

Revisited String Theory

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Abstract

We give an experimental verification of the String Theory and a modification of the black body theory in D – dimensional Universe related to this observation.

1.What is string theory?:

According to Gabriele Veneziano the String Theory is *“the assumption that all truly elementary particles, rather than being point like, are instead one-dimensional objects: strings”*[1].

At the quantum level a specific length is associated to any massive particle (the Compton wavelength):

$$\lambda_c = \frac{h}{mc} \quad (1)$$

Where $h = 6.626196 \cdot 10^{-34}$ joule second : Planck constant;

$c = 2.997924562 \cdot 10^8$ meter/second : speed of light in vacuum

m : mass of the corpuscle.

In string theory there is only one parameter the string tension F which play the role of the mass of the corpuscle. This tension is the energy per unit length of the string. We can also associate to the corpuscle the string length as [2]:

$$\lambda_s = \sqrt{\frac{hc}{F}} \quad (2)$$

At the deepest level of quantum theory there is only a single F_0 for all possible strings and so this tension corresponds to a fundamental inertia ξ_0 which is the basic building block of matter as the Compton wavelength will be a superposition of many fundamental strings λ_0 as:

$$\lambda_0 = \frac{h}{\xi_0 c} \quad (3)$$

And in this level we will have:

$$\frac{h}{\xi_0 c} = \sqrt{\frac{hc}{F_0}} \quad (4)$$

ξ_0 & F_0 : are universal constants.

The following steps will be to determine the constants ξ_0 & F_0 in a classic space time (Minkowski space time) and probably to connect this theory with the theory of gravitation.

Quantum strings acquire through quantum mechanics a characteristic size that permit to the string to respect the principle of uncertainty because a very large size costs a lot of energy and a small size implies a very large momentum which allow us to resolve problems in classical mechanics (including General Relativity) dealing with short distances without any singularities (quantization will enter at work).

A string is a superposition of many universal strings $\lambda_0 = \frac{c}{\nu_0}$ in a certain volume to give them density. So in vacuum those universal strings can be the base to estimate the value of vacuum energy density by statistical manner.

2-Vacuum Energy:

In a quasi-flat space time we can consider the energy $h\nu_0 = \xi_0 c^2$ as an optimum energy for all electromagnetic oscillators in their fundamental states: it is like a temperature of a radiation in a cavity considered as a black body. Then Planck model for black body radiation can be also useful as a model for vacuum energy density with a mean energy of the oscillator at low frequencies to be equal to about $\frac{1}{2}h\nu_0$ where ν_0 is an universal constant. The mean energy of the oscillator will be defined by analogy to Planck model of black body radiation as:

$$U = \frac{\sum_{n=0}^{\infty} \frac{n}{2} h\nu \exp\left(\frac{-\frac{n}{2}h\nu}{\frac{1}{2}h\nu_0}\right)}{\sum_{n=0}^{\infty} \exp\left(\frac{-\frac{n}{2}h\nu}{\frac{1}{2}h\nu_0}\right)} = \frac{\frac{1}{2}h\nu}{\exp\left(\frac{\nu}{\nu_0}\right)-1} \approx \frac{1}{2}h\nu_0 \text{ if } \nu \ll \nu_0 \quad (5)$$

The total energy density of the vacuum is then according to the model of the black body theory:

$$U_0 = \int_0^{\infty} \frac{8\pi\nu^2}{c^3} \cdot U d\nu = \int_0^{\infty} \frac{4\pi h}{c^3} \cdot \frac{\nu^3}{\exp\left(\frac{\nu}{\nu_0}\right)-1} d\nu = \frac{4\pi^5 h}{15 \cdot c^3} \cdot \nu_0^4 \quad (6)$$

The energy density of vacuum as given by General Relativity is as follows [3] :

$$U_0 = \frac{\Lambda \cdot c^4}{8\pi G} \approx 10^{-9} \text{Joule} \cdot \text{m}^{-3} \quad (7)$$

With :

$\Lambda = 1.088 \cdot 10^{-52} \text{m}^{-2}$: Cosmological constant;

$c = 2.997924562 \cdot 10^8 \text{m} \cdot \text{s}^{-1}$: Light celerity in vacuum;

$G = 6.67 \cdot 10^{-11} \text{SI units}$: Gravitationnel constant ;

Equating (6) & (7) we get:

$$\nu_0 = \left[\frac{15\Lambda.c^7}{32.\pi^6 G.h} \right]^{\frac{1}{4}} \approx 0.7149 \cdot 10^{12} \text{Hz} \quad (8)$$

It is an infra red frequency which means that Universe is quasi-flat.

