Emergent Spacetime and Protomatter from Ollivier–Ricci Flow with Discrete Cartan Torsion

A Simplicial Complex Framework with
Feigenbaum Scaling and Hierarchical Renormalization

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Abstract

We propose a discrete geometric model in which classical spacetime and matter emerge from a discrete random network equipped with two combinatorial structures: Ollivier–Ricci curvature (ORC), a metric-free notion of curvature defined via optimal transport on the network, and discrete Cartan torsion, a 2-cochain on a weighted simplicial complex measuring holonomy defects of parallel transport around elementary triangles. Both structures are intrinsically combinatorial and require no background geometry; classical geometry is an output, not an input, of the model. Building on Trugenberger's ORC-based network model, which exhibits a phase transition between a random hyperbolic phase and a geometric phase, we augment the Ricci flow with a nonlinear torsion coupling and demonstrate, in an explicit four-node toy model, that the resulting dynamical system possesses two distinct basins of attraction whose separation is topologically robust and independent of the specific toy model chosen. Regions of the network with vanishing torsion condense into a one-dimensional geometric phase (embryonic spacetime), while regions with non-vanishing torsion condense into a localized, topologically non-trivial configuration identified as a *torsion defect* carrying all quantum numbers equal to zero. A key structural result is that the torsion-bearing fixed point P_T is a *saddle point* of the linearized discrete flow, with one expanding and one contracting direction in the (w, T) plane. The Jacobian entry $J_{Tw} = 8\lambda/9 > 0$ is derived exactly from the Wasserstein optimal transport, yielding real eigenvalues $\mu = 1 \pm \sqrt{8\lambda\eta \operatorname{sech}^2(T^*)/9}$ and a determinant $\det(J) = 1 - (8\lambda\eta/9)\operatorname{sech}^2(T^*) < 1$. We *conjecture*—as a structural analogy motivated by the geometry of Einstein–Cartan theory, but *not* derivable from the linearized dynamics—that this fixed point is geometrically associated with the parametrization of the *Cartan helix*. This conjecture motivates, but does not rigorously imply, the hierarchical level model introduced in Section 7, in which the rotation angle and expansion factor of the helix are tentatively associated with spin halving and mass scaling between levels. We further propose a hierarchical particle spectrum — the *level model* — in which each level is characterized by doubled spacetime dimension, halved spin, and mass scaling by a factor 4α, where α is the electromagnetic fine-structure constant and $\delta_F \approx 4.6692$ is the Feigenbaum period-doubling constant. The conservative flow has a saddle-like structure at P_T and cannot produce a Feigenbaum period-doubling cascade. We introduce a physically motivated *dissipative extension* of the torsion equation — a $-\xi \sin(T)$ restoring term — and show that the resulting effective torsion dynamics reduces, in the strongly dissipative limit, to the sine map $\tilde{T} \mapsto \xi \sin(\tilde{T})$ on $[0, \pi]$, which belongs to the **Feigenbaum universality class**. The Feigenbaum constant $\delta_F \approx 4.6692$ therefore emerges *dynamically* from the flow, and the empirical relation $\alpha \approx 1/(2\pi\delta_F^2)$ is found to be numerically consistent with this scaling, with the factor $2/\pi$ from the sine map amplitude. The physical origin of the dissipation is the geometric coarse-graining at each RG step: information is globally conserved, but the *geometric distinguishability* of sub-Planckian torsion configurations is reduced, creating equivalence classes in the sense of 't Hooft's dissipative quantum gravity program [82]. Our results and 't Hooft's program converge toward the same conceptual conclusion by independent routes: quantum-mechanical behavior can emerge from an effectively dissipative underlying structure, with information globally conserved. The continuum limit of the full construction is argued to reproduce Einstein–Cartan gravity in four dimensions. Falsifiable numerical predictions are formulated for simulation on synthetic networks.

Contents

1 Introduction

The question of how classical spacetime geometry emerges from a more fundamental, pre-geometric substrate is one of the deepest open problems in theoretical physics. Several approaches — loop quantum gravity [1], causal dynamical triangulations [2], tensor models [3], and quantum graphity [4] — share the hypothesis that the smooth manifold of general relativity (GR) is an emergent, low-energy description of a discrete combinatorial structure.

Among these, Trugenberger's network model [5, 6] is particularly elegant: it defines a statistical mechanics of random graphs in which the edge weights evolve under a discrete analogue of Hamilton's Ricci flow [7], guided by the Ollivier–Ricci curvature (ORC) [8, 9]. The model exhibits a sharp phase transition between a *random phase* (negative ORC, hyperbolic geometry, high entropy) and a *geometric phase* (positive ORC, Euclidean/spherical geometry, low entropy). The geometric phase produces networks that approximate smooth Riemannian manifolds with high fidelity.

A natural question is whether this framework can be extended to incorporate *torsion* — the antisymmetric part of the affine connection — which is absent in standard GR but central to the Einstein–Cartan theory [12–14], where torsion couples to the spin density of matter. The Einstein–Cartan theory is the minimal, consistent extension of GR that accounts for intrinsic spin as a source of spacetime geometry, and reduces to GR in vacuum.

In this paper we pursue the following interrelated goals:

(i) **Mathematical framework.** We define discrete Cartan torsion as a 2-cochain on a weighted simplicial complex and couple it nonlinearly to the Ollivier–Ricci flow. We use Bianconi's Network Geometry with Flavor (NGF) framework [10, 11] as the ambient combinatorial structure.

(ii) **Toy model analysis.** On a minimal four-node simplicial complex, we compute ORC values exactly via Wasserstein optimal transport, derive the discrete RG flow equations, and identify all fixed points. We prove, via a Lyapunov function, that the two basins of attraction are separated by an explicit curve, and show that this separation is *topologically robust*: it persists for any simplicial complex admitting a non-trivial torsion 2-cochain. We identify the torsion-free fixed point as having spectral dimension $d_s = 1$ (embryonic spacetime) and the torsion-bearing fixed point as a torsion defect. A central structural result is that the torsion-defect fixed point P_T is a *saddle point* of the linearized flow, with real eigenvalues $\mu = 1 \pm \sqrt{8\lambda\eta\,\mathrm{sech}^2(T^*)/9}$ (Section 5, Remark 4.2). We *conjecture*—as a structural analogy, not a derivation—that the geometric character of this fixed point is related to the Cartan helix, and use this conjecture as a bridge to the hierarchical level model of Section 7.

(iii) **Dissipative extension and dynamical emergence of δ_F.** The conservative ORC+torsion flow has a saddle-like fixed point P_T ($\det(J) < 1$) and cannot produce a Feigenbaum period-doubling cascade, because the reduced torsion circle map is a diffeomorphism of S^1. We introduce a physically motivated $-\xi \sin(T_\sigma)$ dissipative term in the torsion equation (Section 6). In the strongly dissipative limit, the effective torsion dynamics reduces to the Misiurewicz–Thurston sine map [84], which is in the Feigenbaum universality class [15, 17]. The Feigenbaum constant

δ_F therefore emerges dynamically, and $\alpha = 1/(2\pi\delta_F^2)$ follows, with the factor $2/\pi$ arising from the sine map normalization $\langle|\sin T|\rangle_{[0,\pi]} = 2/\pi$.

(iv) **Hierarchical level model and convergence with 't Hooft.** The mass ratio $4\alpha = 2/(\pi\delta_F^2)$ between consecutive levels is a derived consequence of the Feigenbaum cascade (Section 7). The dissipation parameter ξ quantifies geometric coarse-graining at each RG step, creating equivalence classes in the sense of 't Hooft [82]; our model and 't Hooft's program reach the same conceptual conclusion by independent routes.

The continuum limit of the discrete geometry is expected to reproduce Einstein–Cartan gravity (Section 8), though this limit is not yet rigorously established.

The paper is organized as follows. Section 2 reviews ORC, Ricci flow on graphs, and the Trugenberger model. Section 3 introduces the simplicial complex framework and defines discrete Cartan torsion. Section 4 presents the four-node toy model, the exact ORC computation, and the corrected Jacobian analysis revealing the saddle-point structure of P_T. Section 5 proves the topologically robust separation of basins and identifies the torsion defect. Section 6 introduces the dissipative extension, derives the effective sine map, and establishes the dynamical emergence of δ_F. Section 7 presents the hierarchical level model with 4α as a derived result. Section 8 discusses the continuum limit and Einstein–Cartan gravity. Section 9 formulates falsifiable numerical predictions. Section 10 concludes.

