A BRIEF ESSAY ON ELECTROMAGNETIC WAVES

(*TRANSPORTED ENERGY, QUANTUM, PROPAGATION, PHOTON AND WAVE-PARTICLE DUALITY*)

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ABSTRACT

The goal of this essay is to carry out a critical analysis of the extension of the concept of energy applied to stationary electromagnetic fields to traveling electromagnetic fields, that is, electromagnetic waves. In an attempt to establish a connection between the energy equations, obtained from Maxwell's equations and the results of Max Planck's studies, this essay proposes some changes to the conventional electromagnetic wave equations. This essay also seeks to deduce mathematical expressions for the electrical power and voltage associated with the quantum, as well as its subdivision into voltage, current and time components. Finally, a conjecture is proposed about what wave-particle duality could be, and its relationship with the photon as discussed in this essay.

Key words : Transported energy , Quantum , Propagation, Photon , Wave-particle duality

1. INTRODUCTION

The analysis will be started from the expressions for electric and magnetic fields originated from Maxwell's equations. Such expressions will allow the calculation of the total energy stored in a volume and from this energy will be made a critical interpretation of each of its terms considering that the energy function must satisfy the wave equation and conservation law.

Note: All units used in this essay are in accordance with the **SI** (International System of Units).

2. DATA PLUGGED IN THIS ESSAY

. Permittivity ε_0 ………………….. 8.854 x 10^{-12} Farad/m

. Permeability μ_0 ………………… 4π x 10^{-7} Henry/m

. Planck's constant **h** …………….. 6.626 x 10^{-34} J.s

. Velocity of propagation **c** …….. 2.998 x 10^{8} m/s

3. INITIAL DEVELOPMENT

Maxwell's equations provide the harmonic vector fields $\mathbf{E_x}$ and $\mathbf{H_y}$ propagating in vacuum, in the z direction such that:

$$\overrightarrow{\mathbf{E_x}}(\boldsymbol{z},\boldsymbol{t}) = \mathrm{E_0}\sin[\tfrac{2\pi}{\lambda}(\mathrm{z-ct})]\,\overrightarrow{\mathbf{a}_x} \qquad \textbf{Eq. A}$$

$$\overrightarrow{\mathbf{H_y}}(\boldsymbol{z},\boldsymbol{t}) = \mathrm{H_0}\sin[\tfrac{2\pi}{\lambda}(\mathrm{z-ct})]\,\overrightarrow{\mathbf{a}_y} \qquad \textbf{Eq. B}$$

In which:

$$\mathrm{H_0} = \mathrm{E_0}\sqrt{\frac{\varepsilon_0}{\mu_0}}\ ,$$

$$\frac{2\pi}{\lambda}(\mathrm{z-ct}) = (\frac{wz}{\mathrm{c}} - \mathrm{wt})$$

$\mathrm{c} = \boldsymbol{\lambda}\mathrm{f}$ (speed of light)

The energies stored in the electric and magnetic fields are respectively called $\boldsymbol{E_E}$ and $\boldsymbol{E_M}$, and are given by:

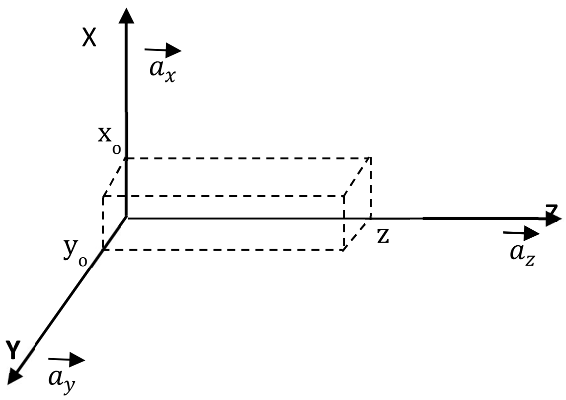
$$E_E = \frac{1}{2}\varepsilon_0 \iiint E_x{}^2 \mathrm{d_v} \qquad \textbf{Eq. 1}$$

$$E_M = \frac{1}{2}\mu_0 \iiint H_y{}^2 \mathrm{d_v} \qquad \textbf{Eq. 2}$$

With $\mathrm{E} = \boldsymbol{E_E} + \boldsymbol{E_M}$ being the total energy presumably transported by the electromagnetic wave moving at the speed of light in the direction of propagation **Z** of the electric and magnetic fields. Using equations **Eq. 1** and **Eq. 2**, it is easily demonstrated that the energies stored in the electric and magnetic fields are equal, with the total energy calculated as follows:

$$E(\mathrm{x,y,z,t}) = \varepsilon_0 \mathrm{E}_0^2 \iiint \sin 2\,(\frac{wz}{c} - \mathrm{wt})\,\mathrm{dv}$$

