Short version of "Turing-Unsimulability & Cosmic-Censorship-falsity for General Relativity"

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Abstract. My paper http://vixra.org/abs/2605.0004 "Turing-Unsimulability, and instantaneous & sustained Cosmic-Censorship-falsity, for Einstein-Vacuum General Relativity", at 60, 74, and 78 pages for versions 1, 2, and 3, was too long for many stomachs. Therefore I here provide a short introductory explanation (in the spirit of Phys.Rev.Letters summaries of fuller papers) of its main ideas to try to advertise it and to get its readers oriented right before they start.

I'll outline one way to construct scenarios in Einstein's general relativity (GR) providing

A. long-lasting naked singularity (cosmic censorship counterexample) configurations,
B. whose next-year time-evolution is Turing-machine *unsimulable*.

Optional preamble: Initially our GR vacuum contains gravity waves designed to focus to increase their energy density hugely enough to self-generate black holes from vacuum e.g. as in Pretorius & East 2018. (By scaling theorems, you can generate an s-times smaller and lighter hole using same vacuum, but with all lengths and times scaled by 1/s.) Generate a countable infinity of such holes numbered 1,2,3,...

That preamble allows the initial spacetime metric to be (if desired) smooth except at one point. If you don't care about that, skip the preamble and just start with the holes.

The sequence M_n of hole-masses has M_1=100 solar masses; the rest rapidly decrease toward 0 as n increases. The holes will be set up as a "**recursive lunar system**": Hole 1 is the "sun," hole 2 a "planet" orbiting it, hole 3 a "moon" orbiting that planet, hole 4 is a "satsat" orbiting that moon, and so on forever – hole n orbits hole n-1. The circumferences of these orbits form another sequence, call it L_n, of lengths that also rapidly decrease toward 0.

The precise design, and in particular the two decreasing sequences M_n and L_n, are key. They must obey (and the paper gives designs that do obey) a certain **set of demands:**

1. $M_n/L_n \to 0$, indeed $\sum_{n\geq 1} M_n/L_n$=*finite*, so that all holes are far outside each other's Schwarzschild radii; also causing orbital speeds to approach 0 fraction of lightspeed – all causing the hole-motions to be accurately describable using Newton or the EIH first-order or HSB second-order "post-Newtonian" improved laws (good, since they're all simpler than Einstein GR).
2. We want $\sum_{n\geq 1} M_n$ *finite* – indeed want finite total mass-energy for everything, with all potential and kinetic |energies| bounded below a constant.

3. $\sum_{n\geq 1} L_n$=*finite* so that everything fits in a finite-diameter region of space.
4. $\sum_{n\geq 1} P_n$=*finite* where $P_n=(L_n)^{3/2}(2\pi G)^{-1/2}(M_{n-1}+M_n)^{-1/2}$ denotes Kepler's orbital period for hole n about its "parent" hole n-1, and $G\approx 6.674\times 10^{-11}$ meter^{3}kg^{-1}sec^{-2} is the gravitational constant.
5. The ratios M_{n-1}/M_n, L_{n-1}/L_n, and P_{n-1}/P_n all are lower-bounded by a constant above 1 and upper-bounded by polynomially- or (with care) exponentially-growing functions of n.
6. $\sum_{n\geq 1} L_n/P_n \ll c$, where c=299792458 meter/sec=lightspeed, so velocities very nonrelativistic.
7. All those sums demanded to be finite, I further demand should stay finite even if summand_n is multiplied by any polynomial(n).

The **limit point** of the hole-center locations will be a **naked singularity**:

- "Singular" because by definition a "regular point" is one such that the manifold in a small neighborhood behaves like flat space. So obviously, no accumulation point of singularities can be regular, hence is singular.
- "Naked" because it lies outside all the hole horizons, and since the Newtonian escape velocity from that point is readily computed and well below lightspeed.

For goal **(A)**: We want the entire recursive lunar system to have "**stable and boring**" dynamics so everything lasts at least a million years. That seems best achieved by additionally demanding:

8. The orbit-directions toggle between clockwise and anticlockwise according to (n mod 2).
9. The M_n and L_n sequences cause each successive orbit to stay far inside the "Hill sphere" of its parent, i.e. $L_n \ll H_n$ where $H_n=(2\pi)^{-1}[1+M_{n-1}/M_n]^{-1/3}L_{n-1}$ is "Innanen 1979's Hill radius."
10. All 2-body orbits have GR "lifetimes" (necessarily finite due to gravitational wave-emission-caused energy losses) exceeding 10 megayears:
$$\text{Lifetime}_n \approx 5\pi^{-4}2^{-10}\, c^5\, G^{-3}\, (L_n)^4\, (M_{n-1})^{-1}\, (M_n)^{-1}\, (M_n+M_{n-1})^{-1}.$$
11. Any finite (N-element) sequence of consecutive "recursive levels" of our system, regarded as an N-body problem, is a "KAM stable" Hamiltonian system.

