives the value of $p_e \cos \phi / (h\nu_i/c) = 0.1875$ which is used as the average momentum ratio for the cross-sectional annulus with radii Ob and Od with 'mid'-point (by circular area) [c] on the negative x axis in Figure: Data points 3.

With the following calculations:

$$\theta = 2 \times \text{atan}\{\cot \phi / (1 + [(h\nu_i)/(m_e c^2)])\} = 0.3803 \text{ radians}$$

$$v_f = c / \{[(h/m_e c) \times (1 - \cos \theta)] + (c/v_i)\} = 21.6232 \times 10^{19} \text{ s}^{-1}$$

$$p_e^2 = [(h\nu_i - h\nu_f + m_e c^2)^2 - m_e^2 c^4] / c^2 = 419.4579 \times 10^{-46} \text{ (kg}^2 \text{m}^2 \text{s}^{-2}\text{)}$$

$$p_e = \sqrt{p_e^2} = 20.4807 \times 10^{-23} \text{ kgms}^{-1}$$

$$p_e \cos \phi = 10.2403 \times 10^{-23} \text{ kgms}^{-1}$$

$$h\nu_i/c = 54.6185 \times 10^{-23} \text{ kgms}^{-1}$$

$$p_e \cos \phi / (h\nu_i/c) = 0.1875 \text{ No units}$$

Calculations with $\phi = 60$ degrees | end.

Momentum ratios | start.

$$\text{Angle YOA } \phi = 30 \text{ degrees gives the value of } p_e \cos \phi / (h\nu_i/c) = 0.7500$$

$$\text{Angle YOC } \phi = 60 \text{ degrees gives the value of } p_e \cos \phi / (h\nu_i/c) = 0.1875$$

So the average momentum transfer (per unit area of the horizontal x-z plane cross-section) from the photon to the electron is the average (momentum out)/(momentum in) =

$$[p_e \cos \phi / (h\nu_i/c)] = (0.7500 + 0.1875)/2 = 0.4687 = 1/2.1336$$

*-----

$1/2.1336 \times 1/8 = 1/17.0685 \rightarrow$ approximately the Coulomb interaction.

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$1/17.0685 \times 1/8 = 1/136.5479 \rightarrow$ approximately the Fine structure constant $1/137.035999$.

The difference between this calculation and the fine structure constant is 0.3562%

By using more angles we can get more precise figures.

We can derive the target figure for the Fine structure constant by changing the multiplier k on the initial photon frequency ν_i from 2.0000 to 2.0190.

To get a more accurate picture of the spread of the results of the photon collisions we need more input points at different angles ϕ . We then see the possibility of photons with integral frequencies equivalent to 4 electron masses as well as equivalent to 2 electron masses and the chance that a 4 electron mass system is a double (2 electron mass) photon entity.

Momentum ratios | end.

*----- Calculations to derive figures for the 2 data points at: $\phi = 30$ degrees and $\phi = 60$ degrees at 2 average points (over each double part of the cross-sectional areas) in 4 equal cross-sectional areas | end.

Calculations with 10 angles. | start.

We can take a look at a cross-sectional area with 20 equal areas and 10 midway (by area) points at $\sqrt{1}/\sqrt{20}$, $\sqrt{3}/\sqrt{20}$, $\sqrt{5}/\sqrt{20}$, $\sqrt{7}/\sqrt{20}$, $\sqrt{9}/\sqrt{20}$, $\sqrt{11}/\sqrt{20}$, $\sqrt{13}/\sqrt{20}$, $\sqrt{15}/\sqrt{20}$, $\sqrt{17}/\sqrt{20}$, $\sqrt{19}/\sqrt{20}$ of the radius ($= 1$) at right angles to the incoming photon - along the x axis of the sphere of the electron.

Average of 10 $p_e \cos \phi / (h\nu_i/c)$ with multiplication factor on photons' $\nu_i = 2.2030$ is 0.4670 where 0.4670 is the target for the first part of the Fine Structure Constant. $\{1/2.1412 \{0.4670\}$ is calculated using the Compton scattering equations and is the first part of the three parts of the Fine Structure Constant: $1/2.1412 \times 1/8 \times 1/8\}$.

We can also have photons operating with integral multiplication factors on ν_i such as:

Average of 10 $p_e \cos \phi / (h\nu_i/c)$ with multiplication factor on $\nu_i = 2$ is 0.4831

Average of 10 $p_e \cos \phi / (h\nu_i/c)$ with multiplication factor on $\nu_i = 4$ is 0.3716

Average of these ratios is 0.4274

We then:

Assume: probability of interaction for multiplication factor on $\nu_i = 2$ is 0.8560

Assume: probability of interaction for multiplication factor on $v_i = 4$ is 0.1440
 $0.8560 + 0.1440 = 1.0000$ (total probability of interaction)

$(0.4831 \times 0.8560) + (0.3716 \times 0.1440) = 0.4670$
where 0.4670 is the target for the first part of the Fine Structure Constant.

$0.8560/0.1440 = 5.9444$ to 1 which is an approximate ratio of 6 to 1 for the ratio of photons with (multiplication factor $v_i = 2$) to (multiplication factor $v_i = 4$) involved in the interactions of 1 electron to another electron.

This suggests a vacuum photon momentum ratio for the multiplication factors of ($v_i = 2 / (v_i = 4)$) of 6 to 1.

Calculations with 10 angles. | end.

Calculations with 20 angles. | start.

We can take a look at a cross-sectional area with 40 equal areas and 20 midway (by area) points at $\sqrt{1}/\sqrt{40}, \sqrt{3}/\sqrt{40}, \sqrt{5}/\sqrt{40}, \sqrt{7}/\sqrt{40}, \sqrt{9}/\sqrt{40}, \sqrt{11}/\sqrt{40}, \sqrt{13}/\sqrt{40}, \sqrt{15}/\sqrt{40}, \sqrt{17}/\sqrt{40}, \sqrt{19}/\sqrt{40}, \sqrt{21}/\sqrt{40}, \sqrt{23}/\sqrt{40}, \sqrt{25}/\sqrt{40}, \sqrt{27}/\sqrt{40}, \sqrt{29}/\sqrt{40}, \sqrt{31}/\sqrt{40}, \sqrt{33}/\sqrt{40}, \sqrt{35}/\sqrt{40}, \sqrt{37}/\sqrt{40}, \sqrt{39}/\sqrt{40}$ of the radius ($= 1$) - at right angles to the incoming photon - along the x axis of the sphere of the electron.

Average of $20 p_e \cos\phi / (h\nu_i/c)$ with multiplication factor on photons' $v_i = 2.2100$ is 0.4670 where 0.4670 is the target for the Fine Structure Constant.

We can also have photons operating with integral multiplication factors on v_i such as:

Average of $20 p_e \cos\phi / (h\nu_i/c)$ with multiplication factor on $v_i = 2$ is 0.4836

Average of $20 p_e \cos\phi / (h\nu_i/c)$ with multiplication factor on $v_i = 4$ is 0.3724

Average of these is 0.4280

We then:

Assume probability of interaction for multiplication factor on $v_i = 2$ is 0.8510

Assume probability of interaction for multiplication factor on $v_i = 4$ is 0.1490

$0.8510 + 0.1490 = 1.0000$ (total probability of interaction)

$(0.4836 \times 0.8510) + (0.3724 \times 0.1490) = 0.4670$

where 0.4670 is the target for the Fine Structure Constant.

$0.8510/0.1490 = 5.7114$ to 1 for (multiplication factor $v_i = 2$) to (multiplication factor $v_i = 4$) involved in the interactions of 1 electron to another electron.

This suggests a vacuum photon momentum ratio for the multiplication factors of ($v_i = 2 / v_i = 4$) of 5.7114 to 1.