Let the volume be in $\mathbf{x_o}$, $\mathbf{y_o}$ and **z** as shown in **Figure 1**, below:



<u>Figure 1</u> - Energy contained in the volume $\mathbf{x_o}$, $\mathbf{y_o}$ and **z**

The energy stored in the volume $\mathbf{x_o}$, $\mathbf{y_o}$ and **z**, with **z** being the wave propagation direction, is calculated as follows:

$E(z,t) = \varepsilon_0 E_0^2 \int_0^z \int_0^{y_0} \int_0^{x_0} sin^2(\frac{wz}{c} - wt)$ dxdydz , resulting in:

$$E(z,t) = \frac{x_0 y_0 \varepsilon_0 E_0^2}{2}\left[z - \frac{c}{2w} sin2wt - \frac{c}{2w} sin2(\frac{wz}{c} - wt)\right] \quad \textbf{Eq. 3}$$

As this energy is presumably transported by electromagnetic waves, it can be heuristically inferred that the equation **Eq. 3** must obey the wave equation, then:

$$\frac{\partial^2}{\partial z^2} E(z,t) = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} E(z,t) \quad \textbf{Eq. 4}$$

Taking:

$$\frac{x_0 y_0 \varepsilon_0 E_0^2}{2} = A$$

$$E(z,t) = Az - \frac{Ac}{2w} sin2wt - \frac{Ac}{2w} sin2(\frac{wz}{c} - wt)$$

Plugging **E(z,t)** in equation **Eq. 4**, the following condition remains:

$$\frac{2wA}{c} sin2wt = 0$$

However $\frac{2wA}{c} \neq 0$, resulting in that, for the energy equation **E(z,t)** to satisfy the wave equation (**Eq.4**), the condition below must be imposed:

$sin2wt = 0$ **Eq. 5**

4. CONSEQUENCES AND CONCLUSIONS

Consequence 1

For the energy equation transported by electromagnetic waves to satisfy the wave equation that originated it, it must be temporally restricted to the condition established in equation **Eq. 5**, that is:

$sin2wt = 0$ => $2wt = k_1\pi$, for $k_1 = 0,1,2,3...$ (the variable **t** takes only positive integer values)

Hence, $t = \frac{k1\pi}{2w} = \frac{k1T}{4}$, where T is the period of the wave.

The equation **Eq. 3** therefore becomes:

$$E(z,t) = \frac{x_0 y_0 \varepsilon_0 E_0^2}{2}\left[z - \frac{c}{2w} sin2(\frac{wz}{c} - wt)\right] \quad \textbf{Eq.6}$$

and **t** $= \frac{k1T}{4}$, $k_1 = 0,1,2,3...$ **Eq. 6.1**

Conclusion 1

The equations **Eq. 1** and **Eq. 2**, for the energies stored in electric and magnetic fields, respectively, do not apply in an unrestricted way to electromagnetic fields that propagate in a vacuum.

Conservation law

The instantaneous power flow through a closed surface **S**, involving a volume **V**, applied to electromagnetic fields in lossless propagation media, establishes that:

$\oiint \bar{E} \times \bar{B} \, . d\bar{S} = - \frac{\partial}{\partial t} \iiint (\frac{\mu_0}{2} \, H^2 + \frac{\varepsilon_0}{2} E^2) dV$ **Eq. 7**

Integrating both sides of **Eq. 7** over time, yields:

$\int_0^t (\oiint \bar{E} \times \bar{B} \, . d\bar{S}) dt = - \iiint (\frac{\mu_0}{2} \, H^2 + \frac{\varepsilon_0}{2} E^2) dV$ **Eq. 8**

