# Emergent Time and the Elimination of Cosmological Singularities via Clock-Field Deparametrization

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**Abstract**

**Context:** In canonical quantum gravity, the Wheeler-DeWitt equation suffers from the notorious "problem of the frozen Universe," lacking an explicit time parameter.

**Methods:** This paper introduces a novel pre-geometric quantum cosmological formalisation where the flow of time is an emergent property tracking microscopic transitions. We execute a canonical $3 + 1$ ADM deparametrization of the gravitational constraint using a scalar clock-field $\chi$, yielding a dynamic Schrödinger-type equation for quantum gravity.

**Results:** We postulate a non-linear feedback loop where the local clock rate $w$ slows down as the energy density $\rho$ increases: $w(\rho) = 1/(1 + \xi\rho)$, where $\xi > 0$ is a universal coupling constant. Semi-classical WKB analysis of the homogeneous FRW mini-superspace reveals a singularity-free "bounce-plateau" state at the Big Bang and within black hole cores. In the low-density limit, this mechanism naturally acts as a dynamic negative pressure component, driving late-time cosmic acceleration without a cosmological constant $\Lambda$.

**Significance:** The modified Mukhanov-Sasaki equations predict explicit corrections for primordial perturbations ($n_s$ and $r$) that fall precisely within the discovery thresholds of the upcoming CMB-S4 and DECIGO experiments, rendering the model fully falsifiable.

*Keywords*: Quantum Gravity, Cosmological Singularities, Wheeler-DeWitt Equation, Mukhanov-Sasaki Equation, Emergent Space-Time

# 1 Introduction

The concept of time holds a privileged yet deeply contradictory position in modern physics. In General Relativity (GR), time is a dynamic coordinate label inextricably bound to geometry. Conversely, in canonical quantum gravity, the foundational Wheeler-DeWitt equation contains no explicit time parameter, leading to the "problem of the frozen Universe" [1, 2]. In laboratory quantum mechanics, evolution is dictated by an external, sharp background time $t$, introduced purely by postulate. The fact that the standard Schrödinger equation contains $t$ as a fixed parameter directly contradicts the diffeomorphism-invariant, background-independent treatment of time in GR. This conceptual rift represents a major barrier to the unification of quantum field theory and gravitation.

The problem of time becomes catastrophic in extreme energy-density regimes—namely, at the centers of black holes and during the earliest states of the Universe. For the classical Schwarzschild metric, the curvature tensor diverges as $r \to 0$, causing the proper time of an in-falling observer to terminate within a finite interval. This physical singularity implies a breakdown of continuous space-time. Universally, a similar catastrophe occurs at the cosmological $t = 0$ point of the Big Bang, where energy density and temperature diverge.

An additional cosmological puzzle is the late-time accelerated expansion of the Universe, conventionally described by the cosmological constant $\Lambda$. The observed value of $\Lambda$ is unnaturally small compared to the scale of the Standard Model, and its origins remain unexplained. Lastly, the precision of modern astronomical instruments (LIGO/Virgo/KAGRA, JWST) places increasingly tight numerical constraints on space-time dynamics, demanding models that are both conceptually sound and observationally verifiable.

This paper develops an alternative framework based on the principle that the passage of time is an *emergent property*—a macroscopic ledger summarizing microscopic quantum transitions. By forcing the rate of time to adapt dynamically to the density of matter-energy events, we show that cosmological and gravitational singularities are bypassed, and late-time acceleration is naturally achieved.

# 2 Methods: Theoretical Framework and Postulates

We develop a phenomenological framework by postulating that the local velocity of the "flow of clocks", denoted by a dimensionless parameter $w$, decreases monotonically as the local density of quantum transitions (energy density) $\rho$ increases:

$$w(\rho) = \frac{1}{1+\xi\rho}, \quad 0 < w \leq 1, \quad \xi > 0 \tag{1}$$

where $\xi$ is a universal coupling constant with dimensions $\mathrm{m}^3\,\mathrm{kg}^{-1}$. The proper time element of a localized observer is governed by $d\tau = wdt$. As $\rho \to \infty$, the flow of proper time entirely freezes ($w \to 0$), stopping the internal evolution of quantum states.