2 Background

2.1 Ollivier–Ricci Curvature

Let (G, w) be a weighted graph with vertex set V, edge set E, and positive weights $w : E \to \mathbb{R}_{>0}$. For each vertex $x \in V$, the *lazy random walk measure* with idleness parameter $\alpha_0 \in [0, 1]$ is

$$\mu_x^{\alpha_0}(y) = \begin{cases} \alpha_0 & \text{if } y = x, \\ \dfrac{(1-\alpha_0)\, w(x,y)}{\sum_{z\sim x} w(x,z)} & \text{if } y \sim x, \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

Definition 2.1 (Ollivier–Ricci curvature [9]). *The* Ollivier–Ricci curvature *of an edge* $(x, y) \in E$ *is*

$$\kappa(x,y) = 1 - \frac{W_1(\mu_x^{\alpha_0}, \mu_y^{\alpha_0})}{w(x,y)}, \tag{2}$$

where $W_1(\mu,\nu) = \inf_{\pi\in\Pi(\mu,\nu)} \sum_{a,b} d(a,b)\,\pi(a,b)$ *is the* L^1 *Wasserstein (earth-mover) distance and* $\Pi(\mu,\nu)$ *is the set of all couplings of* μ *and* ν*.*

The ORC generalizes the classical Ricci curvature of Riemannian geometry. In a smooth Riemannian n-manifold, along a geodesic with unit tangent vector $v = (y - x)/d(x,y)$, the Ollivier–Ricci curvature satisfies

$$\kappa(x,y) = \frac{1-\alpha_0}{2(n+2)} \operatorname{Ric}(v,v)\, d(x,y)^2 + O(d(x,y)^4), \tag{3}$$

where $\alpha_0 \in [0,1]$ is the idleness parameter of the random walk measure [9]. For $\alpha_0 = 0$ this reduces to the formula of [8], Theorem 1. Positive ORC indicates locally convergent geodesics (positively curved geometry); negative ORC indicates locally divergent geodesics (negatively curved, hyperbolic geometry).

2.2 Discrete Ricci Flow

Hamilton's Ricci flow [7] deforms the metric of a Riemannian manifold by $\partial_t g_{\mu\nu} = -2R_{\mu\nu}$. Its discrete analogue on weighted graphs, introduced by Lin, Lu, and Yau [21] and further developed by Ni et al. [23], updates edge weights at each step as

$$w_{ij}^{(n+1)} = w_{ij}^{(n)} \left(1 - \epsilon\, \kappa_{ij}^{(n)}\right), \tag{4}$$

where $\epsilon > 0$ is the step size. Edges with positive curvature shrink; edges with negative curvature grow. The flow drives the network toward a geometry of constant curvature.

2.3 Trugenberger's Network Model

Trugenberger [5, 6] defines a statistical mechanics of random graphs in which the Hamiltonian is a functional of the ORC. The partition function is

$$Z = \sum_G e^{-\beta H[G]}, \qquad H[G] = -\sum_{(i,j)\in E} \kappa_{ij}(G), \tag{5}$$

and the dynamics is governed by a Metropolis Monte Carlo algorithm that accepts or rejects edge rewirings according to ΔH.

The model exhibits a phase transition at a critical inverse temperature β_c:

- **Random phase** ($\beta < \beta_c$): networks are sparse, ORC is predominantly negative, geometry is hyperbolic, Hausdorff dimension is non-integer.
- **Geometric phase** ($\beta > \beta_c$): networks are denser, a smooth manifold-like geometry emerges; ground-state graphs are discretizations of negatively curved Cartan–Hadamard manifolds, with ORC converging to negative Ricci curvature at large scales [6].

This transition is analogous to a topological percolation transition and is the discrete counterpart of the formation of classical spacetime from a pre-geometric substrate.

3 Mathematical Framework

3.1 Weighted Simplicial Complexes

We generalize the graph-based setting to a weighted simplicial complex, following Bianconi's NGF framework [10, 11].

Definition 3.1 (Weighted simplicial complex)**.** *A weighted simplicial complex* (K, w) *consists of:*

- *A finite simplicial complex* K *of dimension* d*, with* k*-skeleton* $K_k = \{\sigma \in K : \dim \sigma = k\}$.

- *A weight function $w : K_1 \to \mathbb{R}_{>0}$ on 1-simplices (edges).*
- *Boundary operators $\partial_k : C_k(K;\mathbb{R}) \to C_{k-1}(K;\mathbb{R})$ defined by $\partial_k[v_0,\ldots,v_k] = \sum_{i=0}^{k}(-1)^i[v_0,\ldots,\hat{v}_i,\ldots,v_k]$.*

The *Hodge Laplacians* are defined as

$$L_k = \partial_k^T \partial_k + \partial_{k+1}\partial_{k+1}^T, \tag{6}$$

generalizing the graph Laplacian L_0 to higher-dimensional simplices. The spectral dimension of (K, w) is defined via the density of states of the graph Laplacian near zero eigenvalue [75, 76]:

$$d_s = -2 \left.\frac{d\log\rho(\lambda)}{d\log\lambda}\right|_{\lambda\to 0^+}, \tag{7}$$

where $\rho(\lambda)$ is the density of states of L_0.

3.2 Discrete Cartan Torsion

In the continuum, the torsion tensor is the antisymmetric part of the affine connection:

$$T^\lambda{}_{\mu\nu} = \Gamma^\lambda_{\mu\nu} - \Gamma^\lambda_{\nu\mu}. \tag{8}$$

On a simplicial complex, the natural discrete analogue of the connection is the parallel transport operator between adjacent simplices.

Definition 3.2 (Discrete Cartan torsion)**.** *Let $\sigma = [v_0, v_1, v_2] \in K_2$ be a 2-simplex with oriented boundary $\partial\sigma = [v_1, v_2] - [v_0, v_2] + [v_0, v_1]$. Assign to each oriented edge $e_{ij} = [v_i, v_j]$ a phase (connection) $\phi_{ij} \in \mathbb{R}/2\pi\mathbb{Z}$. The* discrete torsion *of σ is the holonomy defect*

$$T_\sigma = \phi_{01} + \phi_{12} - \phi_{02} \pmod{2\pi}. \tag{9}$$

This defines a 2-cochain $T \in C^2(K; \mathbb{R}/2\pi\mathbb{Z})$. When $T_\sigma = 0$ for all $\sigma \in K_2$, the parallel transport is path-independent (flat connection); non-vanishing T measures the failure of the triangular holonomy to close — the discrete analogue of a dislocation in the Cartan sense [12]. The identification of dislocations with torsion, and in particular the geometric construction of the Cartan helix as a homogeneous isotropic distribution of torsion built from helical motions, is treated in detail by Lazar and Hehl [37], to which we refer for the continuum background underlying our discrete framework.

Remark 3.3. *In the continuum limit, $T_\sigma \to \frac{1}{2}\int_\sigma T^\lambda{}_{\mu\nu} dx^\mu \wedge dx^\nu$ when the simplicial complex approximates a smooth manifold. The identification of T_σ with the area integral of the torsion tensor is consistent with the definition of torsion as the antisymmetrization of the connection 1-form [33].*

3.3 Discrete Hodge Star and Self-Duality

The discrete Hodge star $\star_k : C^k(K;\mathbb{R}) \to C^{n-k}(K;\mathbb{R})$ is defined via the weighted inner product

$$(\star_k\omega)_\tau = \sum_{\sigma:\sigma\supset\tau} w(\sigma,\tau)\,\omega_\sigma, \tag{10}$$

where $w(\sigma, \tau)$ are geometric weights derived from the simplicial volumes. On a 2-dimensional complex embedded in 4D, the self-duality condition is

$$T = \star T \iff T_\sigma = \textstyle\sum_{\tau\in K_2} w(\sigma,\tau)\,T_\tau. \tag{11}$$

Solutions to (11) are *torsion defects.*

3.4 Extended Ricci Flow with Torsion

We augment the standard discrete Ricci flow (4) with a torsion coupling. The *extended Ricci flow with Cartan torsion* is the coupled dynamical system

$$w_{ij}^{(n+1)} = w_{ij}^{(n)}\Big(1 - \epsilon\,\kappa_{ij}^{(n)} + \eta\tanh\Big(T_{ij}^{(n),\mathrm{eff}}\Big)\Big), \tag{12}$$

$$T_\sigma^{(n+1)} = T_\sigma^{(n)} + \lambda \sum_{e\in\partial\sigma} \kappa_e^{(n)}\, w_e^{(n)} \pmod{2\pi}, \tag{13}$$

where:

- $\epsilon > 0$ is the Ricci flow step size,
- $\eta > 0$ is the torsion–curvature coupling,
- $\lambda > 0$ is the torsion feedback strength,
- $T_{ij}^{\mathrm{eff}} = |\mathcal{T}_{ij}|^{-1}\sum_{\sigma\ni e_{ij}} T_\sigma$ is the effective torsion on edge e_{ij}, averaged over the triangles containing it.

The nonlinear tanh in (12) ensures that the torsion contribution is bounded — large torsion saturates at $\pm\eta$ rather than diverging. Several physically and mathematically motivated reasons support this choice:

(i) *Boundedness.* The torsion $T_\sigma \in \mathbb{R}/2\pi\mathbb{Z}$ is a phase, so its physical effect on curvature should be periodic and bounded. The function tanh maps $\mathbb{R} \to (-1, 1)$, providing a smooth interpolation between $-\eta$ (large negative torsion) and $+\eta$ (large positive torsion).

(ii) *Correct symmetry.* tanh is odd, ensuring that the flow treats positive and negative torsion symmetrically. Any odd function with bounded range and monotone behavior would share this property.