$\int_0^t \frac{x_0 y_0 E_0^2}{c \, \mu_0} sin^2 \, (\frac{wz}{c} - \text{wt}) \; dt = - \iiint (\frac{\mu_0}{2} \, H^2 + \frac{\varepsilon_0}{2} E^2) dV$ **Eq. 9**

The term on the left side of equation **Eq. 9**, calculated from the Poynting vector gives:

$E(z,t) = \frac{x_0 y_0 \varepsilon_0 E_0^2}{2} \left[ct - \frac{c}{2w} sin2 \frac{wz}{c} + \frac{c}{2w} sin2(\frac{wz}{c} - wt) \right]$ **Eq. 10**

The result of the term on the right side of **Eq. 9**, regardless the (-) sign, is that already presented in **Eq. 3**.

Comparing **Eq. 10** with **Eq. 3**, it results:

$\frac{x_0 y_0 \varepsilon_0 E_0^2}{2} \left[ct - \frac{c}{2w} sin2 \frac{wz}{c} + \frac{c}{2w} sin2(\frac{wz}{c} - wt) \right] = \frac{x_0 y_0 \varepsilon_0 E_0^2}{2} \left[\text{z} - \frac{\text{c}}{2\text{w}} \text{sin2}wt - \frac{\text{c}}{2\text{w}} \text{sin2}(\frac{\text{wz}}{\text{c}} - \text{wt}) \right]$ **Eq. 11**

Consequence 2

An analysis of both sides of **Eq. 11** indicates a seemingly inconsistency, which would violate the energy flow continuity theorem. To get around this apparent inconsistency, it is necessary that the variables **t** and **z** are interchangeable, such that:

z = ct e t = z/c

The requirement to make permutations between variables also indicates the need for the variable **z** to also be discrete, such that:

z = ct

As the discrete variables **t** and **z** operate independently in the electromagnetic wave equations, the discrete variable **z** will be associated with **k2**, instead of **k1**, which was associated with the discrete variable **t**.

$z = \frac{k2cT}{4}$, $k_2 = 0,1,2,3...$

In terms of wavelengths, **z** will be as follows:

$z = \frac{k2\lambda}{4}$, $k_2 = 0,1,2,3...$ **Eq. 11.1**

Conclusion 2

The restrictions imposed on the variables $t = \frac{k1T}{4}$ and $z = \frac{k2\lambda}{4}$ reduce the energy equation **Eq. 6** (or **Eq. 3**) into:

$E(z,t) = \frac{x_0 y_0 \varepsilon_0 E_0^2}{2} z$ Eq. 12

$E(z,t) = \frac{x_0 y_0 \varepsilon_0 E_0^2}{2} \frac{k2\lambda}{4}$, $k_2 = 0,1,2,3...$ **Eq. 13**

In other words, at each fraction $\frac{\lambda}{4}$, the electromagnetic wave accumulates a volume of energy equal to $\frac{x_0 y_0 \varepsilon_0 E_0^2}{2} \frac{\lambda}{4}$

Consequence 3

Considering the energy accumulated in 1 second, for a wave of frequency f, crossing the surface $\mathbf{x_0 y_0}$, the expressions below will be obtained:

$f = c/\lambda$, then $k_2 = 4f$

$E(z,t)/x_0 y_0 = \frac{\varepsilon_0 E_0^2}{2} \lambda f = \frac{\varepsilon_0 E_0^2 c}{2}$ [w/m²] **Eq. 14**

Eq. 14 equals the mean value of the Poynting vector.

Therefore, for the exact calculation of electromagnetic energy from the mean value of the Poynting vector, the time value to be used must be defined up to the limit of $t = \frac{k1T}{4}$, $k_1 = 0,1,2,3...$

5. POWER ASSOCIATED WITH THE ENERGY OF A QUANTUM (E = hf)

From **Figure 1** and the results of **Eq. 13**, we obtain **Figure 2**, below:

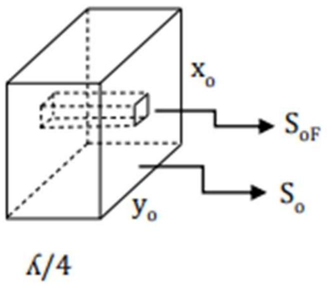


Figure 2 ($z = \lambda/4$ e $x_o y_o = S_o$)