Calculations with 20 angles. | end.

Calculations with 40 angles. | start.

We can take a look at a cross-sectional area with 80 equal areas and 40 midway (by area) points at $\sqrt{1}/\sqrt{80}, \sqrt{3}/\sqrt{80}, \sqrt{5}/\sqrt{80}, \sqrt{7}/\sqrt{80}, \sqrt{9}/\sqrt{80}, \sqrt{11}/\sqrt{80}, \sqrt{13}/\sqrt{80}, \sqrt{15}/\sqrt{80}, \sqrt{17}/\sqrt{80}, \sqrt{19}/\sqrt{80}, \sqrt{21}/\sqrt{80}, \sqrt{23}/\sqrt{80}, \sqrt{25}/\sqrt{80}, \sqrt{27}/\sqrt{80}, \sqrt{29}/\sqrt{80}, \sqrt{31}/\sqrt{80}, \sqrt{33}/\sqrt{80}, \sqrt{35}/\sqrt{80}, \sqrt{37}/\sqrt{80}, \sqrt{39}/\sqrt{80}, \sqrt{41}/\sqrt{80}, \sqrt{43}/\sqrt{80}, \sqrt{45}/\sqrt{80}, \sqrt{47}/\sqrt{80}, \sqrt{49}/\sqrt{80}, \sqrt{51}/\sqrt{80}, \sqrt{53}/\sqrt{80}, \sqrt{55}/\sqrt{80}, \sqrt{57}/\sqrt{80}, \sqrt{59}/\sqrt{80}, \sqrt{61}/\sqrt{80}, \sqrt{63}/\sqrt{80}, \sqrt{65}/\sqrt{80}, \sqrt{67}/\sqrt{80}, \sqrt{69}/\sqrt{80}, \sqrt{71}/\sqrt{80}, \sqrt{73}/\sqrt{80}, \sqrt{75}/\sqrt{80}, \sqrt{77}/\sqrt{80}, \sqrt{79}/\sqrt{80}$ of the radius ($= 1$) - at right angles to the incoming photon - along the x axis of the sphere of the electron.

Average of 40 $p_e \cos \phi / (h\nu_i/c)$ with multiplication factor on photons' $\nu_i = 2.2110$ is 0.4670 where 0.4670 is the target for the Fine Structure Constant.

We can also have photons operating with integral multiplication factors on ν_i such as:

Average of 40 $p_e \cos \phi / (h\nu_i/c)$ with multiplication factor on $\nu_i = 2$ is 0.4837

Average of 40 $p_e \cos \phi / (h\nu_i/c)$ with multiplication factor on $\nu_i = 4$ is 0.3726

Average of these is 0.4282

We then:

Assume probability of interaction for multiplication factor on $\nu_i = 2$ is 0.8500

Assume probability of interaction for multiplication factor on $\nu_i = 4$ is 0.1500

$0.8500 + 0.1500 = 1.0000$ (total probability of interaction)

$(0.4837 \times 0.8500) + (0.3726 \times 0.1500) = 0.4670$

where 0.4670 is the target for the Fine Structure Constant.

$0.8500/0.1500 = 5.6667$ to 1 multiplication factor $\nu_i = 2$ to multiplication factor $\nu_i = 4$ involved in the interactions of 1 electron to another electron.

This suggests a vacuum photon momentum ratio for the multiplication factors of ($\nu_i = 2/ \nu_i = 4$) of 5.6667 to 1 which is $\{5^2/3 = 17/3\}$.

Calculations with 40 angles. | end.

Calculations with 80 angles. | start.

Take a look at a cross-sectional area with 160 equal areas and 80 midway (by area) points at $\sqrt{1}/\sqrt{160}, \sqrt{3}/\sqrt{160}, \sqrt{5}/\sqrt{160}, \sqrt{7}/\sqrt{160}, \sqrt{9}/\sqrt{160}, \sqrt{11}/\sqrt{160}, \sqrt{13}/\sqrt{160}, \sqrt{15}/\sqrt{160}, \sqrt{17}/\sqrt{160}, \sqrt{19}/\sqrt{160}, \sqrt{21}/\sqrt{160}, \sqrt{23}/\sqrt{160}, \sqrt{25}/\sqrt{160}, \sqrt{27}/\sqrt{160}, \sqrt{29}/\sqrt{160}, \sqrt{31}/\sqrt{160}, \sqrt{33}/\sqrt{160}, \sqrt{35}/\sqrt{160}, \sqrt{37}/\sqrt{160}, \sqrt{39}/\sqrt{160}, \sqrt{41}/\sqrt{160}, \sqrt{43}/\sqrt{160}, \sqrt{45}/\sqrt{160}, \sqrt{47}/\sqrt{160}, \sqrt{49}/\sqrt{160}, \sqrt{51}/\sqrt{160}, \sqrt{53}/\sqrt{160}, \sqrt{55}/\sqrt{160}, \sqrt{57}/\sqrt{160}, \sqrt{59}/\sqrt{160}, \sqrt{61}/\sqrt{160}, \sqrt{63}/\sqrt{160}, \sqrt{65}/\sqrt{160}, \sqrt{67}/\sqrt{160}, \sqrt{69}/\sqrt{160}, \sqrt{71}/\sqrt{160}, \sqrt{73}/\sqrt{160}, \sqrt{75}/\sqrt{160}, \sqrt{77}/\sqrt{160}, \sqrt{79}/\sqrt{160}, \sqrt{81}/\sqrt{160}, \sqrt{83}/\sqrt{160}, \sqrt{85}/\sqrt{160}, \sqrt{87}/\sqrt{160}, \sqrt{89}/\sqrt{160}, \sqrt{91}/\sqrt{160}, \sqrt{93}/\sqrt{160}, \sqrt{95}/\sqrt{160}, \sqrt{97}/\sqrt{160}, \sqrt{99}/\sqrt{160}, \sqrt{101}/\sqrt{160}, \sqrt{103}/\sqrt{160}, \sqrt{105}/\sqrt{160}, \sqrt{107}/\sqrt{160}, \sqrt{109}/\sqrt{160}, \sqrt{111}/\sqrt{160}, \sqrt{113}/\sqrt{160},$

$\sqrt{115}/\sqrt{160}, \sqrt{117}/\sqrt{160}, \sqrt{119}/\sqrt{160}, \sqrt{121}/\sqrt{160}, \sqrt{123}/\sqrt{160}, \sqrt{125}/\sqrt{160}, \sqrt{127}/\sqrt{160},$
 $\sqrt{129}/\sqrt{160}, \sqrt{131}/\sqrt{160}, \sqrt{133}/\sqrt{160}, \sqrt{135}/\sqrt{160}, \sqrt{137}/\sqrt{160}, \sqrt{139}/\sqrt{160}, \sqrt{141}/\sqrt{160},$
 $\sqrt{143}/\sqrt{160}, \sqrt{145}/\sqrt{160}, \sqrt{147}/\sqrt{160}, \sqrt{149}/\sqrt{160}, \sqrt{151}/\sqrt{160}, \sqrt{153}/\sqrt{160}, \sqrt{155}/\sqrt{160},$
 $\sqrt{157}/\sqrt{160}, \sqrt{159}/\sqrt{160}$ of the radius ($= 1$) - at right angles to the incoming photon - along the x axis of the sphere of the electron.

Average of $80 p_e \cos\phi / (h\nu_i/c)$ with multiplication factor on photons' $\nu_i = 2.2110$
is 0.4670 where 0.4670 is the target for the Fine Structure Constant.
 $\nu_i = 2.2110$ is the same as for 40 points.