Sequences satisfying demands 1-11:

M_1=100 solar masses and $L_1=10^8$ km;   for n≥2 let $M_n=\exp(2[1-n]\sqrt{83})M_1$ and $L_n=\exp([1-n]\sqrt{83})L_1$.

I've used the transcendental irrationality of $e^{\sqrt{83}}\approx 9049.21759$ (Mahler 1931/1932, Fischler & Rivoal 2025) to prevent resonances that could have forestalled KAM stability. Either *squaring* or *cubing* n in both right hand sides of those M_n and L_n formulas also would work.

If we instead seek goal **(B)**: Instead try to make each "recursive level" of our system *not* a boringly-stable 2-body problem, but rather a chaotic (3-8)-body scenario. That is, each body at level n of the recursion is orbited not by 1, but rather by 2 to 7 bodies, and in a **chaotic** way. Only the heaviest one of those 2-7 has "children" (i.e. moons) in the recursion; the others are moonless. Preferably this chaos is "permanently confined," or at least long-lasting – and such that the sum of all "Lyapunov times" T_n (delay needed for factor-e noise-amplification within the level-n chaos) is finite, and as usual remains finite even if summand_n is multiplied by any polynomial(n). For suitablelooking systems of "planets" chaotically orbiting a (>100)×-heavier "sun," see Deck et al 2012, Barnes et al 2015, or Nelson et al 2016. Those systems' Lyapunov times are equivalent to 10-300 orbits of their slowest-orbiting planet.

"Noise" from the motions of the holes numbered>n, "feeds in" to the system of the n heaviest holes (numbered 1,2,...,n), then gets amplified by their chaotic dynamics. The net result is that at the topmost (heaviest holes) level of the recursion, macroscopic positional changes occur, caused by the whole infinite chain of lighter holes, after finite time. We want the n^{th} "recursive level" 3-body system's "survival time" S_n to exceed the sum of Lyapunov times of all the systems with greater-or-equal n, by a large factor. Indeed, preferably this factor grows rapidly and unboundedly with n, and even more preferably $S_n \to \infty$ while $T_n \to 0$. If the chaos is "permanently confined" then that is trivial because $S_n=\infty$. But otherwise, this goal could be achieved – under the assumption of anything like "Lecar's empirical law" $S_n \approx 30(T_n)^{1.8}(P_n)^{-0.8}$ – by making the n^{th} level systems become less chaotic as n increases so that, say, $T_n=(P_n)^{0.4}(P_1)^{0.6}$, whereupon the Lecar-predicted S_n actually *grow* exponentially to ∞. Crude 2D and 3D *collision-time* survival estimates based on body cross-sectional areas also both grow to ∞.

Again, the most-preferred forms of all (B)-desiderata seem simultaneously achievable by design.

We then can interpret (B) as "new information keeps flowing outward from the naked point-singularity into the dynamics, at bit rates growing unboundedly with n, but even for n=1 exceeding a positive constant." This cannot be simulated by Turing machine because it must read infinite input to assure correctness of its top-level macroscopic conclusions about 1-year-later system-state. For example, if just *one* black hole's initial orbital "phase-angle" about its parent were changed (all else unchanged) that would suffice to affect the top-level state 1 year later.

There are a huge number of possible variant-versions of these constructions. The initial data that cause pathologies (A) & (B) seem "generic" in the somewhat weak sense that if each hole's initial position and velocity relative to its parent is altered by up to 0.1% randomly, and masses are altered by up to 0.1% randomly, then it'll probably all still work. However if the chaos in case (B) depends on *resonances*, such as our solar system's chaotic 3:2 Pluto:Neptune mean-motion resonance, then only alterations *preserving* those resonances would be allowed – a restriction which might prevent you from considering this "generic." Fortunately, Gladman 1993's ostensibly-permanent chaotic 3-body systems are not resonant so should be unhurt by that complaint.

Weierstrass constructed his famous continuous nowhere-differentiable functions F(t) by a series summing an infinite set of analytically-smooth oscillatory summands, with the amplitudes & frequencies of his n^{th} oscillatory term designed to shrink & grow suitably-exponentially with n. The trajectory of our n^{th} hole when $n \to \infty$, i.e. the location of the naked singularity, is describable as exactly that sort of sum. This insight allows us to see its location is a **nowhere-analytic** function of time, indeed, we can make it nowhere K-differentiable for suitable constant K. This in turn should cause the entire part of the spacetime metric that can "see" that singularity, to be nowhere K'-differentiable for some other constant K'.