(iii) *Analogy with the nonlinear sigma model.* In field theory, the *nonlinear sigma model* (NLSM) is a scalar field theory on a target manifold $\mathcal{M}$ (rather than a flat space), defined by the action $S = \int g_{ab}(\phi)\partial_\mu\phi^a\partial^\mu\phi^b$, where g_{ab} is the metric on $\mathcal{M}$ [78]. When $\mathcal{M} = S^1$ (the circle), the target-space metric introduces a nonlinear coupling between the field and its gradient that saturates as the field approaches the poles. Our torsion variable $T \in \mathbb{R}/2\pi\mathbb{Z}$ lives on S^1, and the tanh coupling is the discrete analogue of this saturation: it is the natural nonlinear response function for a field taking values on a circle.

(iv) *Statistical mechanics interpretation.* The function tanh is proportional to the derivative of log cosh, which is a smooth approximation to the absolute value. In statistical mechanics, the partition function of the Ising model on a graph at inverse temperature β with nearest-neighbor interaction strength J contains the factor $\tanh(\beta J)$ as the bond weight in the high-temperature expansion [79, 80]. This is the natural saturable response function for binary spin interactions, mapping the real line to $(-1, 1)$ and encoding ferromagnetic ($\beta J > 0$) or antiferromagnetic ($\beta J < 0$) couplings symmetrically. Identifying the torsion T_σ with an effective spin variable and η with the interaction strength, the ORC+torsion flow has the structure of gradient descent on a discrete free energy.

We emphasize that while tanh is the natural and most robust choice, the qualitative results of Section 5 (existence of two basins, topological robustness) hold for any odd, bounded, monotone function $f : \mathbb{R} \to (-1, 1)$ with $f(0) = 0$, since these properties are determined by the sign structure of ΔV rather than by the specific form of f. The tanh choice is thus convenient but not essential for the main results.

The action functional corresponding to this flow is

$$S[K, w, T] = \sum_{e \in K_1} \kappa(e)\, w(e) + \alpha_T \sum_{\sigma \in K_2} |T_\sigma|^2 + \beta_T \sum_{\sigma \in K_2} f(T_\sigma, \kappa(\partial\sigma))\,, \tag{14}$$

where f is the nonlinear torsion–curvature coupling term and α_T, β_T are coupling constants.

Remark 3.4 (Functional measure and gauge structure)**.** *The partition function associated with* (14) *is formally*

$$Z = \int \mathcal{D}\phi\, \mathcal{D}w\; e^{-S[K,w,T]}, \quad \mathcal{D}\phi = \prod_{e \in K_1} \frac{d\phi_e}{2\pi}, \tag{15}$$

where $\phi_e \in (-\pi, \pi]$ is the U(1) connection on each edge and w denotes the collection of edge weights. The measure $d\phi_e/(2\pi)$ is the normalized Haar measure on U(1), ensuring gauge invariance under $\phi_e \to \phi_e + \chi_{v_j} - \chi_{v_i}$ for any vertex function χ. Discrete gauge fixing can be achieved by fixing the connection on a spanning tree of K_1 [77]. The Ricci flow equations (12)*–*(13) *are the gradient flow of S with respect to w and T, and their fixed points are the stationary points of* (14)*.*

Remark 3.5 (Connection between ORC and torsion)**.** *The two geometric structures — Ollivier-Ricci curvature and discrete Cartan torsion — enter the model in complementary roles. ORC measures the* convergence or divergence of geodesics*: positive ORC means neighborhoods are closer in Wasserstein distance than in graph distance (spherical/Euclidean geometry), negative ORC means they diverge (hyperbolic geometry). Discrete torsion measures the* failure of parallel transport to close*: a non-vanishing holonomy defect T_σ signals a dislocation in the combinatorial geometry [37]. In the extended flow* (12)*–*(13)*, torsion acts as a* source of curvature*: the $\tanh(T_{ij}^{\text{eff}})$ term in* (12) *modifies the curvature evolution, while the $\kappa_e w_e$ term in* (13) *feeds curvature back into the torsion. This mutual coupling is the discrete analogue of the Einstein–Cartan field equations, in which torsion couples algebraically to spin density and back-reacts on curvature [59].*

4 Four-Node Toy Model

4.1 Setup

We consider the minimal non-trivial simplicial complex with two triangles:

$$K = \big(\{0, 1, 2, 3\},\ K_1 = \{01, 02, 12, 13, 23\},\ K_2 = \{012, 123\}\big). \tag{16}$$

The two triangles share the edge $(1, 2)$. Initial weights are uniform: $w_{ij}^{(0)} = 1$ for all $(i, j) \in K_1$. Initial torsion values are $T_{012}^{(0)} = T_0$ and $T_{123}^{(0)} = T_1$, with $T_0, T_1 \in [0, 2\pi)$.

The degree sequence is $d_0 = d_3 = 2$, $d_1 = d_2 = 3$. The edge $(1, 2)$ is contained in both triangles; all other edges in exactly one.

4.2 ORC Computation

We compute the ORC for each edge using the Ollivier formula (2) with $\alpha_0 = 0$ (non-lazy walk) and uniform weights.

Proposition 4.1 (ORC values, uniform weights)**.** *With $w_{ij} = 1$ for all edges and non-lazy random walk ($\alpha_0 = 0$), the ORC values are:*

$$\kappa_{01} = \kappa_{02} = \kappa_{13} = \kappa_{23} = \frac{1}{3}, \qquad \kappa_{12} = \frac{2}{3}. \tag{17}$$

Proof. We compute directly from the Wasserstein optimal transport plan (2).

Edge $(1, 2)$. The uniform random walk measures are $\mu_1 = \frac{1}{3}(\delta_0 + \delta_2 + \delta_3)$ and $\mu_2 = \frac{1}{3}(\delta_0 + \delta_1 + \delta_3)$. The optimal coupling matches $\delta_0 \to \delta_0$ (cost 0), $\delta_3 \to \delta_3$ (cost 0), and $\delta_2 \to \delta_1$ (cost 1): $W_1(\mu_1, \mu_2) = \frac{1}{3}$, giving $\kappa_{12} = 1 - \frac{1}{3} = \frac{2}{3}$.

Edge $(0, 1)$. $\mu_0 = \frac{1}{2}(\delta_1 + \delta_2)$, $\mu_1 = \frac{1}{3}(\delta_0 + \delta_2 + \delta_3)$. The optimal plan matches $\frac{1}{3}$ of δ_2 to δ_2 (cost 0); the residual $\frac{1}{6}$ of δ_2 goes to δ_0 (cost 1); $\frac{1}{3}$ of δ_1 goes to δ_3 (cost 1) and $\frac{1}{6}$ of δ_1 goes to δ_0 (cost 1): $W_1(\mu_0, \mu_1) = \frac{1}{6} + \frac{1}{3} + \frac{1}{6} = \frac{2}{3}$, giving $\kappa_{01} = 1 - \frac{2}{3} = \frac{1}{3}$. By the symmetry of K, the same value holds for $(0, 2), (1, 3), (2, 3)$. □

All curvatures are positive, confirming that the uniform-weight configuration lies in the geometric phase. The weighted mean curvature is $\bar{\kappa} = (4 \cdot \frac{1}{3} + 1 \cdot \frac{2}{3})/5 = \frac{2}{5}$.

4.3 Effective Torsion on Edges

The effective torsion values on each edge are:

$$\begin{aligned} T_{01}^{\text{eff}} &= T_{02}^{\text{eff}} = T_{012}, \\ T_{12}^{\text{eff}} &= \tfrac{1}{2}(T_{012} + T_{123}), \\ T_{13}^{\text{eff}} &= T_{23}^{\text{eff}} = T_{123}. \end{aligned} \tag{18}$$

4.4 Explicit Flow Equations

Substituting (17) and (18) into (12)–(13), the flow equations at $w = 1$ become:

$$w'_{01} = 1 - \tfrac{\epsilon}{3} + \eta \tanh(T_{012}), \tag{19}$$

$$w'_{12} = 1 - \tfrac{2\epsilon}{3} + \eta \tanh\left(\tfrac{T_{012}+T_{123}}{2}\right), \tag{20}$$

$$T'_{012} = T_{012} + \lambda(\kappa_{01} + \kappa_{02} + \kappa_{12}) = T_{012} + \lambda\left(\tfrac{1}{3} + \tfrac{1}{3} + \tfrac{2}{3}\right) = T_{012} + \tfrac{4\lambda}{3}, \tag{21}$$

$$T'_{123} = T_{123} + \tfrac{4\lambda}{3}. \tag{22}$$

(Equations for w_{02}, w_{13}, w_{23} follow by symmetry from (19).)

4.5 Jacobian Analysis and the Cartan Helix

We compute the Jacobian of the flow map $F : (w, T) \mapsto (w', T')$ at the fixed point (w^*, T^*).

Diagonal entry J_{ww}. The linearization of (12) yields the diagonal response:

$$\partial_w(\kappa w)\big|_{w^*=1} = \kappa^* + w^* \partial_w \kappa. \tag{23}$$

For the ORC on this graph, the weighted mean curvature is $\bar{\kappa} = (4 \cdot \frac{1}{3} + 1 \cdot \frac{2}{3})/5 = 2/5$ (this value is *exact*). The response $\partial_w \kappa|_{w=1}$ is approximately $-2/5$ (numerical finite-difference), giving $J_{ww} \approx 1$.

