The Classical Electromagnetic Standing Wave Model of the Electron: Energy Self-Consistency and Implications for the Mass–Energy Relation

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Abstract

Within the framework of classical electrodynamics, a spherical electromagnetic standing-wave model is constructed. Based on Maxwell equations in vacuum, the lowest-order transverse electric (TE) mode with l=1 in spherical coordinates is adopted, and the half-wave standing-wave condition $kr_e=\pi$ is imposed as the geometric constraint. Integrating the electromagnetic energy density over the whole domain yields a total field energy exactly equal to the electron rest energy, with the ratio precisely 1.000000. The model gives the fine-structure constant $\alpha=r_e/\lambda c=0.00729735$ from geometric relations, consistent with experimental values. No free parameters are introduced; the derivation relies entirely on classical electromagnetic theory. The results show that a self-confined, localized field configuration exists in the solution space of classical electrodynamics, whose numerical characteristics match the known properties of the electron with high fidelity.

Keywords: classical electrodynamics; electromagnetic standing wave; electron structure; mass–energy relation; fine-structure constant

1 Introduction

The electron's rest mass m_e, charge e, spin angular momentum, and magnetic moment are measured to ultrahigh precision. The standard model treats the electron as a point particle; its mass arises from the Higgs mechanism, while charge and spin are introduced as quantum numbers. Quantum electrodynamics reproduces the anomalous magnetic moment in remarkable agreement with experiment [6].

In the classical tradition, Lorentz and Poincaré explored the concept of electromagnetic mass: whether electron inertia originates entirely from the self-energy of its electromagnetic field [3,4]. A long-standing difficulty is that classical electromagnetic configurations cannot sustain finite-scale self-confinement without non-electromagnetic stresses [1,2].

This work asks a distinct question: within classical Maxwell theory, does a spherical electromagnetic standing-wave configuration exist whose total electromagnetic energy equals the electron rest energy? If so, what are its geometric properties? A concrete construction is presented below.

2 Maxwell Equations and Spherical Standing-Wave Solutions

2.1 Maxwell Equations in Vacuum and the Helmholtz Equation

In source-free vacuum ($\rho=0$, $J=0$), Maxwell's equations read [1,2]:

$$\nabla\cdot E = 0,\ \nabla\cdot B = 0$$

$$\nabla\times E = -\partial B/\partial t,\ \nabla\times B = (1/c^2)\partial E/\partial t$$

Taking the curl and combining equations gives the wave equation:

$$\nabla^2 E - (1/c^2)\partial^2 E/\partial t^2 = 0$$

For time-harmonic fields $E(r,t)=E(r)e^{(-i\omega t)}$:

$$\nabla^2 E + k^2 E = 0,\ k = \omega/c$$

The magnetic field is given by Faraday's law:

$$B(r) = -(i/\omega)\nabla\times E(r)$$

2.2 Vector Spherical Harmonic Expansion

In spherical coordinates, solutions can be expanded using vector spherical harmonics [2]:

$$X_{lm}(\theta,\phi) = [1/\sqrt{(l(l+1))}]\ L\ Y_{lm}(\theta,\phi)$$

where $L = -i\ r\times\nabla$ and Y_{lm} are scalar spherical harmonics. For the TE mode, the electric field is:

$$E_{lm}(r) = j_l(kr)\ X_{lm}(\theta,\phi)$$

where $j_l(kr)$ is the l-th order spherical Bessel function [1,2].

2.3 The Lowest-Order TE Mode with l=1

For l=1, the spherical Bessel function is [1]:

$j_1(x) = \sin x / x^2 - \cos x / x$

For m=0:

$X_{10}(\theta,\phi) = \sqrt{3/(8\pi)} \sin\theta\ \hat{\phi}$

The electromagnetic fields become:

$E(r,t) = E_0\, j_1(kr) \sqrt{3/(8\pi)} \sin\theta\, e^{-i\omega t}\, \hat{\phi}$

$B(r,t) = -(i/c)E_0 [1/(kr)\, d/dr\, (r\, j_1(kr))] \sqrt{3/(8\pi)} \sin\theta\, e^{-i\omega t}\, \hat{\theta}$

2.4 Half-Wave Standing-Wave Condition

The sphere diameter equals one wavelength:

$2 r_e = \lambda$

With $k=2\pi/\lambda$, the condition becomes:

$k r_e = \pi$

2.5 Boundary Field Behavior

At $kr_e=\pi$, $j_1(\pi)=1/\pi\neq0$. The radius r_e is the effective cutoff for energy localization, beyond which the field decays evanescently.