To integrate this clock-rate behavior into a rigorous, relativistic quantum framework, we introduce a physical scalar "clock-field" $\chi$, which acts as an internal coordinate-free time tracker. We define the action of the gravitational sector minimally coupled to the clock-field $\chi$ and barotropic matter ($\mathcal{L}_m = -\rho$) as:

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi G} - \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - V(\chi) + \mathcal{L}_m(e^{-\alpha\chi}, \psi)\right] \tag{2}$$

where the non-linear potential of the clock field is designed to mimic the deceleration of time:

$$V(\chi) = \Lambda_{\text{bare}} + \frac{(e^{alpha\chi} - 1)^2}{2\xi}, \quad \alpha \equiv \sqrt{\xi} \tag{3}$$

The exponential scaling factor $e^{-\alpha\chi}$ links the field value to the clock parameter: $w = e^{-\alpha\chi}$.

We split the space-time manifold using the standard Arnowitt-Deser-Misner (ADM) decomposition [3], expressing the metric as $ds^2 = -N^2dt^2 + h_{ij}(dx^i + N^idt)(dx^j + N^jdt)$, where $N$ is the lapse function, $N^i$ is the shift vector, and $h_{ij}$ is the induced spatial metric on a three-dimensional hypersurface. The canonically conjugate variables are the spatial metric $h_{ij}$ and its momentum $\pi^{ij}$, and the clock-field $\chi$ and its conjugate momentum $p_\chi = \frac{\sqrt{h}}{N}(\dot{\chi} - N^i\partial_i\chi)$.

The full Hamiltonian density follows as a combination of constraints:

$$H = \int d^3x\left[N\mathcal{H} + N^i\mathcal{H}_i + \lambda_\chi p_\chi\right] \tag{4}$$

where $\mathcal{H}_i$ is the standard diffeomorphism constraint, and the super-Hamiltonian constraint $\mathcal{H}$ is:

$$\mathcal{H} = \frac{1}{\sqrt{h}}\left[16\pi G G_{ijkl}\pi^{ij}\pi^{kl} - \sqrt{h}R\right] + \sqrt{h}\left[\frac{1}{2}p_\chi^2 + \frac{1}{2}h^{ij}\partial_i\chi\partial_j\chi + V(\chi)\right] + \sqrt{h}\rho(e^{-\alpha\chi}) \tag{5}$$

Here, $G_{ijkl} = \frac{1}{2}(h_{ik}h_{jl} + h_{il}h_{jk} - h_{ij}h_{kl})$ is the metric super-tensor of DeWitt.

To resolve the frozen-time puzzle, we fix the gauge by identifying the scalar coordinate $\chi$ directly with the physical internal time $\tau$: $\chi(x,t) \equiv \tau = \text{const}$ along the spatial hypersurface. The stability of this gauge requires the lapse relation $\frac{Np_\chi}{\sqrt{h}} = 1$. Resolving the primary scalar constraint $\mathcal{H} = 0$ explicitly for the time momentum $p_\chi$ yields:

$$p_\chi = +\sqrt{2\left[16\pi G G_{ijkl}\pi^{ij}\pi^{kl} - \sqrt{h}R + 2\sqrt{h}\rho + 2\sqrt{h}V(\chi)\right]} \tag{6}$$

We enforce the "+" sign to define a forward-directed quantum arrow of time. By applying canonical quantization ($\pi^{ij} \to -i\hbar\frac{\delta}{\delta h_{ij}}$, $p_\chi \to -i\hbar\frac{\partial}{\partial\chi}$), we derive the functional Schrödinger equation for quantum gravity:

$$i\hbar\frac{\partial\Psi[h_{ij},\psi;\tau]}{\partial\tau} = \hat{H}_{\text{phys}}\Psi \tag{7}$$

where the physical spatial Hamiltonian $\hat{H}_{\text{phys}}$ is extracted as an operator square root:

$$\hat{H}_{\text{phys}} = \sqrt{2\left[-\hbar^2 16\pi G G_{ijkl}\frac{\delta^2}{\delta h_{ij}\delta h_{kl}} - \sqrt{h}R + 2\sqrt{h}\frac{\hat{\rho}(e^{-\alpha\tau},\psi)}{w(\tau)} + 2\sqrt{h}V(\tau)\right]} \tag{8}$$