We can also have photons operating with integral multiplication factors on ν_i such as:

Average of $80 p_e \cos\phi / (h\nu_i/c)$ with multiplication factor on $\nu_i = 2$ is 0.4837

Average of $80 p_e \cos\phi / (h\nu_i/c)$ with multiplication factor on $\nu_i = 4$ is 0.3727

Average of both is 0.4282

We then:

Assume probability of interaction for multiplication factor on $\nu_i = 2$ is 0.8491

Assume probability of interaction for multiplication factor on $\nu_i = 4$ is 0.1509

$0.8491 + 0.1509 = 1.0000$ (total probability of interaction)

$(0.4837 \times 0.8491) + (0.3727 \times 0.1509) = 0.4670$

where 0.4670 is the target for the Fine Structure Constant.

$0.8491/0.1509 = 5.6269$ to 1 multiplication factor $\nu_i = 2$ to multiplication factor $\nu_i = 4$ involved in the interactions of 1 electron to another electron.

This suggests a vacuum photon momentum ratio for the multiplication factors of ($\nu_i = 2/ \nu_i = 4$) of 5.6269 to 1.

Calculations with 80 angles. | end.

Conclusion. | start.

Either we can use a photon frequency of $2.2110 \times$ the base frequency ($12.3559 \times 10^{19} \text{ s}^{-1}$) to obtain 0.4670 which is the target for the Fine Structure Constant;

or

separate calculations with $\nu_i = 2 \times$ and $\nu_i = 4$ in the ratio $0.8491/0.1509 = 5.6269$ to 1 to obtain 0.4670 which is the target for the Fine Structure Constant.

* ----- Unanswered question | start.

Are there other ratios different from $\nu_i = 2, 4$ with higher integers that comply with the Fine structure constant?

* ----- Unanswered question | end.

Conclusion. | end.

Data points | end.

Electron - kinetic picture. Electron 'radius'. Compton wavelength | start.

Electron - kinetic picture | start.

What happens if we compare an electron to a classical system? An electron may be represented as a particle of mass m_e , travelling at light speed c , and held by elastic collisions from particles in the vacuum in an approximately circular orbit of radius r with angular momentum (spin = $\frac{1}{2}$) of $m_e c r = h/4\pi$ where h = Planck's constant.

In quantum mechanics, spin is referred to as intrinsic angular momentum which is regarded as different from classical angular momentum.

An electron is a fermion, that is a half spin particle {half-integer spin (spin quantum number = $\frac{1}{2}$)}

with spin (intrinsic angular momentum) = $\frac{1}{2} \times$ Planck's constant (h) divided by $2\pi = (h/4\pi)$.

Electric charge may be regarded as an electron's ability to redirect vacuum momentum, possibly vacuum photons, outwards and to be the receiver of vacuum momentum.

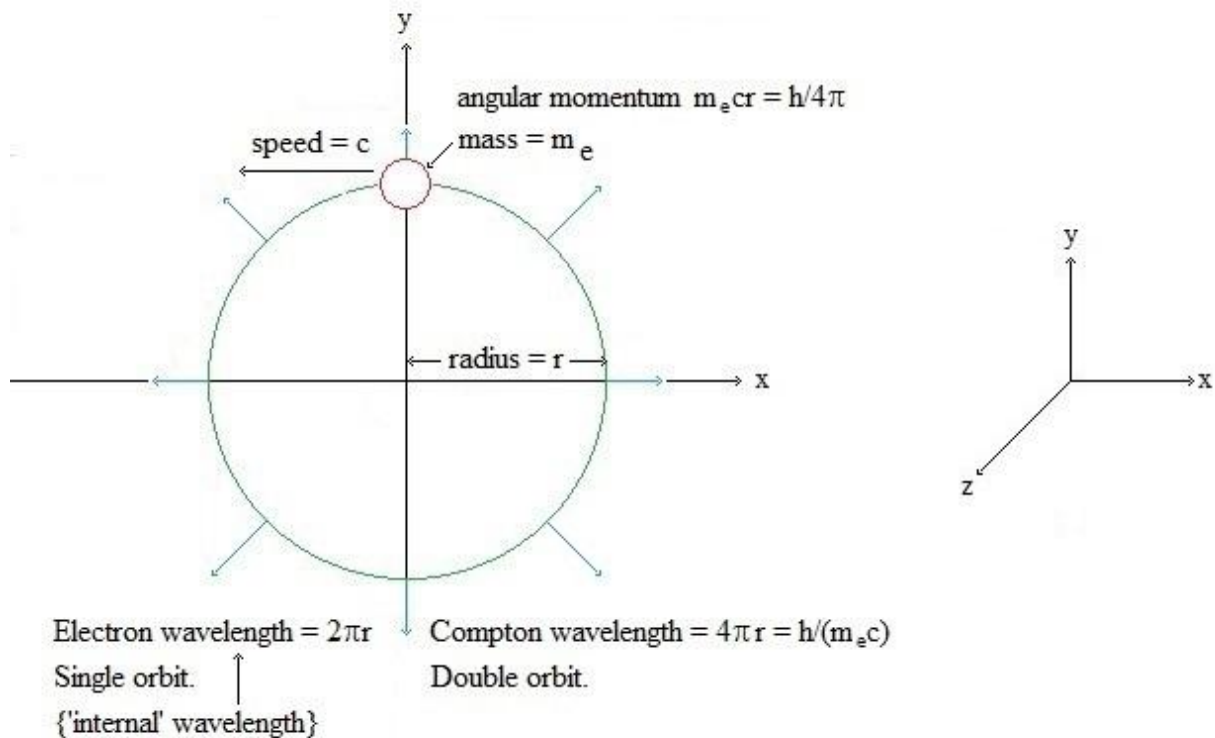


Figure: Electron - kinetic picture.

We can see in this schematic picture of the 'electron' its diffuse and dual nature - being both a particle and having a wavelength: wave-particle duality. This helps to explain why it is not possible to specify an exact location for an electron.

*----- Unanswered question | start.

Can we assume that a key part of the electron is always 'pointing out' towards the containing momentum and that this is where the contact is made by the vacuum momentum on the electron? The frequency of the orbit is then equal to the internal 'rotational' frequency of the electron.

*----- Unanswered question | end.

Electron - kinetic picture | end.

Electron self-orbital 'radius' | start.

We can calculate the radius r , of the orbit of the 'electron' from its angular momentum {mass of electron (m_e) x speed of light (c) x radius of orbit (r) equal to Planck's constant (h) divided by (4π):

Angular momentum: $m_e c r = h/4\pi$

Planck's constant: $h = 6.6261 \times 10^{-34} \text{ kgm}^2\text{s}^{-1}$

Mass of the electron, $m_e = 9.1094 \times 10^{-31} \text{ kg}$

Speed of light (in a vacuum), $c = 2.9979 \times 10^8 \text{ ms}^{-1}$

Angular momentum of the electron: $m_e c r = h/4\pi$

Therefore: $r = h/(4\pi m_e c)$

$r \sim (6.6261 \times 10^{-34} \text{ kgm}^2\text{s}^{-1}) / (4 \times 3.1416 \times 9.1094 \times 10^{-31} \text{ kg} \times 2.9979 \times 10^8 \text{ ms}^{-1})$

Radius of the electron's self-orbit $r = 1.9308 \times 10^{-13} \text{ m}$

This is between the nuclear radius of $\sim 10^{-15} \text{ m}$ (femtometer) and the atomic radius of $\sim 10^{-10} \text{ m}$

Electron self-orbital 'radius' | end.

Compton wavelength | start.

The Compton wavelength is equal to twice the circumference of the electron's 'self' orbit.

Compton's equations have been used earlier in this paper to calculate the amount of momentum transferred from the vacuum photons to the electron.

Compton wavelength | end.

Electron - kinetic picture. Electron 'radius'. Compton wavelength | end.

=====

Deriving the fraction 1/8 | start.