Universe is also considered as black body at a temperature $T = 2.72548 \text{ K}$. If we consider the fundamental blocks of matter as a perfect gas filling the Universe than this “atom or molecule” of this gas will have an energy equal to $\frac{1}{2} k_B T$ along one space direction.

Considering the Universe as having $D - \text{space directions}$ than:

$$h\nu_0 = \frac{D-1}{2} k_B T \quad (9)$$

For $D = 3$ we get the particular Planck mean energy of an oscillator in the three space dimensional black body at an equilibrium temperature T .

We should not worry about that vacuum has a temperature because in equation (5) there is full quanta emission of $h\nu_0$. Remember that gases have spontaneous emission of radiation due to absorption of quanta from vacuum.

With $k_B = 1.38 \cdot 10^{-23} \text{ Joule Kelvin}$: Boltzmann constant ;

It comes that:

$$D - 1 = \frac{2h\nu_0}{k_B T} = \frac{2 \times 6.626196 \cdot 10^{-34} \times 0.7149 \cdot 10^{12}}{1.38 \cdot 10^{-23} \times 2.72548} = 25.19 \approx 25 \quad (10)$$

It is the same number of space dimensions predicted by String Theory in a quasi flat space-time (Minkowski space-time) [4], for a Universe filled with a perfect gas of bosons particles having the energy $h\nu_0$.

By taking $D = 26$ in equation (9) and going back we can determine all the other universal constants. This is an experimental verification of the coherence of the String Theory. For the Universe we can say that there is no big bang , not a pre-big bang but only a separation and agglomeration of the basic blocks of matter which gives us elementary particles, atoms, molecules , black holes , planets & galaxies.

3-Black body radiation in $D - \text{Dimensional Universes}$:

The mean energy of a Planck oscillator at low frequencies should be as given by equation (9) in $D - \text{dimensional Universe}$ and in general the mean energy of Planck oscillator is as follows:

$$E_{med} = \frac{h\nu}{\exp\left(\frac{h\nu}{k'_B T}\right) - 1} \quad (11)$$

With $:k'_B = \frac{D-1}{2} k_B$: Boltzmann constant in $D - \text{dimensional Universe}$.

In the form it is the same Planck mean energy by replacing Boltzmann constant k_B with k'_B in the ordinary Planck formulae. Of course we can obtain this formulae by statistical manner.

The total energy density in a cavity at equilibrium in a temperature T will be as follows [5]:

$$\rho_T = a_D T^{D+1} \quad (12)$$

With :

$$a_D = \left(\frac{2}{hc}\right)^D (\sqrt{\pi})^{D-1} \left(\frac{D-1}{2}\right)^{D+1} k_B^{D+1} D(D-1) \Gamma\left(\frac{D+1}{2}\right) \zeta(D+1) \quad (13)$$

$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt$: Gamma function.

$\zeta(x) = \frac{1}{\Gamma(x)} \int_0^\infty \frac{t^{x-1}}{e^t - 1} dt$: Riemann Zeta function (or $\zeta(x) = \sum_{n=1}^\infty n^{-x}$, $Re(x) > 1$)

$D > 1$: number of space dimensions of the Universe.

The generalized Stefan-Boltzmann law or the total energy emitted per unit time and per unit area of the cavity surface known as the *radiance* is as follows:

$$R_T = \sigma_D T^{D+1} \quad (14)$$

With :

$$\sigma_D = \left(\frac{2}{c}\right)^{D-1} (\sqrt{\pi})^{D-2} \left(\frac{D-1}{2}\right)^{D+1} \frac{k_B^{D+1}}{h^D} D(D-1) \Gamma\left(\frac{D}{2}\right) \zeta(D+1) \quad (15)$$

References:

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- [3] M. P. Hobson, G. P. Efstathiou and A. N. Lasenby 2006 "General Relativity" page 187, Cambridge University. [https://wigner.hu/~barta/GRcourse2020/Hobson%20M.P.,%20Efstathiou%20G.P.,%20Lasenby%20A.N.%20-%20GR%20-%20An%20Introduction%20for%20Physicists\(Cambridge,2006\).pdf](https://wigner.hu/~barta/GRcourse2020/Hobson%20M.P.,%20Efstathiou%20G.P.,%20Lasenby%20A.N.%20-%20GR%20-%20An%20Introduction%20for%20Physicists(Cambridge,2006).pdf)
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- [5] Tatiana R. Cardoso & Antonio S. de Castro "The blackbody radiation in D-Dimensional Universes" arxiv 0510002v1, <https://arxiv.org/abs/quant-ph/0510002>