Exact derivation of the off-diagonal entry J_{Tw}. The torsion update for triangle $\{012\}$ is

$$T'_{012} = T_{012} + \lambda\, S(w), \qquad S(w) \equiv \kappa_{01}(w)\, w_{01} + \kappa_{02}(w)\, w_{02} + \kappa_{12}(w)\, w_{12}. \tag{24}$$

The Jacobian entry is $J_{Tw} = \lambda\, dS/dw|_{w=1}$. To compute this we must account for the fact that varying $w_{12} = w$ (the shared edge) changes not only κ_{12} but also κ_{01} and κ_{02}, because the degree of nodes 1 and 2 depends on w.

With $w_{12} = w$ and all peripheral weights $w_{01} = w_{02} = w_{13} = w_{23} = 1$, the non-lazy Wasserstein calculation gives:

$$\kappa_{12}(w) = 1 - \frac{w}{2+w} = \frac{2}{2+w}, \tag{25}$$

$$\kappa_{01}(w) = \kappa_{02}(w) = \frac{w}{2+w}. \tag{26}$$

Equation (26) follows from the Wasserstein plan for edge $(0,1)$: the degree of node 1 is $2+w$, giving $\mu_1 = \frac{1}{2+w}\delta_0 + \frac{w}{2+w}\delta_2 + \frac{1}{2+w}\delta_3$, and the optimal transport cost is $W_1 = 2/(2+w)$, so $\kappa_{01} = 1 - 2/(2+w) = w/(2+w)$. (At $w = 1$ both formulas reproduce $\kappa_{01} = 1/3$ and $\kappa_{12} = 2/3$ as required by Proposition 4.1.)

Substituting into $S(w)$ with $w_{01} = w_{02} = 1$ and $w_{12} = w$:

$$S(w) = \frac{w}{2+w} \cdot 1 + \frac{w}{2+w} \cdot 1 + \frac{2}{2+w} \cdot w = \frac{4w}{2+w}. \tag{27}$$

The derivative is

$$\frac{dS}{dw} = \frac{4(2+w) - 4w}{(2+w)^2} = \frac{8}{(2+w)^2}, \tag{28}$$

and at $w^* = 1$:

$$\left.\frac{dS}{dw}\right|_{w=1} = \frac{8}{9}. \tag{29}$$

Therefore the off-diagonal Jacobian entry is

$$\boxed{J_{Tw} = \frac{8\lambda}{9} > 0.} \tag{30}$$

Note that this value is *positive*, not negative: increasing the central edge weight increases the sum of curvatures around the triangle, which accelerates the torsion.

The corrected Jacobian. The full Jacobian at (w^*, T^*) is:

$$J = \begin{pmatrix} 1 & \eta\, \mathrm{sech}^2(T^*) \\ +\dfrac{8\lambda}{9} & 1 \end{pmatrix}, \tag{31}$$

with characteristic equation $(1-\mu)^2 - \frac{8\lambda\eta}{9}\, \mathrm{sech}^2(T^*) = 0$ and eigenvalues

$$\mu_{1,2} = 1 \pm \sqrt{\frac{8\lambda\eta}{9}\, \mathrm{sech}^2(T^*)}. \tag{32}$$

Since all parameters are positive, the square root is real and positive. Both eigenvalues are *real*: $\mu_1 = 1 + r > 1$ and $\mu_2 = 1 - r$, where $r = \sqrt{8\lambda\eta\,\mathrm{sech}^2(T^*)/9}$. At the toy-model values $\eta = \lambda = 1$, $T^* = \ln 2$ ($\mathrm{sech}^2(\ln 2) = 16/25$): $r = \sqrt{128/225} = 8\sqrt{2}/15 \approx 0.754$. Thus $\mu_1 \approx 1.754 > 1$ and $\mu_2 \approx 0.246 \in (0, 1)$.

The fixed point P_T is therefore a *saddle* of the linearized discrete flow: perturbations along the unstable eigendirection grow ($\mu_1 > 1$) while perturbations along the stable eigendirection shrink ($\mu_2 < 1$). There is *no rotation* in the (w, T) plane.

The determinant is

$$\det(J) = 1 - \frac{8\lambda\eta}{9}\,\mathrm{sech}^2(T^*) < 1, \tag{33}$$

so the map is *area-contracting* at P_T even in the conservative case. Numerically, $\det(J) \approx 0.43$ at the toy-model fixed point.

Remark 4.2 (The Cartan helix as a conjecture)**.** *The corrected linearized analysis shows that P_T is a saddle, not a spiral source. There is therefore* no derivation *of the Cartan helix from the linearized flow: the orbit structure is hyperbolic, not helical.*

Nevertheless, the Cartan helix remains a geometrically motivated conjecture*: Lazar and Hehl [37] showed that the helicoid $(\rho\cos\theta, \rho\sin\theta, p\theta/2\pi)$ represents, in the continuum, a homogeneous and isotropic distribution of torsion with constant internal torque in 3d Einstein–Cartan gravity. Since our model produces torsion defects embedded in a 4D Einstein–Cartan continuum limit (Section 8), the Cartan helix is the natural geometric object to associate with a localized torsion configuration. We retain the identification of the expansion factor ρ with mass scaling and of the angle θ with spin halving as an* external conjecture *motivated by this geometric reasoning, and present it as a research program rather than a derived result (Section 7). Whether a non-linear or global extension of the flow produces genuine spiral structure remains an open problem.*

5 Fixed Points, Basins of Attraction, and Torsion Defects

5.1 Classification of Fixed Points

The fixed-point conditions $(w', T') = (w^*, T^*)$ reduce to:

$$\kappa^*_{ij} = \eta\tanh\left(T^{*,\mathrm{eff}}_{ij}\right) \quad \forall (i,j) \in K_1, \tag{34}$$

$$\sum_{e\in\partial\sigma} \kappa^*_e w^*_e = 0 \pmod{2\pi/\lambda} \quad \forall \sigma \in K_2. \tag{35}$$

Fixed point P_0 (torsion-free sector). Setting $T^* = 0$, equation (34) requires $\kappa^* = 0$ for all edges. The network sits at flat geometry. We denote this sector K_0.

Fixed point P_T (torsion-bearing sector). For the symmetric ansatz $T^*_{012} = T^*_{123} = T^*$, condition (35) on triangle $\{012\}$ with the correct curvature values $\kappa^*_{01} = \kappa^*_{02} = \kappa^*$, $\kappa^*_{12} = 2\kappa^*$ gives:

$$\kappa^*(1 + 1 + 2)w^* = 4\eta\tanh(T^*) = 0 \pmod{2\pi/\lambda}, \tag{36}$$

with solutions

$$T_n^* = \tanh^{-1}\left(\frac{\pi n}{2\eta\lambda}\right), \quad n \in \mathbb{Z}. \tag{37}$$

These solutions exist and are real if and only if $2\eta\lambda \geq \pi$, i.e.,

$$\eta\lambda \geq \eta\lambda_c := \frac{\pi}{2}. \tag{38}$$

Proposition 5.1 (Existence of torsion-bearing fixed point)**.** *The fixed point P_T with non-vanishing torsion exists if and only if*

$$\eta\lambda \geq \eta\lambda_c := \frac{\pi}{2}. \tag{39}$$

5.2 Lyapunov Function and Basin Separation

Theorem 5.2 (Separation of basins)**.** *For $\eta\lambda > \pi/2$, the extended Ricci flow (12)–(13) has two distinct basins of attraction in the (T_{012}, T_{123}) plane, separated by the curve*

$$T_{012} + T_{123} = T_c^{\text{sep}} \equiv \frac{2\epsilon}{3\lambda} + \frac{3}{8}. \tag{40}$$

Configurations with $T_{012} + T_{123} < T_c^{\text{sep}}$ converge to P_0; configurations with $T_{012} + T_{123} > T_c^{\text{sep}}$ are repelled from P_T along the unstable manifold and eventually condense into the torsion-bearing torsion-defect sector. (The linearized flow near P_T is hyperbolic with real eigenvalues; the global evolution is conjectured to map onto the Cartan helical structure, but this is not derivable from the local dynamics — see Remark 4.2.)

Proof. Define the Lyapunov functional

$$V(w, T) = \sum_{(i,j)\in K_1} w_{ij}(\kappa_{ij})^2 + \mu \sum_{\sigma\in K_2} (T_\sigma)^2, \tag{41}$$

with $\mu > 0$ to be determined.

Along the flow, $\Delta V = V^{(n+1)} - V^{(n)}$ has two contributions.

Curvature term. $\Delta(\sum w\kappa^2) \approx -2\epsilon \sum w(\kappa^*)^2 + O(\epsilon^2)$. With the correct values (17): $\sum w\kappa^2 = 4 \cdot (1/3)^2 + (2/3)^2 = 4/9 + 4/9 = 8/9$. So $\Delta(\sum w\kappa^2) \approx -16\epsilon/9 + O(\epsilon^2) \leq 0$.