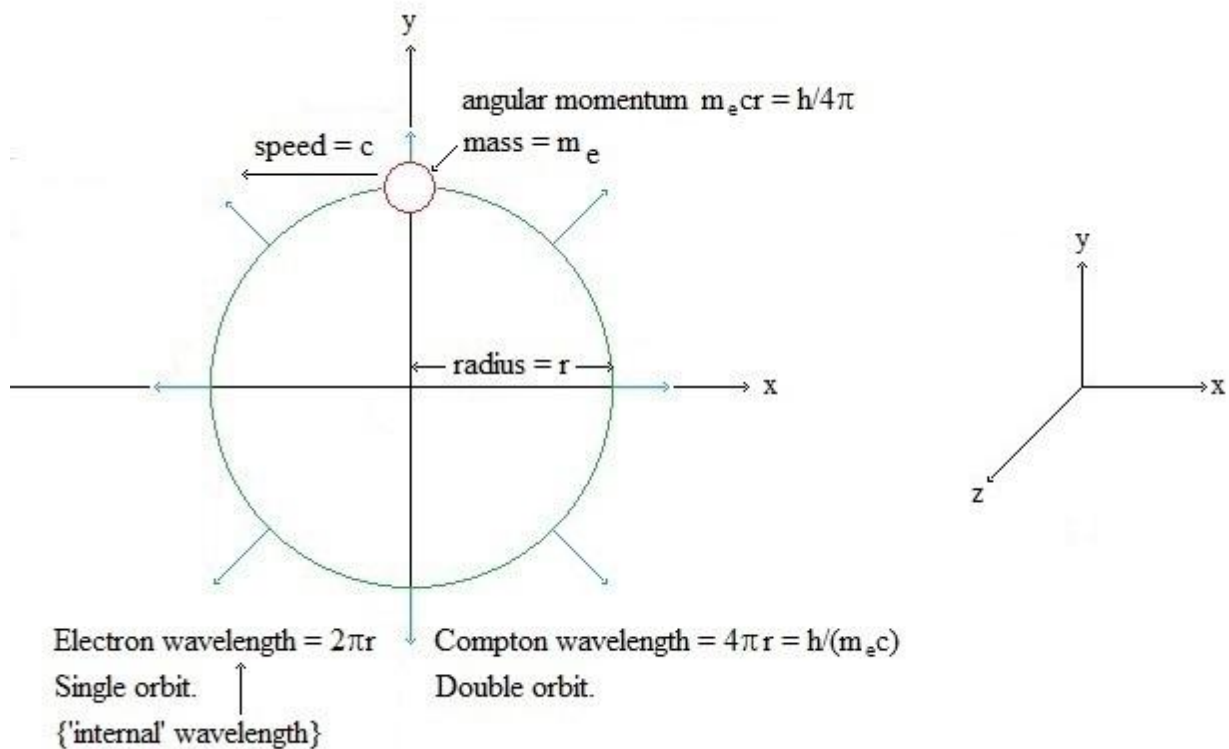


Figure: Deriving the fraction 1/8 (Figure 1).

*----- Contents | start.

Electron kinetics. Photon/electron interaction. Deriving the fraction 1/8.

*----- Contents | end.

Please note: Here we are applying the Compton equations to the electron as it travels in its approximately circular self-orbit as in the above section 'Electron - kinetic picture' and not just to a 'stationary' electron.

We need to do this to derive the probability ratio of 1/4 (cross section/surface area) from the geometry of a sphere.

An electron may be compared to a constrained particle moving about a fixed centre in three dimensions at a distance r from the centre (Figure: Deriving the fraction 1/8 (Figure 1). above). Call this constrained particle E.

The surface area of a sphere with radius r is $4\pi r^2$ and E is located somewhere on this area.

A photon may be compared to a rotating electron/positron pair travelling through space. Call this rotating, travelling pair P .

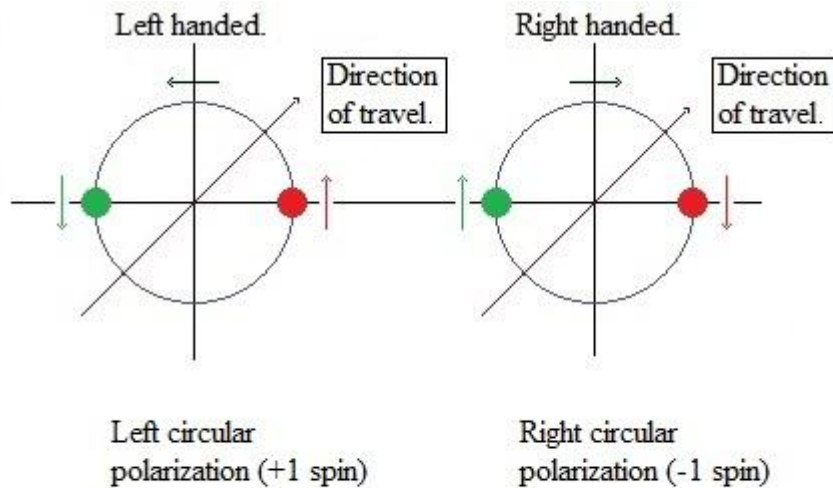


Figure: Deriving the fraction 1/8 (Figure 2).

In Figure: Deriving the fraction 1/8 (Figure 1) above assume that P travels somewhere down (in the $-y$ direction) through the cross-sectional area πr^2 in the xz plane surrounding the y axis as it interacts with the system containing E .

The actual location of particle E is somewhere on the surface of its sphere of average location with area $4\pi r^2$.

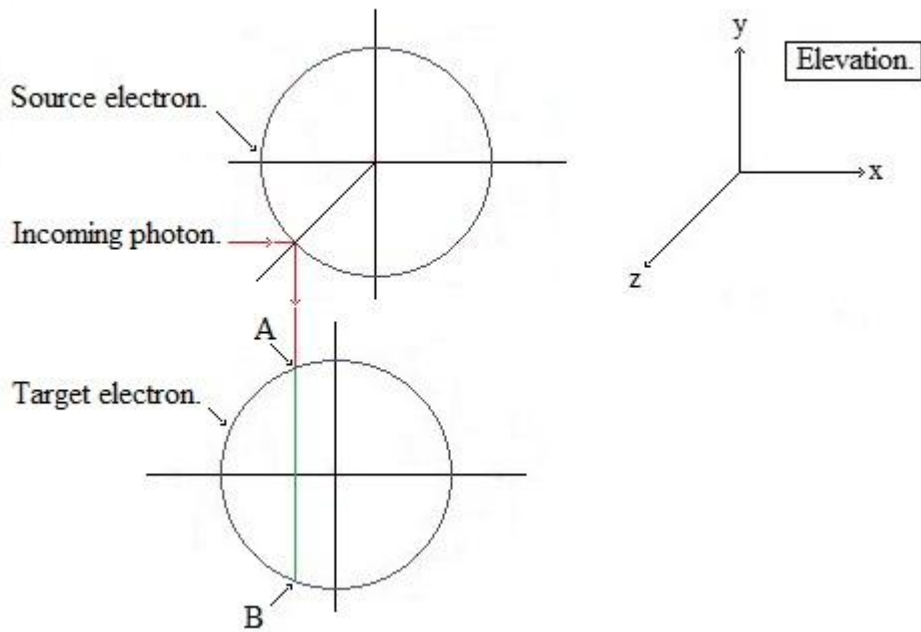
The chance of P interacting with E is proportional to $\pi r^2 / 4\pi r^2 = 1/4$

* ----- Further reduction to 1/8 | start.

Assume that an electron is contained in its 'self-orbit' by incoming photons in the sub-atomic vacuum.