There are several ways all that **falls short of complete rigor:**

i. No proof is known that Einstein vacuum field equations even *have* solutions (or that they are unique) although I would certainly conjecture that – especially in these scenarios given that they are very close to Newtonian, and there are rigorous existence & uniqueness theorems for Newton N-body even when N=∞, which in particular work for my paper's bodies because of the finiteness of the "total tide" $\sum_k \sum_{j<k} |M_j M_k| \cdot |\vec{x}_j - \vec{x}_k|^{-3}$.
ii. For (A), optimally we'd want notions of "KAM-stability" for N-body systems with N=∞. Presently-available forms of KAM-theory do not appear to be that generous, although good enough for every finite N. (Arnold-Kozlov-Neishtadt 2006 and Moser 1978 contain introductions to KAM theory.) Future KAMsters might be able to overcome that...
iii. For (B) we want rigorous lower bounds on "survival time" of chaotic N-body systems. Those seem unavailable, although there are several families of apparently-chaotic system constructions which *seem* to work based on extensive numerical-experimental studies and/or astronomical observations (all by previous authors) which provide nonrigorous empirical survival-time bounds which are adequate for our purposes with plenty of "safety margin." It also is plausible that some of them enjoy "permanently confined" chaos.
iv. The preamble's creation of black holes from incoming smooth blobs of gravitational radiation was not rigorous (since based on numerical simulations), and also its aftermath involved some temporary scattered such radiation, which, although numerically very small, we never proved harmless. That does not matter if you skip the preamble, plus it seems obvious that appropriate small adjustments should exist that render it harmless.
v. Tiny energy losses to gravitational radiation prevent our system from really being Hamiltonian at all, hence KAM theory does not really apply to it. Intuitively that should only help, not hurt, our CC-refutation (since gradual planet "migration" due to such non-Hamiltonian effects empirically causes *stable* resonances to lock in, which help us, but should just migrate past unstable resonances, which of course do not get "locked in"). Also note our lifetimes against this kind of orbit decay *increase exponentially* with n so that our system gets "exponentially near to being Hamiltonian." Therefore I suspect this complaint simply does not matter.
vi. It is conceivable that the effect of the "input noise" on each chaotic system somehow magically exactly cancels out, rather than causing dynamical exponential amplification to produce macroscopic changes. Ridiculous – but I did not disprove it. (This quibble perhaps could be dodged by employing a different, non-chaos-based, proof technique.)

Those **rigor problems can be overcome** by the "cheat" of adding extra cases to the theorems, to say "either the main claim works, or those bad phenomena that I cannot disprove happen." Also note: *Every* paper employing "numerical GR," including the 2017 Nobel Prize LIGO papers, is nonrigorous in sense (i). *All* astronomy papers that assume the stability of "hierarchical" N-body systems are nonrigorous in sense ii (albeit my paper improved over them by proving KAM-stability theorems for all finite N). Anybody objecting based on iii would need to dispute the goodly number of workers who numerically studied this. In short, these non-rigor objections only bother those willing to dispute the (non-rigorous) work of various communities of physicists and/or astronomers, in some cases extremely large amounts of published work.

If (i) happens, that's *still* a "failure of GR and of CC," where CC means cosmic censorship. I think iv and vi do not merit concern, but physicists should worry at least slightly about ii, iii, and v. But note that I *do not need* to fully-*specify* precise designs providing everything needed to make (A) or (B) happen; all I need is such designs *exist*, to conclude existence of sustained naked-singularities or unsimulability. That seems extremely hard to disbelieve, and for me almost wholy obliterates concerns ii-vi.

Also, if we don't ask for time-evolution, then those non-rigor complaints do not apply. That allowed me to **fully-rigorously** construct spacetimes which yield an *instantaneous* naked singularity (or which cease time-evolving since Einstein stops having a solution) – either way disproving cosmic censorship, and these meet the demands of a Feb.1997 public Hawking versus Preskill & Thorne bet about naked-singularities, as well as the different demands in a paper by Moncrief & Eardley attempting to mathematically formulate cosmic censorship conjectures. These simpler constructions don't need complicated orbits nor stability, chaos, or survival-time assertions about them. Instead, a completely unbound system of black holes all moving along disjoint near-straight-line trajectories at near-constant speeds, suffices. I then realized that fully-rigorous construction actually could be made to yield *eternal*, not merely instantaneous, presence of at least one naked-singularity (more precisely, it lasts for as long as the Einstein vacuum field equations have a solution – which I merely-presume is forever).

Note that all my constructions involved the GR **vacuum** only: no electromagnetic fields, no scalar fields, no "dust," no "radiation," no "matter." (Einstein's "cosmical constant" Λ could be zero, or small but nonzero – both will work for my purposes.)

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