Torsion term. $\Delta(\sum T^2) = 2\lambda \sum_\sigma T_\sigma \cdot \sum_{e\in\partial\sigma} \kappa_e w_e + O(\lambda^2)$. With values (17): $\sum_{e\in\partial\{012\}} \kappa_e = 1/3 + 1/3 + 2/3 = 4/3$. So:

$$\Delta\left(\sum T^2\right) = \frac{8\lambda}{3}(T_{012} + T_{123}) + O(\lambda^2). \tag{42}$$

Setting $\Delta V = 0$ with $\mu = 1$:

$$\frac{16\epsilon}{9} = \frac{8\lambda}{3}(T_{012} + T_{123}), \tag{43}$$

which rearranges to $T_{012} + T_{123} = 2\epsilon/(3\lambda)$. Including the $O(\lambda^0)$ contribution from the fixed-point torsion threshold (38) gives (40). Below the separatrix $\Delta V < 0$: the flow converges to P_0. Above the separatrix the curvature term is dominated by the torsion term and the system is driven toward the torsion-bearing sector. □

Remark 5.3 (Topological independence)**.** *The existence of two basins $\mathcal{B}_0$ and $\mathcal{B}_T$ is a consequence of the sign structure of ΔV and does not depend on the specific numerical values of κ or on the topology of the toy model. Any simplicial complex admitting a non-trivial 2-cochain T and a Ricci flow with torsion coupling satisfying $\eta\lambda > \eta\lambda_c$ will exhibit the same bifurcation into a torsion-free and a torsion-bearing sector. The precise location of the separatrix (40) is model-dependent; the existence of the two sectors is not.*

5.3 The Discrete Sequence of Torsion-Defect Sectors

The torsion-bearing fixed points form a discrete countable family. From (37), with the symmetric ansatz, the n-th torsion-defect sector corresponds to torsion value

$$T_n^* = \tanh^{-1}\left(\frac{\pi n}{2\eta\lambda}\right), \quad n = 1, 2, 3, \ldots \tag{44}$$

This sequence is finite: real solutions exist only for $n < 2\eta\lambda/\pi$, so for any finite coupling $\eta\lambda$ there are only finitely many torsion-defect sectors.

For large n, the sequence is asymptotically equally spaced: $T_n^* \approx \frac{n-1}{2} \ln 2 + \text{const}$, giving a step $\Delta T^* = (\ln 2)/2$ per level. However, this asymptotic approximation is *not* internally consistent with the exact formula (44) at small n. The exact formula gives $\tanh(T_n^*) = \pi n/(2\eta\lambda)$, which grows *linearly* in n; the inverse hyperbolic tangent of a linear sequence is not itself linear, but grows logarithmically as $\tanh(T^*) \to 1$. For the parameter constraint of Section 7.6, we therefore use the *exact* values from (44) rather than the asymptotic approximation. Each sector $P_{T,n}$ is a saddle point (Remark 4.2) with the same hyperbolic orbit geometry but increasing torsion amplitude. The torsion-defect action at the n-th sector is

$$S_n = 5\eta^2 \tanh^2(T_n^*) + 2\mu(T_n^*)^2, \tag{45}$$

where the factor 5 is the total number of edges in K_1, all carrying the same curvature $\kappa^* = \eta \tanh(T_n^*)$ at the symmetric fixed point (see Section 7.6 for details).

Remark 5.4 (The torsion map is non-period-doubling)**.** *The reduced torsion map* $\theta \mapsto \theta + (4\lambda\eta/3)\tanh\theta \pmod{2\pi}$ *has derivative* $f'(\theta) = 1 + (4\lambda\eta/3)\,\text{sech}^2\,\theta > 0$ *everywhere for* $\lambda, \eta > 0$*. It is therefore a strictly orientation-preserving diffeomorphism of* S^1*, which cannot exhibit period-doubling bifurcations for any positive parameter values. The discrete sequence* (44) *arises from the fixed-point conditions of the full 2D flow* (w, T)*, not from a period-doubling cascade of the reduced map.*

5.4 Spectral Dimension and the Embryonic Spacetime

The torsion-free fixed point P_0 is, in our opinion, the most physically significant result of the paper: it is a candidate for the geometric origin of spacetime itself. We now characterize it carefully.

Spectral dimension. The spectral dimension d_s of a network is defined via the random-walk return probability $P(t) \sim t^{-d_s/2}$ as $t \to \infty$ [75, 76], or equivalently via the density of states of the graph Laplacian near zero eigenvalue. For a d-dimensional regular lattice, $d_s = d$ exactly.

At the torsion-free fixed point P_0, the curvature vanishes ($\kappa^* = 0$) and all edge weights relax to a uniform value. The resulting network has a nearly tree-like, path-like topology. For a path graph on N nodes, the Laplacian eigenvalues are $\lambda_k = 2(1 - \cos(k\pi/N)) \approx k^2\pi^2/N^2$ for small k, giving a density of states $\rho(\lambda) \sim \lambda^{-1/2}$. The spectral dimension formula (7) then gives $d_s = 2 \times (1/2) \times (-1) \times (-1) = 1$.

More precisely, by the Ollivier–Villani convergence theorem, the ORC of the condensed network approaches the Laplacian eigenvalue gap as $N \to \infty$, and a network with Laplacian gap $\Delta \sim 1/N$ (path-like) has $d_s \to 1$ in this limit [5].

Corollary 5.5 (Embryonic spacetime)**.** *The torsion-free basin $\mathcal{B}_0$ condenses, under the extended Ricci flow, to a network of spectral dimension $d_s = 1$. This is the* embryonic spacetime *of the level model: the one-dimensional geometric substrate (level 1) from which higher-dimensional spacetime emerges through iterative RG steps.*

Why dimension 1, not 4? The spectral dimension $d_s = 1$ at P_0 is not the observed $3 + 1$ dimensions of macroscopic spacetime. Rather, it is the *starting point* of the hierarchical level model: the ORC flow produces a one-dimensional condensate (level 1), and the full four-dimensional spacetime emerges through the iteration of the level-doubling RG operator $\mathcal{D} : \mathrm{Dim} \mapsto 2\,\mathrm{Dim}$ applied twice (levels $1 \to 2 \to 4$). In this picture, the two massless levels (scalar graviton at level 1 and photon at level 2) represent intermediate geometric phases that exist only at the Planck scale; the observed four-dimensional spacetime at level 3 carries the electron as the lightest massive constituent.

Topological robustness. The identification $d_s(P_0) = 1$ is independent of the specific toy model geometry (Remark 5.3). Any simplicial complex whose torsion-free sector condenses to a tree-like network will exhibit $d_s \to 1$, because tree-like networks universally have one-dimensional spectral geometry. This robustness is what makes the result physically meaningful: it is not an artifact of the four-node setup but a structural consequence of the ORC flow dynamics.

5.5 The Torsion-Bearing Fixed Point as a Torsion Defect

We now show that P_T satisfies the self-duality condition (11).

Proposition 5.6 (Self-dual fixed points of the discrete torsion flow)**.** *Let K be a finite oriented simplicial complex embedded in a 4-dimensional Euclidean space, and let $T \in C^2(K, \mathbb{R})$ evolve according to the torsion flow (13). Assume a discrete Hodge star $\star : C^2(K) \to C^2(K)$ induced by the embedding. Then any configuration T^* satisfying:*

(i) T^* is a fixed point of the dynamics; and

(ii) $\star T^* = T^*$ (self-duality),

is called a self-dual stationary configuration of the discrete flow. *The set of self-dual stationary configurations is an invariant subset of the dynamical system (whenever the dynamics is continuous in $C^2(K)$).*

With uniform weights $w^*_{ij} = 1$ and area forms $A_{012} = A_{123} = 1$, the Hodge star on K (embedded in $\mathbb{R}^4$) gives $(\star T)_{012} = T^*_{012}$ and $(\star T)_{123} = T^*_{123}$, so the self-duality condition is satisfied at P_T.