We can picture a photon (redirected by a 'source' electron to a 'target' electron) as passing through a 'target' electron's self-orbit. There are 2 possible points of contact. Label the photon's first possible point of contact A and the second possible point of contact B .

Please see: Figure: Deriving the fraction 1/8 (Figure 3) on the next page.



Source and target electrons and contact points A and B.

Figure: Deriving the fraction 1/8 (Figure 3).

At the second point of contact B, the electron is contained by vacuum photons with components moving inwards in the opposite direction to the redirected photon. We assume that this collision does not affect the target electron's 'downward' momentum.

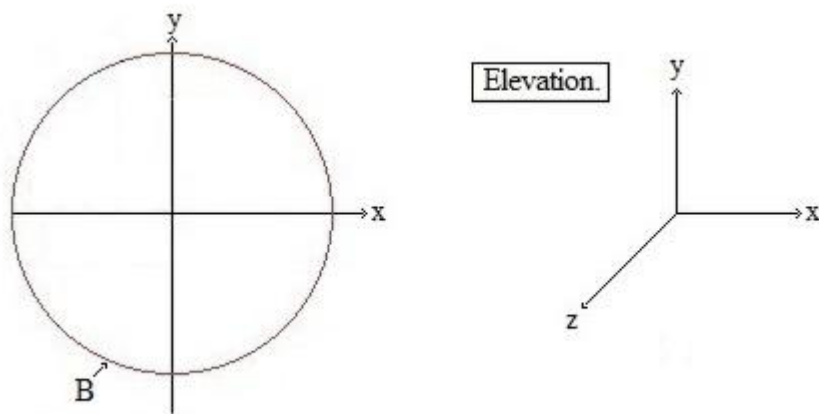


Figure: Deriving the fraction 1/8 (Figure 4).

At the point of contact A the redirected photon has the ability to interact with the electron and move it.

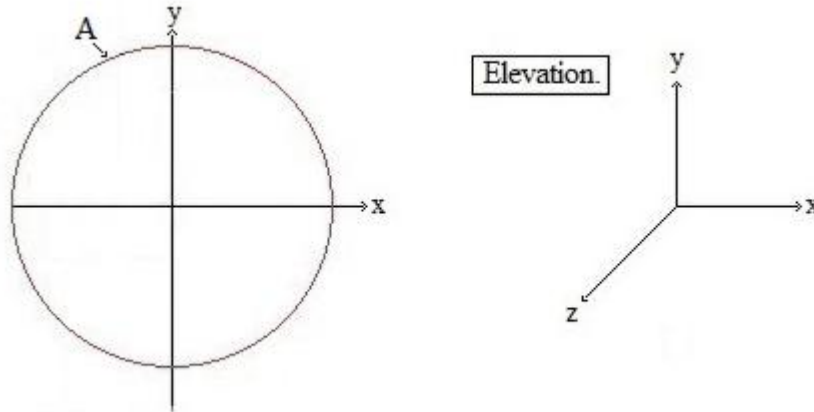


Figure: Deriving the fraction 1/8 (Figure 4).

If P can only interact with E when P's component (positive or negative) matches the compatible property of E (negative) at the point of collision then the probability of interaction is further reduced by $\frac{1}{2}$ bringing the overall probability of interaction to $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$.

E negative P positive Does not match: not compatible.

E negative P negative Does match: compatible.

or

E negative P positive Does match: compatible.

E negative P negative Does not match: not compatible.

* ----- Further reduction to 1/8 | end.

* ----- Unanswered question | start.

Can we assume that the physical size of the key component of the photon is the same as the physical size of the electron?

* ----- Unanswered question | end.

* ----- Conclusion. | start.

The containment interaction between a photon and an electron has a 1/8 probability.

* ----- Conclusion. | end.

* ----- Summary of ratios. | start.

In the 'Data points' section (pages 14 and 15) the ratio of $p_e \cos \phi / (h v_i / c)$ with a multiplication factor on photons' frequency $v_i = 2.2110$ is 0.4670.

$$0.4670 = 1/2.1412$$

*-----

$$1/2.1412 \times 1/8 = 1/17.1295$$

$(1/17.1295) \times \pi r^2$ (where r is the electron's radius of self-orbit) is Coulomb's Law.

Please see Reference 2 on pages 25 to 29.

*-----

$$1/17.1295 \times 1/8 = 1/137.0360 \leftarrow \text{this is the Fine Structure Constant.}$$

*----- Summary of ratios. | end.

*----- Additional. | start.

Notice that in this section the focus was on negative charges. The calculations may also apply to positive charges (for example positrons and protons) or a mixture of positive and negative charges (for example the hydrogen atom).

How does a target electron identify a photon as having come from a source electron and not from a source of positive charge? Can we assume that an electron responds to a photon because of its handedness (spin angular momentum).

For example:

an electron identifies with a left-handed spin photon; and
a positive charge identifies with a right-handed spin photon.

or

an electron identifies with a right-handed spin photon; and
a positive charge identifies with a left-handed spin photon.

*----- Additional. | end.

*----- Unanswered questions | start.

If the 'charge' on an electron is caused by a physical spin (angular momentum) and the charge on a positron is due to an opposite physical spin (angular momentum) then is conservation of electric charge merely a subset of conservation of angular momentum?

Can the source particle be within the self-orbit of the target particle?

*----- Unanswered questions | end.

Deriving the fraction 1/8 | end.

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*----- Reference 1 | start.

Reference 1. Compton equations.

*=====

Please note: m_0 is used in this reference book but m_e is used in this paper.

Source (1). Electromagnetic Radiation F. H. Read, ISBN 0 471 27714 2

m_0 = electron's rest mass. { m_e is used in this document for the electron's rest mass.}

m = electron's kinetic mass.

v = electron's velocity

T_e = electron's recoil kinetic energy (after collision)

p_e = electron momentum (after collision)

ν_i = photon initial (before collision) frequency

ν_f = photon final (after collision) frequency

θ = angle between the final direction of the photon and the initial direction of the photon

ϕ = angle between the final direction of the electron and the initial direction of the photon

$\gamma = h\nu_i/m_0c^2 = [\text{photon initial energy}] / [\text{electron (initial) rest mass energy}]$

λ_c = Compton wavelength of the electron

λ_i = initial wavelength of the photon

λ_f = final wavelength of the photon

Page 230 (a) Electron rest energy $E = m_e c^2$

Page 230 (b) Relativistic mass increase of an electron: $m = m_e / (1 - v^2/c^2)^{1/2}$

Page 230 (c) Total electron energy $E = mc^2$

Page 231 (d) Equation 8.12: with electron momentum p_e : $E^2 = m_e^2 c^4 + p_e^2 c^2$

Page 231 (e) Equation 8.13: Electron's recoil kinetic energy $T_e = (m - m_e)c^2 = h\nu_i - h\nu_f$

Page 231 (f) Equation 8.14: $h\nu_i/c = (h\nu_f/c)\cos\theta + p_e\cos\phi$

Page 231 (g) Equation 8.15: $(h\nu_f/c)\sin\theta = p_e\sin\phi$

Page 231 (h) $p_e^2 = (h\nu_i/c)^2 - 2(h\nu_i/c)(h\nu_f/c) + (h\nu_f/c)^2 + 2m_e h(\nu_i - \nu_f)/c$

Page 231 (i) $p_e^2 = (h\nu_i/c)^2 + (h\nu_f/c)^2 - 2(h\nu_i/c)(h\nu_f/c)\cos\theta$

Page 231 (j) Equation 8.16: $\nu_f/\nu_i = 1/[1 + \gamma(1 - \cos\theta)]$

$\nu_f = \nu_i / [1 + \gamma(1 - \cos\theta)]$ $\nu_f = \nu_i / [1 + (h\nu_i/m_e c^2)(1 - \cos\theta)]$

$\nu_f = c / \{ [(h/m_e c) \times (1 - \cos\theta)] + (c/\nu_i) \}$

Page 231 (k) Equation 8.17: $\gamma = h\nu_i/m_e c^2$

Page 231 (l) Equation 8.18: $\lambda_f - \lambda_i = (h/m_e c) \times (1 - \cos\theta)$

Page 231 (m) Equation 8.19: $\lambda_c = (h/m_e c) = 2.4263 \times 10^{-12} \text{ m}$ {Compton wavelength of the electron}.