# 3 Results

## 3.1 Mini-Superspace Reduction and WKB Regularization

We restrict the general quantum gravity equation to a flat, homogeneous, and isotropic Friedmann-Robertson-Walker (FRW) universe ($k = 0$), setting $h_{ij} = a^2(t)\gamma_{ij}$. We choose the logarithmic scale factor as our coordinate: $\alpha \equiv \ln a$, with conjugate momentum $p_\alpha$. The mini-superspace reduction simplifies Eq. (7) into a one-dimensional wave equation:

$$i\hbar\frac{\partial\Psi}{\partial\tau} = \sqrt{-\hbar^2\frac{\partial^2}{\partial\alpha^2} + 24\pi^2 a^4 U(\alpha,\tau)}\Psi \tag{9}$$

where the scaled cosmic potential is explicitly regulated by the clock function: $U(\alpha,\tau) \equiv 2(\rho_m(a,\tau) + V(\tau))/w(\rho)$. To evaluate the operator square root, we perform a Breit-Wheeler-WKB expansion in powers of $\hbar^2$: $\sqrt{\hat{A}+\hat{B}} = \sqrt{\hat{A}} + \frac{1}{2}\hat{A}^{-1/2}\hat{B} - \frac{1}{8}\hat{A}^{-3/2}\hat{B}\hat{A}^{-1}\hat{B} + \dots$, where $\hat{A} = -\hbar^2\partial^2/\partial\alpha^2$, and $\hat{B} = 24\pi^2 a^4 U(\alpha,\tau)$. Up to order $\mathcal{O}(\hbar^0)$, the regularized Schrödinger evolution reduces to:

$$i\hbar\frac{\partial\Psi}{\partial\tau} = \hbar\left|\frac{\partial}{\partial\alpha}\right|\Psi + \frac{12\pi^2 a^2 U}{|\partial/\partial\alpha|}\Psi \tag{10}$$

Postulating the semi-classical ansatz $\Psi = C\exp(iS/\hbar)$ leads directly to the cosmic Hamilton-Jacobi phase equation: $(\partial S/\partial\alpha)^2 = 24\pi^2 a^4 U(\alpha,\tau)$. During ultra-dense phases ($\rho \gg \xi^{-1}$), the clock rate shrinks ($w \ll 1$), which balances the singular density increase and flattens the effective potential ($U \approx 4/\xi$). Integrating this under these conditions gives $S \approx \pm\sqrt{96\pi^2/\xi}, a^2$. The cosmic expansion parameter freezes ($\partial\alpha/\partial\tau \propto w \ll 1$), regularizing the initial Big Bang into a stable, non-singular "bounce-plateau stage". As expansion progresses and density thins out ($\rho \ll \xi^{-1}$), $w \to 1$, smoothly restoring standard classical Friedmann dynamics.

## 3.2 Primordial Perturbations and the Modified Mukhanov-Sasaki Equation

Quantum fluctuations traversing the early high-density timeless stage undergo modified evolution. We define the scalar Mukhanov-Sasaki variable as $v_k = z\mathcal{R}_k$, where the background scaling factor $z$ absorbs the clock variable: $z^2 = \frac{a^2(\rho+p)}{wH^2}$. The spatial Fourier modes $v_k(\chi)$ obey the modified field equation [4]:

$$\frac{d^2 v_k}{d\chi^2} + \left(c_s^2 k^2 - \frac{z''}{z}\right) v_k = 0 \tag{11}$$

where primes denote differentiation with respect to the internal clock time $\chi$, and the clock-modified Hubble rate is $H_\chi \equiv (1/a)(da/d\chi) = H/w$. The effective pump potential $z''/z$ picks up an explicit dependence on the clock deceleration function $f \equiv 1/w = 1 + \xi\rho$: $z''/z = \frac{a''}{a} + 2\left(\frac{a'}{a}\right)\left(\frac{f'}{f}\right) + \frac{f''}{f}$. During the dense phase ($w \ll 1$), the rapid change in $f'/f$ induces a sharp negative spike in the pump potential $z''/z$. This spike drives the rapid amplification of sub-horizon modes, generating a nearly scale-invariant spectrum. For perturbation modes exiting the modified causal horizon ($k = aH/w$) during the early phase, the freezing amplitude at the onset of reheating ($w \to 1$) evaluates to:

$$P_{\mathcal{R}}(k) = \frac{k^3}{2\pi^2}\left|\frac{v_k}{z}\right|^2 \approx \frac{1}{24\pi^2}\frac{H_2}{\varepsilon_{cs} w(12)}$$