Remark 5.7 (Relation to gauge-theoretic instantons)**.** *The self-dual fixed points of Proposition 5.6 are* not *instantons in the sense of gauge theory or Euclidean general relativity. Standard instantons require: a principal bundle with gauge group, equations of motion derived from an action functional, finite Euclidean action, and an integer-valued topological charge [73, 74, 93]. The self-dual fixed points satisfy only the self-duality condition, and constitute a purely formal analogy limited to this property. We use the shorter term* torsion defect *throughout the paper, while keeping "self-dual" as the precise mathematical descriptor.*

Remark 5.8 (Dimensional consistency of self-duality)**.** *In standard Discrete Exterior Calculus (DEC) on a purely 2-dimensional complex, the Hodge star maps 2-cochains to*

0-cochains of the dual complex: $\star_2 : C^2(K) \to C^0(K^*)$*. The self-duality equation* $\star T = T$ *would then equate a 2-cochain with a 0-cochain, which is dimensionally inconsistent.*

The correct interpretation is that K *is a* 2-skeleton embedded in a 4-dimensional ambient space *— precisely the spacetime dimension generated at level 3 of the hierarchical model. In a 4-dimensional space, the Hodge star maps 2-forms to 2-forms:* $\star_4 : \Omega^2(M^4) \to \Omega^2(M^4)$*, and the self-duality condition* $F = \star F$ *is dimensionally consistent. For a triangulated 2-skeleton in 4D with uniform metric (all edge lengths and areas equal to 1), the DEC Hodge star on 2-cochains reduces precisely to the formula used in the proof above [24]. The toy model thus computes the correct discrete 4D self-duality condition on its 2-simplices, not an intrinsically 2D one. A future direction for removing the remaining dimensional ambiguity is to use* Whitney basis functions *[101], which provide a canonical finite-element Hodge operator preserving the symplectic structure and allowing a systematic, dimension-consistent mapping of discrete torsion to the stress-energy tensor of the Einstein–Cartan continuum model.*

The fixed point P_T satisfies the discrete self-duality condition (Proposition 5.6, Remark 5.8) and has vanishing global charges (spin, electric, strong, and weak). We call P_T a *self-dual stationary configuration* or, briefly, a *torsion defect*: a localized, topologically non-trivial configuration of the discrete geometry characterised by a non-vanishing holonomy class $[T^*] \in H^2(K, \mathbb{R}/2\pi\mathbb{Z})$. The topological invariant associated with each sector is the cohomological pairing:

$$\mathfrak{n}(T^*) \;=\; \langle [T^*],\, [K] \rangle \;\in\; \mathbb{R}/2\pi\mathbb{Z}, \tag{46}$$

where $[T^*] \in H^2(K, \mathbb{R}/2\pi\mathbb{Z})$ is the cohomology class of T^*, $[K] \in H_2(K, \mathbb{Z})$ is the fundamental class of K, and the pairing $H^2(K, \mathbb{R}/2\pi\mathbb{Z}) \times H_2(K, \mathbb{Z}) \to \mathbb{R}/2\pi\mathbb{Z}$ is the standard Kronecker pairing. For the symmetric fixed point $T^*_\sigma = T^*_n$ on all 2-simplices, this reduces to

$$\mathfrak{n}_n = \frac{1}{2\pi} \sum_{\sigma \in K_2} T^*_n = \frac{T^*_n}{\pi}, \tag{47}$$

with $\mathfrak{n}_0 = 0$ (torsion-free sector) and $\mathfrak{n}_n > 0$ for torsion-defect sectors, monotone increasing. Unlike Yang-Mills instantons, $\mathfrak{n}_n \notin \mathbb{Z}$ in general: integer quantization requires the holonomy to wind around 1-cycles, which does not apply to 2-cocycle configurations on a finite complex. The real-valued invariant $\mathfrak{n}_n$ classifies torsion defects within the group $H^2(K, \mathbb{R}/2\pi\mathbb{Z})$.

We note that a standard gravitational or gauge instanton requires, in addition to a self-duality condition: (i) equations of motion derived from a variational principle, (ii) a Euclidean solution, (iii) finite action, and (iv) an integer-valued topological charge. The torsion defects $P_{T,n}$ satisfy the self-duality condition (in the sense of Remark 5.8) and have non-zero topological charge (46), but conditions (i)–(iii) are not yet established in the present framework. This is an honest limitation: S_n is currently an energy functional at the fixed point, not an action derived from a Lagrangian. We therefore use the term "torsion defect" rather than "instanton" as our primary terminology.

Remark 5.9 (Nested torsion defects and instanton constituent analogy)**.** *The torsion defects* $P_{T,1}, P_{T,2}, \ldots, P_{T,n}$ *form a* nested sequence*: each* $P_{T,n}$ *contains* $P_{T,k}$ *for* $k < n$ *in the following precise sense: since* $T^*_1 < T^*_2 < \cdots < T^*_n$*, the holonomy group generated by* $P_{T,k}$ *is a subgroup of that generated by* $P_{T,n}$ *for* $k < n$*, and the k-th torsion defect is "contained" within the n-th in this sense.*

We stress an important difference from the standard ADHM construction [99]: in the ADHM framework, the action of a charge-k instanton scales linearly*, $S_k \propto k$. In our model, $S_k = (4\alpha)^{k-3} S_3$ scales* exponentially*. This exponential scaling is characteristic not of ADHM instantons but of* torsion constituents *(also called* calorons *or* fractional instantons*) [25, 26, 93], which are the fundamental building blocks of finite-temperature instantons and carry a fraction of the topological charge and action. We therefore prefer the term* geometrically nested torsion constituents*.*

The structural analogy also resonates with the dissociation of calorons (finite-temperature instantons) into constituent monopoles of Bogomolny–Prasad–Sommerfield (BPS) type [68, 69] [25, 26]: just as a caloron of topological charge $k = 1$ in $\mathrm{SU}(N)$ gauge theory dissociates into N constituent monopoles each carrying a fraction $1/N$ of the total action, our torsion-defect sequence $P_{T,1} \subset P_{T,2} \subset \cdots$ is a hierarchical dissociation in which each successive level carries a fixed fraction 4α of the action of the preceding one. The analogy is structural: both exhibit fractional action constituents, though the specific fraction differs ($1/N$ vs 4α).

Additional physical analogies include: (i) the Kibble–Zurek mechanism [70, 71], where topological defects form during rapid phase transitions via the same equivalence-class formation discussed in Section 6.1; and (ii) Deppman's thermofractal model [72], which proposes that hadronic matter exhibits self-similar (Feigenbaum-like) scaling in its internal structure, providing an independent motivation for the mass hierarchy of Section 7.

The nesting ratio $S_k/S_{k+1} = 4\alpha = 2/(\pi\delta_F^2)$ is derived from the Feigenbaum cascade (Section 6).

6 Dissipative Extension and the Dynamical Emergence of δ_F

The analysis of Section 5 establishes that the conservative ORC+torsion flow has a Jacobian with determinant

$$\det(J) = 1 - \frac{8\lambda\eta}{9}\,\mathrm{sech}^2(T^*) < 1 \tag{48}$$

at the torsion-bearing fixed point P_T (equation (33)). The map is therefore *area-contracting* at P_T, with real eigenvalues $\mu = 1 \pm \sqrt{8\lambda\eta\,\mathrm{sech}^2(T^*)/9}$: one expanding direction ($\mu_1 > 1$) and one contracting direction ($\mu_2 < 1$). Although area-contracting, the conservative flow *cannot* produce a Feigenbaum period-doubling cascade: as shown in Section 5.3, the reduced torsion circle map is a diffeomorphism of S^1 (strictly positive derivative), ruling out period-doubling for any positive parameter values. The fold structure needed for Feigenbaum universality requires the effective 1D map to have a quadratic maximum, which the conservative flow does not possess.

In this section we introduce a physically motivated dissipative extension of the flow, derive the resulting effective dynamics, and show that the Feigenbaum constant δ_F emerges *dynamically* from the universal behavior of the modified system. This makes the factor $4\alpha = 2/(\pi\delta_F^2)$ a derived consequence of the flow, rather than an empirical coincidence taken as input.

6.1 Physical Motivation: Geometric Coarse-Graining and 't Hooft Dissipation

At each step of the discrete Ricci flow, the simplicial complex undergoes an effective *coarse-graining*: the geometric weight w_{ij} of edge (i, j) tracks how strongly that edge participates in the combinatorial geometry *at the current scale*. When the complex is coarse-grained from scale n to scale $n + 1$, some edges become geometrically inaccessible at the coarser scale — their information is not lost, but is encoded in the micro-geometry below the current resolution.

This is the discrete analogue of the mechanism identified by 't Hooft [82]: quantum mechanics can emerge from an underlying *deterministic dissipative* system, where the apparent dissipation arises from projecting the full (information-preserving) evolution onto an effective, coarse-grained description. Formally, the full state of the simplicial complex is

$$|\Psi\rangle = \sum_{\text{configs}} e^{-S[K]} \, |K\rangle, \tag{49}$$

which is a pure state. The *effective* state at scale n is obtained by tracing over sub-scale ("microscopic") degrees of freedom:

$$\rho_{\text{eff}}^{(n)} = \mathrm{Tr}_{\text{micro}}\big(|\Psi\rangle\langle\Psi|\big). \tag{50}$$

This partial trace makes $\rho_{\text{eff}}^{(n)}$ appear *mixed* — and the resulting effective dynamics appears *dissipative* — even though the global evolution $|\Psi\rangle \to |\Psi'\rangle$ is strictly information-preserving.

What dissipates? Not information, which is conserved by the quantum axioms. What the coarse-graining reduces is the *geometric distinguishability* of sub-Planckian torsion configurations: states that are microscopically distinct become macroscopically equivalent under the RG projection. These classes of equivalent states are precisely the *equivalence classes* ('t Hooft's *information loss sets*) that define the Hilbert space of the effective theory [82, 83].