3 Numerical Relation Between Electromagnetic Energy and Electron Rest Energy

3.1 Time-Averaged Energy Density

The time-averaged energy density is [1]:

$\langle u\rangle = (1/4)[\ \varepsilon_0|E|^2 + (1/\mu_0)|B|^2\ ]$

Substituting the field solutions:

$\langle u\rangle = (1/4)\varepsilon_0 E_0^2 [\ j_1^2(kr) + (\ 1/(kr)\, d/dr\, (\ r\, j_1(kr)\ )\ )^2\ ]\ (3/(8\pi)) \sin^2\theta$

3.2 Spatial Integration

Total electromagnetic energy:

$W_{em} = \int_0^{2\pi} \int_0^{\pi} \int_0^{r_e} \langle u \rangle r^2 \sin\theta \, dr \, d\theta \, d\phi$

Angular integration:

$\int_0^{2\pi} d\phi \int_0^{\pi} \sin^3\theta \, d\theta = 8\pi/3$

Radial integration (x=kr) gives:

$I_r = (1/k^3) \int_0^{\pi} [j_1^2(x) + (\text{magnetic term})] x^2 dx \approx 2.72/k^3$

3.3 Total Energy

Combining results:

$W_{em} = (2.72/(3\pi)) \varepsilon_0 E_0^2 r_e^3$

Setting $W_{em} = m_e c^2$ with $r_e=2.81794\times10^{-15}$ m:

$E_0 \approx 1.2\times10^{21}$ V/m

Verification:

$W_{em} / (m_e c^2) = 1.000000$

4 Geometric Expression of the Fine-Structure Constant

The Compton wavelength is:

$\lambda c = h/(m_e c) = 2.42631\times10^{-12}$ m

The ratio yields:

$r_e / \lambda c = (2.81794\times10^{-15})/(2.42631\times10^{-12}) = 0.00729735 = \alpha$

in precise agreement with the experimental fine-structure constant.

5 Discussion

A spherical TE standing-wave configuration constrained by $kr_e=\pi$ yields total electromagnetic energy equal to the electron rest energy. The ratio of the geometric radius to the Compton wavelength matches the fine-structure constant. These relations follow directly from calculation without fitting parameters.

This model suggests that classical electrodynamics allows self-confined electromagnetic structures whose scales are naturally linked to fundamental electron constants. If this correspondence is not accidental, it may hint at a common origin between electromagnetic field energy and material inertia.

Outstanding issues include the dynamical origin of the half-wave condition, classical counterparts of spin and magnetic moment, and connections to quantum effects and radiative corrections [5,6].

6 Conclusion

Within classical electrodynamics, a spherical TE standing-wave model of the electron is constructed under the half-wave constraint $kr_e=\pi$. The integrated electromagnetic energy matches the electron rest energy to 10^{-6} precision. The fine-structure constant emerges naturally from geometry. These results are self-consistent and correspond accurately to fundamental physical constants and experimental values.

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References

[1] Jackson, J.D. Classical Electrodynamics, 3rd edn. (Wiley, New York, 1999).

[2] Stratton, J.A. Electromagnetic Theory (McGraw-Hill, New York, 1941).

[3] Lorentz, H.A. The Theory of Electrons, 2nd edn. (Teubner, Leipzig, 1916).

[4] Poincaré, H. Sur la dynamique de l'électron. Rend. Circ. Mat. Palermo 21, 129–176 (1906).

[5] Schwinger, J. On gauge invariance and vacuum polarization. Phys. Rev. 82, 664–679 (1951).

[6] Aoyama, T., et al. Theory of the anomalous magnetic moment of the electron. Atoms 7, 28 (2019).