Let P_o be the power that arrives at the surface S_o and I_o the power of the electromagnetic wave per unit area. I_o will be defined as follows:

$I_o = \frac{Po}{So}$ Eq. 15

Let n_F be the number of quanta in the volume S_o. In this way, S_o can be related to S_{oF} (area of incidence of a quantum of energy) as follows:

$S_o = n_F S_{oF}$ Eq. 16

$I_o = \frac{Po}{nF\ SoF}$ Eq. 17

Let $P_{oF} = \frac{Po}{nF}$ be the power of a single quantum incident on the area S_{oF} , then:

$I_o = \frac{PoF}{SoF} = I_{oF}$ (the quantum power per unit area S_{oF})

$I_{oF} = \frac{PoF}{SoF}$ Eq. 18

The energy E contained in the volume defined in **Figure 2** ($S_o\,\lambda/4$) will be:

$E = \frac{Po\,T}{4} = \frac{Po}{4f}$ Eq. 19

$E = n_F hf$ Eq. 20

Comparing equations **Eq. 19** and **Eq. 20**, we obtain:

$\frac{Po}{4f} = n_F hf$ Eq. 21

As already defined above, $\frac{Po}{nF} = P_{oF}$

$P_{oF} = 4hf^2$ Eq. 22

P_{oF} *is the power associated with a quantum of energy.*

The same result for the power relative to a quantum can also be obtained by dividing the energy of a quantum hf by $\frac{T}{4}$, where T is the period of the wave of frequency f.

6. QUANTUM AS AN ASSOCIATED COMPONENTS MODEL

Starting from **Eq. 13**, it results:

$$E(z,t) = \frac{x_0 y_0 \varepsilon_0 E_0^2}{2}\frac{k2\lambda}{4}, \qquad k_2 = 0,1,2,3...$$

For $k_2 = 1$, $E(z,t) = \frac{x_0 y_0 \varepsilon_0 E_0^2}{2}\frac{\lambda}{4} = \frac{x_0 y_0 c \varepsilon_0 E_0^2}{2}\frac{T}{4}$

$x_o.y_o E_0^2 = V_0^2$ (V_0 is the voltage associated with area $x_0.y_0$)

$$E(z,t) = \frac{c\varepsilon_0 V_0^2}{2}\frac{T}{4} \qquad \text{Eq. 23}$$

$$E(z,t) = c\varepsilon_0 \frac{V_0}{\sqrt{2}}\frac{V_0}{\sqrt{2}}\frac{T}{4}$$

Defining $Z_o = \frac{1}{c\varepsilon_0}$ (intrinsic impedance of the medium), and $\frac{V_0}{\sqrt{2}} = V_{0RMS}$, we obtain:

$$E(z,t) = \frac{1}{Z_o}\frac{V_0}{\sqrt{2}}\frac{V_0}{\sqrt{2}}\frac{T}{4} = \frac{V_{0RMS}}{Z_o} V_{0RM} \frac{T}{4} = I_{0RMS}.V_{0RMS}\frac{T}{4}$$

$$E(z,t) = I_{0RMS} V_{0RMS}\frac{T}{4} \quad \text{Eq. 24}$$

Considering just one quantum, **Eq. 23** becomes:

$$hf = \frac{c\varepsilon_0 V_{0F}^2}{2}\frac{T}{4} \qquad \text{Eq. 25}$$

V_{0F} in **Eq. 25** is the voltage associated with a single quantum of energy.

From **Eq. 25**, it can be deduced that:

$V_{0F} = \sqrt{\frac{\boldsymbol{8h}}{c\varepsilon_0}}.\boldsymbol{f} = 0.9992x\sqrt{2}\text{x}10^{-1}$ f **Eq. 26**

Note: V_{0F} depends only on the constants **h**, **c** and $\boldsymbol{\varepsilon_0}$ and the frequency **f** of the wave.