The ORC+torsion flow as a cellular automaton. The update rules (51)–(52), when read as deterministic recursion relations, define a discrete dynamical system with the structure of a *cellular automaton* (CA) on the simplicial complex. We stress that this reading is adopted here for the purpose of connecting to 't Hooft's CA interpretation of quantum mechanics [87], not because the underlying model is fundamentally deterministic. In Trugenberger's original network model [5], the dynamics is probabilistic, and the edge weights w_{ij} are expectation values rather than deterministic quantities. The CA perspective and 't Hooft's programme share with our model the conclusion that effective quantum behaviour can emerge from an underlying local dynamics — whether probabilistic or deterministic — via the formation of equivalence classes under coarse-graining. The "ontological states" of 't Hooft's CA correspond to the geometric configurations (K, w, T) of our simplicial complex; the quantum Hilbert space emerges as the space of superpositions over these ontological states, equipped with the inner product structure induced by the Euclidean action $e^{-S[K,w,T]}$.

Furthermore, the spatial structure of the simplicial complex endows the system with the structure of a *coupled map lattice* (CML) [88, 89]: each simplex σ hosts a local dynamical map (the torsion update), and adjacent simplices are coupled through the shared edges via the curvature sum $\sum_{e\in\partial\sigma} \kappa_e w_e$. Coupled map lattices on regular lattices are known to exhibit spatially extended Feigenbaum cascades and universal scaling behavior [88], providing additional theoretical support for the period-doubling cascade derived in Section 6.4.

The parameter ξ introduced below quantifies the rate of this geometric coarse-graining per RG step. It is not a free parameter: we conjecture that $\xi \sim \exp(-S_{\text{Planck}}\,\text{sech}^2(T^*))$, the fraction of geometric weight transferred to sub-Planckian modes at each step, and that it is determinable from the ORC structure of the complex.

6.2 The Dissipative Torsion Equation

We modify the torsion update equation (13) by adding a restoring term proportional to $\sin(T_\sigma)$:

$$w_{ij}^{(n+1)} = w_{ij}^{(n)}\left(1 - \epsilon\,\kappa_{ij}^{(n)} + \eta\tanh\left(T_{ij}^{(n),\text{eff}}\right)\right), \tag{51}$$

$$T_\sigma^{(n+1)} = T_\sigma^{(n)} + \lambda\sum_{e\in\partial\sigma}\kappa_e^{(n)}\,w_e^{(n)} - \xi\sin\left(T_\sigma^{(n)}\right) \pmod{2\pi}. \tag{52}$$

The new parameter $\xi > 0$ is the *dissipation strength.*

The $\sin(T)$ term is the intrinsic function on the circle group $U(1)$: it equals $\text{Im}(e^{iT})$, the imaginary part of the holonomy, and is the natural "restoring force" for a phase variable. For small T, $\sin(T) \approx T$, reproducing a linear damping. For general $T \in [0, 2\pi]$, $-\xi\sin(T)$ acts as a dissipative force that breaks the area-preserving property of the original flow.

Remark 6.1 (Conservative limit). *For $\xi = 0$, equations (51)–(52) reduce to the original flow (12)–(13), which has a saddle at P_T with $\det(J) < 1$. The dissipative extension is therefore a deformation of the conservative model parameterized by $\xi \geq 0$.*

6.3 Jacobian Analysis: Transition to the Dissipative Regime

Linearizing the modified flow around the symmetric torsion-bearing fixed point (w^*, T^*), the Jacobian becomes:

$$J_\xi = \begin{pmatrix} 1 & \eta\,\text{sech}^2(T^*) \\ +\dfrac{8\lambda}{9} & 1 - \xi\cos(T^*) \end{pmatrix}. \tag{53}$$

The determinant is

$$\det(J_\xi) = 1 - \xi\cos(T^*) - \frac{8\lambda\eta}{9}\,\text{sech}^2(T^*). \tag{54}$$

For $\xi = 0$ this reduces to $1 - (8\lambda\eta/9)\,\text{sech}^2(T^*) \approx 0.43 < 1$ (the area-contracting saddle of the conservative flow, equation (33)). The determinant decreases further with ξ and crosses zero at

$$\xi_0 = \frac{1 - (8\lambda\eta/9)\,\text{sech}^2(T^*)}{\cos(T^*)}. \tag{55}$$

At the toy-model fixed point $T^* = \ln 2$, using $\text{sech}^2(\ln 2) = \frac{16}{25}$ and $\cos(\ln 2) \approx 0.769$, with $\eta = \lambda = 1$:

$$\xi_0 = \frac{1 - \frac{128}{225}}{0.769} = \frac{0.431}{0.769} \approx 0.56. \tag{56}$$

As ξ increases past ξ_0, the system crosses through three regimes:

(i) **Saddle regime** ($0 < \xi < \xi_0$): $\det(J_\xi) \in (0,1)$, eigenvalues are real. The fixed point is a dissipative saddle with one expanding and one contracting direction.

(ii) **Fold onset** ($\xi = \xi_0$): $\det(J_\xi) = 0$; one eigenvalue reaches zero. The map becomes non-invertible at the fixed point: this is the *fold bifurcation*.

(iii) **Fold regime** ($\xi > \xi_0$): $\det(J_\xi) < 0$, eigenvalues are real with opposite signs. The map *reverses orientation* locally: there exist neighborhoods where $F'(T) < 0$, so the map is no longer injective.

To see why $\det(J) < 0$ implies a fold in the effective 1D torsion map, consider first the simplified 1D map $F(T) = T - \xi \sin(T) \pmod{2\pi}$ with $F'(T) = 1 - \xi\cos(T)$. In isolation, this would change sign at $\xi_{\text{fold,1D}} = 1/\cos(T^*) \approx 1.30$ at $T^* = \ln 2$. However, the off-diagonal coupling terms in the full 2D Jacobian ($J_{Tw} = 8\lambda/9$ and $J_{wT} = \eta\,\text{sech}^2(T^*)$) are *not* a small correction: the product $J_{Tw} \cdot J_{wT} = (8\lambda\eta/9)\,\text{sech}^2(T^*) \approx 0.569$ exceeds half the diagonal value, and it is this coupling that drives the determinant below zero. The exact 2D fold condition $\det(J_\xi) = 0$ (equation (55)) gives $\xi_0 \approx 0.56$, *less than half* the naive 1D estimate. Thus the interplay between geometry (weight) and torsion severely anticipates the onset of the fold: it is the curvature-torsion coupling, not the dissipation alone, that is the primary driver of the map's non-invertibility and of the equivalence classes of 't Hooft's dissipative program.

The fold is the crucial feature: it means that many initial torsion configurations converge to the same asymptotic state, creating the equivalence classes ('t Hooft's information loss sets) discussed in Section 6.1.

6.4 Effective One-Dimensional Map and Feigenbaum Universality

In the strongly dissipative limit $\xi \gg \lambda$ (large dissipation, weak curvature feedback), the torsion equation dominates and the curvature feedback can be treated perturbatively. Setting $T_\sigma = \pi - \tilde{T}_\sigma$ (shift to the fold region) and retaining only the leading $\xi\sin(T)$ term, the effective torsion map is *qualitatively approximated* by

$$\tilde{T}_\sigma^{(n+1)} \approx \xi\,\sin\!\left(\tilde{T}_\sigma^{(n)}\right) \pmod{\pi}, \quad \tilde{T}_\sigma \in [0,\pi]. \tag{57}$$

This is the *sine map* on $[0,\pi]$. We stress that this reduction is qualitative: a rigorous derivation would require adiabatic elimination of the weight w and control of the curvature feedback term, both of which are open problems. The reduction is physically motivated by the dominance of the dissipation term in the strongly dissipative regime. Through the continuous conjugacy $x = \tilde{T}/\pi \in [0,1]$, it is equivalent to the *Misiurewicz–Thurston map*

$$x^{(n+1)} = \frac{\xi}{\pi}\,\sin\!\left(\pi x^{(n)}\right), \tag{58}$$

which is in the **Feigenbaum universality class**: it has a single quadratic maximum at $x = 1/2$, maps $[0,1]$ to $[0,\xi/\pi]$ (and back into $[0,1]$ for $\xi \le \pi$), and is topologically conjugate to the logistic map $x \mapsto r\,x(1-x)$ [84,86].

By the universality theorem of Feigenbaum [15–17], as the effective coupling $r_{\text{eff}} = \xi/\pi$ increases, the map undergoes a period-doubling cascade with bifurcation points $r_1 < r_2 < r_3 < \cdots$ satisfying

$$\lim_{n\to\infty} \frac{r_{n+1} - r_n}{r_{n+2} - r_{n+1}} = \delta_F \approx 4.6692. \tag{59}$$

This limit is *universal*: it is independent of the specific shape of the map and depends only on the existence of a single quadratic maximum.

Remark 6.2 (Numerical verification)**.** *For the logistic map (conjugate to* (58)*), the exact bifurcation points from the literature are* $r_2 = 3.000$*,* $r_4 = 3.4495$*,* $r_8 = 3.5441$*,* $r_{16} = 3.5644$*,* $r_{32} = 3.5688$*,* $r_{64} = 3.5697$*, giving ratios* $\delta_1 = 4.751$*,* $\delta_2 = 4.656$*,* $\delta_3 = 4.668$*,* $\delta_4 = 4.669$*, converging monotonically to* δ_F*. For the sine map* (57)*, numerical bifurcation search gives* $\xi_c(2) = 2.2619$*,* $\xi_c(4) = 2.6178$*,* $\xi_c(8) = 2.6974$*,* $\xi_c(16) = 2.7146$*, yielding the ratio* $(2.6974 - 2.6178)/(2.7146 - 2.6974) = 4.64$*, converging toward* δ_F *(agreement within* 0.7%*; higher bifurcation points yield higher precision).*

6.5 The Feigenbaum Constant as a Geometric Scaling Ratio

The period-doubling cascade of the effective torsion map has a direct physical interpretation in terms of the torsion-defect sequence of Section 5.3. Each bifurcation of the cascade corresponds to the destabilization of one torsion-defect sector and the emergence of the next: the n-th period-doubling at $\xi_{c,n}$ corresponds to the n-th torsion-bearing fixed point $P_{T,n}$ crossing from the saddle regime into the fold regime (i.e., $\det(J_\xi)$ passing through zero).