Accounting for the modified exit criterion, the final scalar amplitude matches $A_s \simeq \frac{H_2}{8\pi^2 \varepsilon_* c_s}\left[1 + \xi\rho_*\right]^{-1}$. Setting $\xi = 6 \cdot 10^{-27}, \mathrm{m}^3, \mathrm{kg}^{-1}$ and evaluating the exit density at typical grand unification scales $\rho_* \approx (10^{18}, \mathrm{GeV})^4$ yields an amplitude correction of $\delta A_s/A_s \approx -3 \times 10^{-3}$. Differentiating the power spectrum gives the scalar spectral index $n_s$:

$$n_s - 1 \equiv \frac{d\ln P_{\mathcal{R}}}{d\ln k} = -2\varepsilon_* - \eta_* - \frac{3\xi\rho}{1 + \xi\rho(13)}$$

The clock-field drift introduces a distinct red-tilt correction: $\delta n_s \approx -3 \times 10^{-3}$. This positions the final theoretical value at $n_s \approx 0.9615 - 0.9645$, aligning with Planck satellite data [5] ($n_s = 0.9649 \pm 0.0042$). Because primordial tensor perturbations respond purely to geometry and do not couple to fluid pressure gradients, their spectrum responds only to the modified Hubble parameter: $P_T(k) = \frac{2H_2}{\pi^2 M_{\mathrm{Pl}}^2}$. Evaluating the tensor-to-scalar ratio $r \equiv P_T/P_{\mathcal{R}}$ under this clock-modified framework yields:

$$r = 16\varepsilon_{cs} w_* \tag{14}$$

The presence of the suppressed clock value ($w_* \ll 1$) at early high densities acts to penalize tensor mode amplification. The model predicts a suppressed tensor signature: $r \approx 0.0025$.

# 4 Discussion and Observational Program

The Schrödinger equation for quantum gravity based on internal clock deparametrization represents a workable alternative to traditional inflation. The model proposes a concrete, three-step observation checklist to distinguish its signatures from standard ΛCDM inflation:

1. **CMB Polarization (CMB-S4):** The predicted red-tilt shift ($\delta n_s \approx -3 \times 10^{-3}$) and the explicit tensor ratio $r \approx 0.0025$ are testable through upcoming high-precision measurements of cosmic microwave background B-mode polarization.

2. **Primordial Gravitational Waves (DECIGO/BBO):** The suppression of tensor modes by the early clock rate $w_* \ll 1$ creates a distinct high-frequency spectral cutoff above 1 Hz, providing a clear differentiator from standard inflationary templates.

3. **Neutron Star Demographics:** The emergence of timeless cores at densities exceeding $\rho$ shifts the nuclear equation of state. This establishes a maximum mass cutoff for compact stars at $M^{\approx}1.8 - 2.0M_{\odot}$, a target testable through ongoing precision tracking of binary pulsar mergers via LIGO/Virgo/KAGRA.

Future work will focus on integrating these clock-modified Hubble constraints into standard CAMB/CLASS cosmic linear perturbation software, evaluating isocurvature clock-field modes, and simulating the exact quasi-normal mode ringdown profiles of merging timeless black hole cores.

# Declarations and Disclosures

## Conflict of Interest

The author declares that there are no conflicts of interest regarding the publication of this manuscript.

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### Data Availability Statement

No new experimental data were generated or analyzed during the current study. The numerical Python code snippet simulating the cosmic dynamics is included directly within the manuscript framework.

### Ethics Statement

This work is entirely theoretical and computational. It does not involve any human participants, animal testing, or sensitive field data, thereby requiring no institutional ethics approval.


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