$\frac{V_{0F}}{\sqrt{2}} = V_{0FRMS}$, so we have:

$V_{0FRMS} = \sqrt{\frac{\boldsymbol{4h}}{c\varepsilon_0}}.\boldsymbol{f} = 0.9992x10^{-1}$ f **Eq. 27**

As can be seen in **Eq. 26**, $\sqrt{\frac{\boldsymbol{8h}}{c\varepsilon_0}} = 0.9992\text{x}\sqrt{2}\text{x}10^{-15}$. This provides a more compact way for the following relationships:

$\boldsymbol{c\varepsilon_0} = 1.0016x4h10^{30}$ **Eq. 28**

Notice that $Z_o = \frac{\boldsymbol{1}}{\boldsymbol{c\varepsilon_0}}$ (function of the Planck's constant)

In a similar way, it can be deduced that:

$\boldsymbol{c\mu_0} = 0.9984\ \frac{10^{-30}}{4h}$ **Eq. 29**

However, $H_0 = E_0\sqrt{\frac{\varepsilon_0}{\mu_0}}$, then:

$H_0 = 1.0016x4h10^{30}.E_0$ **Eq. 30**

Taking **Eq. 24** again for a single quantum, we can write:

$hf = I_{0FRMS}V_{0FRMS}\frac{T}{4}$ **Eq. 31**

From equations **Eq. 27** and **Eq. 31**, it can be deduced that:

$I_{0FRMS} = \sqrt{4hc\varepsilon_0}$ f **Eq. 32**

With the results of equations **Eq. 28** and **Eq. 32**, the expression for I_{0FRMS} becomes:

$I_{0FRMS} = 1.0008\text{x}4\text{h}10^{15}\text{f}$ **Eq. 33**

7. QUANTUM AS ENERGY STORED IN A CAPACITOR

The energy E_F stored in a capacitor C_F, subjected to a voltage V_{0F} (**Eq. 26**), is given by the expression below:

$E_F = \frac{1}{2} C_F \boldsymbol{V_{0F}^2}$ **Eq. 34**

Taking $E_F = hf$:

$V_{0F} = 0.9992x\sqrt{2}\text{x}10^{-15}\text{f}$

$C_F = 1.0016\text{x}\frac{\boldsymbol{h}}{\boldsymbol{f}}\ \boldsymbol{10^{30}}$ **Eq. 35**

8. MODIFIED EQUATIONS FOR ELECTROMAGNETIC WAVES

With **t** and **z** defined as in equations **Eq. 6.1** and **Eq. 11.1**, one can write the equations for the electric fields and magnetic intensity as below, considering only a wave segment containing $\mathbf{n}\frac{\Lambda}{4}$ wavelengths $\{n \in k_2\}$, starting in Z_0 and ending in Z_f (see **Figura 3** shown below).

$$\overrightarrow{E_x}(z,t)=E_0 \sin\left[\frac{2\pi}{\lambda}\left(z - \frac{cT}{4}k_1 - z_0\right)\right]\left\{U_{-1}\left[z - \left(\frac{cT}{4}k_1 + z_0\right)\right] - U_{-1}\left[z - \left(\frac{cT}{4}k_1 + z_f\right)\right]\right\}\overrightarrow{a_x} \quad \text{Eq. A.1}$$

$$\overrightarrow{H_y}(z,t)=H_0 \sin\left[\frac{2\pi}{\lambda}\left(z - \frac{cT}{4}k_1 - z_0\right)\right]\left\{U_{-1}\left[z - \left(\frac{cT}{4}k_1 + z_0\right)\right] - U_{-1}\left[z - \left(\frac{cT}{4}k_1 + z_f\right)\right]\right\}\overrightarrow{a_y} \quad \text{Eq. B.1}$$

The energy $\mathbf{E(z,t)}$ calculated from equations **Eq. A.1** and **Eq. B.1**, results in the expression found in equation **Eq. 13**, with $k_2 = n$.

From the equations above, the graph in **Figura 3**, below, can be drawn, considering only the representation of the electric field vector (**Eq. A.1**). For better clarity of the results, the representation of the vector $\overrightarrow{H_y}(z,t)$ was left out.