Page 232 (n) Equation 8.20: $T_e = h\nu_i \{ [\gamma(1 - \cos\theta)] / [1 + \gamma(1 - \cos\theta)] \}$

Page 232 (o) Equation 8.21: $(T_e)_{\max} = h\nu_i [2\gamma / (1 + 2\gamma)]$

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*=====

Source (2). Compton scattering - Wikipedia

{https://en.wikipedia.org/wiki/Compton_scattering}

Equation 1: $p_e^2 c^2 = (h\nu_i - h\nu_f + m_e c^2)^2 - m_e^2 c^4$

The magnitude of the momentum gained by the electron (formerly zero) exceeds the energy/c lost by the photon:

$(1/c) [\sqrt{(h\nu_i - h\nu_f + m_e c^2)^2 - m_e^2 c^4}] > (h\nu_i - h\nu_f)/c$

Equation 2: $p_e^2 c^2 = (h\nu_i)^2 + (h\nu_f)^2 - 2h\nu_i h\nu_f \cos\theta$

Equation 3: $\lambda_f - \lambda_i = (h/m_e c) \times (1 - \cos\theta)$ [Same as Page 231 (l) Equation 8.18 above.]

Equation 4: $\cot\phi = [1 + (h\nu_i/m_e c^2)] \times \tan(\theta/2)$

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*----- Reference 1 | end.

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*----- Reference 2 | start.

<https://vixra.org/abs/1611.0237>

What is a photon? Photon kinetic and electromagnetic structure simplified and explained and how one photon can go through two different holes at the same time. 14th November 2016.

© Colin James 2016. Pages 46 and 47 out of 51.

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*----- Equivalence to Coulomb's Law | start.

Here are some derivations of the interaction cross-section σ (from the calculations in the paper: <https://vixra.org/abs/1611.0237>) to show that the value σ gives an equivalent to Coulomb's Law.

The outward momentum flow per unit time ($\text{kgms}^{-1}/\text{s} = \text{kgms}^{-2}$) (centrifugal force) from one electron (the source electron) is $m_e c^2/r$ ($\text{kgm}^2\text{s}^{-2}/\text{m} = \text{kgms}^{-2}$) where m_e is the rest mass of the electron, c is the speed of light in a vacuum and r is the radius of the electron's self-orbit.

*-----

Radius of the electron's self-orbit $r = 1.9308 \times 10^{-13} \text{ m}$

This is shown in the section: Electron - kinetic picture. Electron 'radius'. Compton wavelength. Sub-section: Electron orbital 'radius'. On page 17.

*-----

The outward momentum flow per unit time (force) spreads outwards from the electron and at a distance d from the electron the force extends over the surface of a spherical area such that the area density of the force is $(m_e c^2/r) / (4\pi d^2)$

The amount of the momentum flow intercepted by another electron (the target electron) is the force F , on the target electron and is characterized by an area (interaction cross-section) of size, σ

$$F = \sigma \times (m_e c^2/r) / (4\pi d^2)$$

σ is the fraction of momentum flow intercepted from one electron/positron to another electron/positron.

If we equate this to the electrostatic force between two electrons that are a distance d apart

$$F = (e^2/(4\pi\epsilon_0 d^2)) \text{ \{Coulomb's Law\}}$$

we obtain:

$$F = \sigma \times [(m_e c^2/r)/(4\pi d^2)] = (e^2/(4\pi\epsilon_0 d^2))$$

Multiplying both sides by $4\pi d^2$ gives:

$$\sigma \times (m_e c^2/r) = e^2/\epsilon_0$$

$$\sigma = (e^2 r)/(\epsilon_0 m_e c^2)$$

$$\sigma = (e^2 r^2)/(\epsilon_0 m_e c^2 r)$$

$$\sigma = (e^2 r^2)/(\epsilon_0 [m_e c r] c) \quad \text{using } m_e c r = h/4\pi \text{ as the angular momentum of the electron.}$$

$$\sigma = (e^2 r^2)/(\epsilon_0 [h/4\pi] c)$$

$$\sigma = 4\pi(e^2 r^2)/(\epsilon_0 h c)$$

$$\sigma = \{(4e^2)/(\epsilon_0hc)\} \pi r^2$$

Calculating $\sigma = \{(4e^2)/(\epsilon_0hc)\} \pi r^2$ | start.

This is the first method of calculating σ .

*----- Calculating $4e^2$ | start.

$$e = 1.6022 \times 10^{-19} \text{ C [C = Coulomb]}$$

$$e^2 = 1.6022^2 \times 10^{-38} \text{ C}^2$$

$$e^2 = 2.5670 \times 10^{-38} \text{ C}^2$$

$$4e^2 = 10.2679 \times 10^{-38} \text{ C}^2$$

*----- Calculating $4e^2$ | end.

*----- Calculating ϵ_0hc | start.

*----- ϵ_0hc Constants | start.

$$\epsilon_0 = 8.8542 \times 10^{-12} \text{ m}^{-3} \text{ kg}^{-1} \text{ s}^4 \text{ C}^2 \text{ s}^{-2}$$

$$h = 6.6261 \times 10^{-34} \text{ kg m}^2 \text{ s}^{-1}$$

$$c = 2.9979 \times 10^8 \text{ ms}^{-1}$$

*----- ϵ_0hc Constants | end.

$$\epsilon_0hc \text{ numbers: } 8.8542 \times 6.6261 \times 2.9979 = 175.8830$$

$$\epsilon_0hc \text{ powers: } 10^{-12} \times 10^{-34} \times 10^8 = 10^{-46} \times 10^8 = 10^{-38}$$

$$\epsilon_0hc \text{ units: } (\text{m}^{-3} \text{ kg}^{-1} \text{ s}^4 \text{ C}^2 \text{ s}^{-2}) \times (\text{kg m}^2 \text{ s}^{-1}) \times (\text{ms}^{-1})$$

$$= (\text{m}^{-3+2+1}) \times (\text{kg}^{-1+1}) \times (\text{s}^{4-2-1-1}) \times \text{C}^2 = (\text{m}^0 \text{ kg}^0 \text{ s}^0 \text{ C}^2) = \text{C}^2$$

$$\epsilon_0hc = 175.8830 \times 10^{-38} \text{ C}^2$$

*----- Calculating ϵ_0hc | end.

$$\sigma = \{(4e^2)/(\epsilon_0hc)\} \pi r^2$$

$$\sigma = \{(10.2679 \times 10^{-38} \text{ C}^2)/(175.8830 \times 10^{-38} \text{ C}^2)\} \pi r^2$$

$$\sigma = \{(10.2679)/(175.8830)\} \pi r^2$$

$$\sigma = 0.05838 \pi r^2$$

$$\sigma = (1/17.1295) \pi r^2$$

This is the interaction cross-section between one electron and another electron or an electron/positron etc. interaction.

Calculating $\sigma = \{(4e^2)/(\epsilon_0hc)\} \pi r^2$ | end.

*----- Calculating $\sigma = (e^2r)/(\epsilon_0m_e c^2)$ | start.