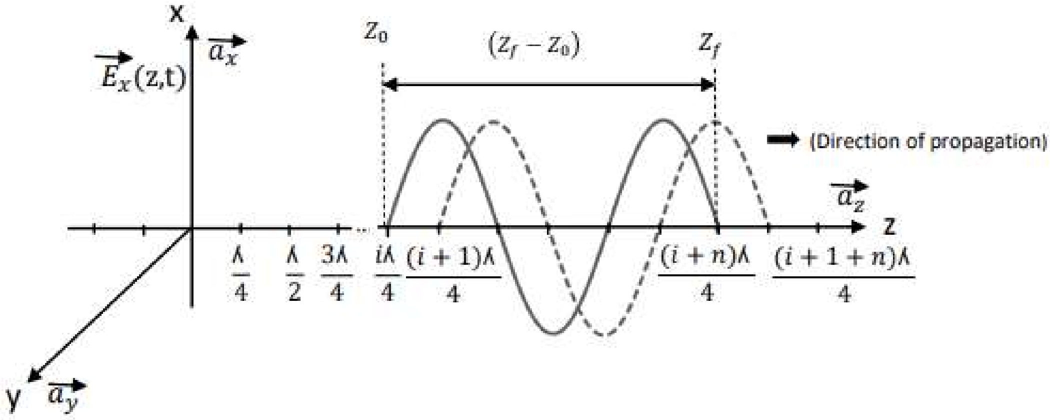


Figura 3

The equation **Eq. A.1** is represented in the graph in **Figure 3**, in two different positions. At each position along the propagation direction **z**, the wave remains "frozen" for a period equal to $\frac{T}{4}$. After this time, the electromagnetic wave jumps with infinite speed to a new position located $\frac{\Lambda}{4}$ ahead. ***Summarizing, electromagnetic waves propagate at the speed of light however they move in quantized jumps of Λ/4 with infinite speed, that is, instantly.*** This wave behavior gives consistency to the fact that each energy packet is indivisible, and at the same time that the energy packet disappears from the last quarter wave at the end of the path, another identical one is immediately established in the following quarter wave, that is, the total energy of the wave segment remains unchanged.

9. CONJECTURE

As demonstrated in this essay, the energy associated with electromagnetic waves accumulates instantly at each quarter of a wavelength, along the path taken by the wave, in discrete amounts equal to $\frac{x_0 y_0 \varepsilon_0 E_0^2}{2}\frac{\Lambda}{4}$.

However, the photoelectric effect as well as the Compton effect suggest that the displacement of electrons occurs as a result of collisions between particles. In both cases, the displacement of electrons occurs as a result of the interaction between a quantum and an electron, based on a certain energy level attributed to the quantum. This energy is given by the product of Planck's constant (h) and the irradiation frequency (f), that is, hf.

And then, how do the mass and velocity arise to generate the momentum in order to produce the impact and displacement of electrons?

In an attempt to answer this question, it is proposed in this essay that each volume of energy that accumulates along the propagation path of the electromagnetic wave, due to the fact that it is established instantly in each quarter of a wavelength, in a first instant manifests itself as an energy equal to the average value $m(x,y,z,t)c^2$, with m(x,y,z,t) being a ***dynamic rest mass***. This initial manifestation of energy is what we will call a ***photon*** and its energy is equal to hf. In the moments that follow, it is converted into pure electromagnetic energy, as given by **Eq.13**. This manifestation of dynamic inertial mass occurs only in the quarter cycle of the wave front. During the emergence of the dynamic inertial mass, the electric and magnetic fields undergo transients until they are fully re-established, and the electromagnetic wave advances to the next quarter cycle. This establishes a direct dependence between the mass and the electromagnetic fields of the wave.

The sudden and momentary emergence of this *dynamic rest mass* m(x,y,z,t) is what defines the corpuscular nature of electromagnetic waves and can produce momentum capable of displacing electrons.

In summary, this essay suggests that the ***photon*** is a *fugitive phenomenon and local*, with *dynamic rest mass* m(x,y,z,t) and possessing the same energy as a quantum, that is, ***the photon is a transient manifestation that will give rise to to quantum.***

Teaser: Can we refer to the region of the wave tail where a quantum of energy has been instantaneously removed as an **anti-photon**? How do the speed of light and the mass behave in that region, in the instant immediately after the sudden loss of energy?

10. COLORARY OF THE CONJECTURE IN ITEM 9, ABOVE

If the particles are a result of electromagnetic waves traveling in closed trajectories, their masses will also obey the same transience as that proposed in the context of Item 9, above. Therefore, it can be inferred that there is a primordial non-local property of empty space, responsible for the instantaneous gravitational entanglement between any two newly created particles, regardless of the distance between them.
Particles with transient masses could also explain quantum tunneling.

11. REFERENCE

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