This is the second method of calculating σ .

$$\sigma = (e^2r)/(\epsilon_0m_e c^2)$$

$$\sigma = [(e^2r)/(\epsilon_0m_e c^2)] \times \pi r / \pi r$$

$$\sigma = [(e^2)/(\epsilon_0m_e c^2 \pi r)] \times \pi r^2$$

Constants.

$$c = \text{Speed of light (in a vacuum), } c = 2.9979 \times 10^8 \text{ ms}^{-1}$$

$$e = \text{Charge on the electron} = 1.6022 \times 10^{-19} \text{ C [C = Coulomb]}$$

$$\epsilon_0 = 8.8542 \times 10^{-12} \text{ m}^{-3} \text{ kg}^{-1} \text{ s}^4 \text{ A}^2 \quad [\text{A = Ampere, 1 Ampere = 1 Coulomb per second}]$$

$$m_e = \text{Rest mass of the electron: } m_e = 9.1094 \times 10^{-31} \text{ kg}$$

$$r = \text{Radius of the electron's self-orbit } r = 1.9308 \times 10^{-13} \text{ m}$$

$$\pi = 3.1416$$

Calculation:

$$\sigma = (e^2r)/(\epsilon_0m_e c^2) = [(1.6022 \times 10^{-19} \text{ C})^2 \times (1.9308 \times 10^{-13} \text{ m})] / [(8.8542 \times 10^{-12} \text{ m}^{-3} \text{ kg}^{-1} \text{ s}^4 \text{ A}^2) \times (9.1094 \times 10^{-31} \text{ kg}) \times (2.9979 \times 10^8 \text{ ms}^{-1})^2]$$

$$\text{Powers of 10: } [10^{-38} 10^{-13} = 10^{-51}] / [(10^{-12} 10^{-31} 10^{16} = 10^{-27})] = (10^{-51}) / (10^{-27}) = (10^{-24})$$

$$\text{Units: } = (\text{C}^2 \text{ m}) / (\text{m}^{-3} \text{ kg}^{-1} \text{ s}^4 \text{ A}^2 \text{ kg m}^2 \text{ s}^{-2})$$

$$\text{Units: } = (\text{C}^2 \text{ m}) / (\text{m}^{-3} \text{ kg}^{-1} \text{ s}^4 (\text{Cs}^{-1})^2 \text{ kg m}^2 \text{ s}^{-2})$$

$$\text{Units: } = (\text{C}^2 \text{ m}) / (\text{C}^2 [\text{kg}^{-1} \text{ kg}] [\text{m}^{-3} \text{ m}^2] [\text{s}^4 \text{ s}^{-2} \text{ s}^{-2}])$$

$$\text{Units: } = (\text{C}^2 \text{ m}) / (\text{C}^2 [\text{kg}^{-1} \text{ kg}] [\text{m}^{-3} \text{ m}^2] [\text{s}^4 \text{ s}^{-4}])$$

$$\text{Units: } = (\text{m}) / (\text{m}^{-1})$$

$$\text{Units: } = \text{m}^2$$

$$\sigma = [(1.6022^2 \times 1.9308)] / [8.8542 \times 9.1094 \times (2.9979)^2] \times 10^{-24} \text{ m}^2$$

$$\sigma = 0.0205 \times 10^{-24} \text{ m}^2$$

$$\sigma = 0.0205 \times 10^{-24} \text{ m}^2$$

Please divide by πr to get $\sigma = [(e^2)/(\epsilon_0m_e c^2 \pi r)] \times \pi r^2$

$$\sigma = (0.0205 \times 10^{-24} \text{ m}^2) / (\pi r)$$

$$\sigma = (0.0205 \times 10^{-24} \text{ m}^2) / (3.1416 \times 1.9308 \times 10^{-13} \text{ m})$$

$$\sigma = (e^2 r) / (\epsilon_0 m_e c^2)$$

$$\sigma = (e^2 r) / (\epsilon_0 m_e c^2) \times (\pi r) / (\pi r)$$

$$\sigma = (e^2) / (\epsilon_0 m_e c^2 \pi r) \times (\pi r^2)$$

$$\sigma = [(e^2) / (\epsilon_0 m_e c^2 \pi r)] \times \pi r^2$$

$$\sigma = \{(e^2) / (\epsilon_0 m_e c^2 \pi r) = [(1.6022 \times 10^{-19} \text{ C})^2] / [(8.8542 \times 10^{-12} \text{ m}^{-3}$$

$$\text{kg}^{-1} \text{ s}^4 \text{ A}^2) \times (9.1094 \times 10^{-31} \text{ kg}) \times (2.9979 \times 10^8 \text{ ms}^{-1})^2 \times 3.1416 \times (1.9308 \times 10^{-13} \text{ m})\} \times \pi r^2$$

Numbers:

$$[1.6022^2] / [8.8542 \times 9.1094 \times 2.9979^2 \times 3.1416 \times 1.9308] = 0.00058380 \times 10^2$$

$$= 0.05838$$

$$\text{Powers of 10: } [10^{-38} = 10^{-38}] / [(10^{-12} 10^{-31} 10^{16} 10^{-13} = 10^{-40})]$$

$$= (10^{-38}) / (10^{-40}) = (10^2)$$

$$\text{Units: } = (\text{C}^2) / (\text{m}^{-3} \text{ kg}^{-1} \text{ s}^4 \text{ A}^2 \text{ kg m}^2 \text{ s}^{-2} \text{ m})$$

$$\text{Units: } = (\text{C}^2) / (\text{m}^{-3} \text{ kg}^{-1} \text{ s}^4 (\text{Cs}^{-1})^2 \text{ kg m}^2 \text{ s}^{-2} \text{ m})$$

$$\text{Units: } = (\text{C}^2) / [\text{C}^2 \text{ m}^{-3} \text{ m}^2 \text{ m kg}^{-1} \text{ kg s}^4 \text{ s}^{-2} \text{ s}^{-2}]$$

$$\text{Units: } = (1) / [\text{m}^{-3} \text{ m}^2 \text{ m}]$$

$$\text{Units: } = (1) / (1) \text{ equivalent to no units.}$$

$$\underline{\sigma = 0.05838 \times \pi r^2 = 1/17.1295 \times \pi r^2}$$

σ is the fraction of momentum flow intercepted from one electron/positron (source) to another electron/positron (target).

This fraction of the cross-sectional area matches the fine structure constant x 8 giving 1/17.1295

*----- Finding the value of $\sigma = (e^2 r) / (\epsilon_0 m_e c^2) | \text{ end.}$

*----- Calculating alpha $\alpha | \text{ start.}$

*----- Constants | start.

$$e = 1.6022 \times 10^{-19} \text{ C [C = Coulomb]}$$

$$\epsilon_0 = 8.8542 \times 10^{-12} \text{ m}^{-3} \text{ kg}^{-1} \text{ s}^4 \text{ A}^2 \quad [\text{A = Ampere, 1 Ampere = 1 Coulomb per second}]$$

Planck's constant: $h = 6.6261 \times 10^{-34} \text{ kgm}^2\text{s}^{-1}$

Speed of light (in a vacuum): $c = 2.9979 \times 10^8 \text{ ms}^{-1}$

$\pi = 3.1416$

*----- Constants | end.

Calculations to show that: $\alpha = e^2/(2\epsilon_0hc) = 1/137.0360$

$e^2/(2\epsilon_0hc) = (1.6022 \times 10^{-19} \text{ C})^2 / \{(2 \times (8.8542 \times 10^{-12} \text{ m}^{-3} \text{ kg}^{-1} \text{ s}^4 \text{ A}^2) \times (6.6261 \times 10^{-34} \text{ kgm}^2\text{s}^{-1}) \times (2.9979 \times 10^8 \text{ ms}^{-1})\}$

No powers of 10: $(10^{-38}) / (10^{-12} 10^{-34} 10^8 = 10^{-38}) = (10^{-38}) / (10^{-38}) = 1$

No units: $(\text{C}^2) / (\text{C}^2 \text{ kg}^{-1} \text{ kg} \text{ m}^{-3} \text{ m}^2 \text{ m} \text{ s}^4 \text{ s}^{-2} \text{ s}^{-1} \text{ s}^{-1}) = 1$

$\alpha = (1.6022^2) / \{(2 \times (8.8542) \times (6.6261) \times (2.9979)\}$

$\alpha = (2.5670) / \{(351.7665)\}$

$\alpha = 0.007297 = 1/137.0360$

*----- Calculating alpha α | end.

*----- Equivalence to Coulomb's Law | end.

*----- Reference 2 | end.

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*----- Reference 3 | start.

References for section: Electron - kinetic picture. Electron 'radius'. Compton wavelength. | start.

* -----

Reference 3 - sub reference 1. Photons and electron-positron pairs. | start.

(2) Bose-Einstein. [Column heading: Obeyed by]. Even number of elementary particles (e.g., deuterons, photons). [Column heading: Restrictions on N]. Any number of particles per state.

It is interesting to note that photons follow Bose-Einstein statistics, suggesting that they are complex particles and recalling the formation of electron-positron pairs from X ray photons.

Source: Physical Chemistry. Walter J Moore. Professor of Chemistry, Indiana University. Fourth Edition. New impression 1968. Page 641. Lines 23 to 25.

Reference 3 - sub reference 1. Photons and electron-positron pairs. | end.

* -----

* -----

Reference 3 – sub reference 2. Electron radius. | start.

The electron radius is between 10^{-18} m and 10^{-22} m.

Search source:

<https://en.wikipedia.org/wiki/Electron#:~:text=Observation%20of%20a%20single%20electron%2Cthe%20uncertainty%20relation%20in%20energy.>

Observation of a single electron in a Penning trap suggests the upper limit of the particle's radius to be 10^{-22} meters. [Source reference 93] The upper bound of the electron radius of 10^{-18} meters [Source reference 94] can be derived using the uncertainty relation in energy.

Source reference 93: Dehmelt, H. (1988). "A Single Atomic Particle Forever Floating at Rest in Free Space: New Value for Electron Radius". Physica Scripta. T22: 102–110.

Bibcode:1988PhST...22..102D. doi:10.1088/0031-8949/1988/T22/016. S2CID 250760629.

Source reference 94: Gabrielse, Gerald. "Electron Substructure". Physics. Harvard University. Archived from the original on 2019-04-10. Retrieved 2016-06-21.

Reference 3 – sub reference 2. Electron radius. | end.

* -----

* -----

Reference 3 – sub reference 3.

Intrinsic angular momentum of the electron is $\frac{1}{2}(h/2\pi)$ or $-\frac{1}{2}(h/2\pi)$. | start.

In 1925, Wolfgang Pauli investigated the problem of why the lines in the spectra of the alkali metals are not single as predicted by the Bohr theory. but actually made up of two closely spaced components. An example was shown in Fig. 12.6. He showed that the doublet in the fine structure could be explained if the electron could exist in two distinct states. G.E Uhlenbeck and S. Goudsmit of Leiden identified these states as two states of different angular momentum. They showed that the spectral multiplets could be explained by introducing a new quantum number s, which could have either of two values $+\frac{1}{2}$ or $-\frac{1}{2}$, so that the intrinsic angular momentum of the electron could be $\frac{1}{2}(h/2\pi)$ or $-\frac{1}{2}(h/2\pi)$.

Physical Chemistry. Walter J Moore. Professor of Chemistry, Indiana University. Fourth Edition. New impression 1968. Page 500. Lines 15 to 24.

Reference 3 – sub reference 3.

Intrinsic angular momentum of the electron is $\frac{1}{2}(h/2\pi)$ or $-\frac{1}{2}(h/2\pi)$. | end.

* -----

References for section: Electron - kinetic picture. Electron 'radius'. Compton wavelength. | end.

*----- Reference 3 | end.

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*----- Reference 4 | start.

Examples of calculations | start.

Fixed.	Fixed.	Fixed.	Fixed.	Fixed.	Fixed.	Fixed.
Entered.	Entered.	Entered.	Entered.	Entered.	Entered.	Entered.
Not	Not	Not	Not	Not	Not	Not
variable	variable	variable	variable	variable	variable	variable
c	h	h/mec	h/(mec^2)	me	mec	mec^2
2.9979	6.6261	0.2426	0.0809	9.1094	27.3092	81.8711
2.9979	6.6261	0.2426	0.0809	9.1094	27.3092	81.8711
2.9979	6.6261	0.2426	0.0809	9.1094	27.3092	81.8711
2.9979	6.6261	0.2426	0.0809	9.1094	27.3092	81.8711

Aim.	Input.	Input.	Calc.	Calc.	Calc.	Input.
						0.0000
						Factor on
				ϕ /radians	ϕ /degrees	v_i
$\sqrt{1/\sqrt{160}}$	1	160	0.0791	0.0791	4.5344	1.0000
$\sqrt{3/\sqrt{160}}$	3	160	0.1369	0.1374	7.8703	1.0000
$\sqrt{5/\sqrt{160}}$	5	160	0.1768	0.1777	10.1821	1.0000
$\sqrt{7/\sqrt{160}}$	7	160	0.2092	0.2107	12.0734	1.0000

Input.	Input.	Calc.	Calc.	Summary.	Summary.	Summary.
Target ->	0.4670					
v_i	ϕ /degrees	ϕ /radians	θ /degrees	θ /radians	v_f	pe^2
0.0000	4.5344	0.0791	161.9748	2.8270	4.1875	1312.0278
0.0000	7.8703	0.1374	149.0914	2.6021	4.3236	1284.8155
0.0000	10.1821	0.1777	140.4824	2.4519	4.4587	1258.0065
0.0000	12.0734	0.2107	133.6782	2.3331	4.5926	1231.5938

Summary.	Summary.	Summary.	Summary.	$[(h\nu_i)/(m_e c^2)]$	$\cot\phi$	$\{\cot\phi/(1 + [(h\nu_i)/(m_e c^2)])\}$
pe	pe cos ϕ	$h\nu_i/c$	$pe \cos\phi/(h\nu_i/c)$			
36.2219	36.1086	27.3092	1.3222	1.0000	12.6095	6.3048
35.8443	35.5067	27.3092	1.3002	1.0000	7.2342	3.6171
35.4684	34.9098	27.3092	1.2783	1.0000	5.5678	2.7839
35.0941	34.3178	27.3092	1.2566	1.0000	4.6752	2.3376

$\theta = 2 \times \text{ATAN}\{\cot\phi/(1 + [(h\nu_i)/(m_e c^2)])\}$	$[(h/mec) \times (1-\cos\theta)]$	$v_f = c/[(h/[(h/mec) \times (1-\cos\theta)])(c/v_i)]$
$\theta/\text{radians}$		v_f
2.8270	0.4733	0.2426
2.6021	0.4507	0.2426
2.4519	0.4297	0.2426
2.3331	0.4101	0.2426

$(h\nu_i - h\nu_f + m_e c^2)$	$(h\nu_i - h\nu_f + m_e c^2)^2$	$[(h\nu_i - h\nu_f + m_e c^2)^2 - m_e^2 c^4]$	$pe^2 = [(h\nu_i - h\nu_f + m_e c^2)^2 - m_e^2 c^4]/c^2$	pe
135.9956	18494.7946	11791.9176	1312.0278	36.2219
135.0934	18250.2224	11547.3454	1284.8155	35.8443
134.1986	18009.2758	11306.3988	1258.0065	35.4684
133.3113	17771.8903	11069.0133	1231.1000	35.0870

Examples of calculations | end.

*----- Reference 4 | end.

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End of file.