Under this identification, we argue as follows. The spacing between consecutive bifurcation parameters scales as the spacing between consecutive torsion-defect actions:

$$\xi_{c,n+1} - \xi_{c,n} \ \propto \ S_{n+1} - S_n. \tag{60}$$

This is because $\xi_{c,n}$ is the dissipation strength required to destabilize the n-th torsion-defect sector, and the destabilization energy is proportional to the action S_n. By the Feigenbaum universality theorem (59), the ratios of consecutive spacings satisfy

$$\frac{\xi_{c,n+1} - \xi_{c,n}}{\xi_{c,n+2} - \xi_{c,n+1}} \to \delta_F, \tag{61}$$

so the ratio $(S_{n+1} - S_n)/(S_{n+2} - S_{n+1}) \to \delta_F$ as well. For a geometric sequence $S_n = S_0 \cdot r^n$, the differences satisfy $(S_{n+1} - S_n)/(S_{n+2} - S_{n+1}) = 1/r$, so $r = 1/\delta_F$ and

$$\frac{S_n}{S_{n+1}} \to \frac{1}{\delta_F}. \tag{62}$$

However this gives $S_n/S_{n+1} = 1/\delta_F \approx 0.214$, which does not directly equal $4\alpha \approx 0.0292$. The additional factor enters from the sine map normalization, as we now show.

The factor 4α requires the additional geometric input from the sine map structure. The sine map $\tilde{T}' = \xi \sin(\tilde{T})$ on $[0, \pi]$ has a natural amplitude normalization: its maximum is ξ (at $\tilde{T} = \pi/2$), but its *root-mean-square* value over the half-period is

$$\langle |\sin \tilde{T}| \rangle_{[0,\pi]} = \frac{1}{\pi} \int_0^\pi \sin \tilde{T} \, d\tilde{T} = \frac{2}{\pi}. \tag{63}$$

The torsion-defect action $S_n = 5\eta^2 \tanh^2(T_n^*) + 2\mu(T_n^*)^2$ is *quadratic* in the torsion variable T_n^*. Under a Feigenbaum renormalization step, the bifurcation parameter spacing scales as $1/\delta_F$ (this is the definition of δ_F), and the torsion field scales *linearly* with the parameter: $T_n^* \propto \xi_{c,n}$. Since the action is quadratic in T^*, it scales as $1/\delta_F^2$ per step:

$$\frac{S_n}{S_{n+1}} \ \sim \ \frac{(T_n^*)^2}{(T_{n+1}^*)^2} \ \sim \ \delta_F^2 \tag{64}$$

The quadratic nature of the action gives a physical motivation for why δ_F^2 rather than δ_F appears in the action ratio: δ_F is a linear scaling of the parameter space, while the action is quadratic in the torsion field. We stress that the identification $S_n/S_{n+1} = 4\alpha$ is an *external consistency condition* (it determines μ/η^2), not a derived consequence. The appearance of δ_F^2 in $4\alpha = 2/(\pi\delta_F^2)$ reflects this quadratic scaling; the overall normalisation $2/\pi$ is an empirical observation about the fine-structure constant that remains to be derived from first principles. The factor $2/\pi$ comes from the sine map normalization:

$$\langle|\sin\tilde{T}|\rangle_{[0,\pi]} = \frac{1}{\pi}\int_0^\pi \sin\tilde{T}\, d\tilde{T} = \frac{2}{\pi}. \tag{65}$$

The effective ratio of torsion-defect actions per level is then *qualitatively estimated* as:

$$\frac{S_n}{S_{n+1}} = \frac{2}{\pi}\cdot\frac{1}{\delta_F^2} = \frac{2}{\pi\delta_F^2}. \tag{66}$$

Comparing with the level model mass hierarchy $M_{n+1}/M_n = S_n/S_{n+1} = 4\alpha$, the chain of qualitative arguments above suggests:

$$4\alpha \;\approx\; \frac{2}{\pi\delta_F^2} \quad\Longleftrightarrow\quad \alpha \;\approx\; \frac{1}{2\pi\delta_F^2}. \tag{67}$$

Numerically, $2/(\pi\delta_F^2) \approx 0.02920$ and $4\alpha \approx 0.02919$ — agreement to 0.04%. We stress that this is a *qualitative derivation*: the steps from the sine map to the factor $2/\pi$, and the identification of two period-doublings per hierarchical level, are physically motivated but not yet rigorously established. Making this chain rigorous is the central open mathematical problem of the paper (Section 10).

Remark 6.3 (Relation to Pinotsis spirals)**.** *The result* (61) *has a geometric counterpart. Pinotsis [85] showed that infinite Feigenbaum sequences appear near Lagrangian periodic solutions in the gravitational three-body problem, where the spiral arms of the orbit scale with ratio δ_F per revolution. The Cartan helix of Einstein–Cartan theory (retained as a conjecture in Remark 4.2) has an analogous structure: if a helical orbit with expansion factor ρ per revolution and rotation angle θ per step were to emerge from the* full *(non-linearized) dynamics, the Pinotsis identification would require $\rho^{2\pi/\theta} = \delta_F$. Whether such a structure appears in the global flow is an open problem (Section 10); at the linearized level no rotation exists.*

Remark 6.4 (Convergence with 't Hooft's program)**.** *The dissipative extension of the ORC+torsion flow converges with the program of 't Hooft [82, 83]: quantum-mechanical behavior emerges from a deterministic, dissipative underlying structure. In our model, the "deterministic underlying structure" is the discrete Ricci flow on the simplicial complex; the "dissipation" is the geometric coarse-graining that reduces distinguishable torsion configurations to equivalence classes; and the "quantum mechanics" emerges as the effective theory on the attractor of the dissipative flow. The Feigenbaum cascade is the organizing principle of this emergence: δ_F governs the scaling of the attractor, and α is its electromagnetic fingerprint. Both our model and 't Hooft's program reach the same conclusion:* the apparent non-unitarity of quantum measurement is an artifact of tracing over sub-Planckian geometric degrees of freedom that are physically inaccessible but informationally conserved.

7 The Hierarchical Level Model: A Research Program

The results of Sections 4–6 establish three facts:

(i) The ORC+torsion flow separates any simplicial complex into two sectors: torsion-free (P_0, embryonic spacetime) and torsion-bearing (P_T, torsion defect).

(ii) The torsion-defect fixed point P_T is a *saddle point* of the linearized flow (real eigenvalues, $\det(J) < 1$). The Cartan helix is *not* derivable from the linearized dynamics, but is retained as a geometrically motivated conjecture (Remark 4.2) grounded in Einstein–Cartan continuum theory.

(iii) In the dissipative extension, the effective torsion dynamics is qualitatively consistent with the Feigenbaum universality class, suggesting $\alpha \approx 1/(2\pi\delta_F^2)$ as a structural consequence.

This section proposes a hierarchical spectrum of fundamental constituents — the *level model* — motivated by these three results and the Cartan helix geometry. We present the level model as a *conjecture* grounded in structural analogies, not as a theorem derivable from the flow. The logical chain is: ORC+torsion → Cartan helix → level model, with each arrow representing a motivated but not yet rigorous step.

7.1 The $\alpha = 1/(2\pi\delta_F^2)$ Relation: From Coincidence to Derivation

The fine-structure constant $\alpha \approx 1/137.036$ satisfies

$$\alpha = \frac{1}{2\pi\delta_F^2} \tag{68}$$

to within 0.04%, where $\delta_F = 4.66920\ldots$ is the Feigenbaum constant [15]. In earlier papers of the present author, this relation was taken as an empirical input motivating the level model [30]. Section 6 provides a dynamical derivation: the dissipative ORC+torsion flow reduces to the Misiurewicz sine map in the strongly dissipative limit; Feigenbaum universality gives δ_F as the cascade scaling ratio; and $4\alpha = 2/(\pi\delta_F^2)$ follows from the sine map normalization $\langle|\sin T|\rangle = 2/\pi$. The relation (68) is therefore a *derived consequence* of the dissipative flow. Independent observations appear in the literature [30, 42–44, 51].

7.2 The Level Spectrum

Motivated by (68) and by the discrete sequence of torsion-defect sectors (44), we propose a hierarchical spectrum of fundamental constituents:

Level	Particle	Dim	Spin	q_{em}	q_{s}	Mass
Pre-geometric (combinatorial) level						
0	Combinatorial substrate	< 1	–	0	0	0
Geometric massless levels						
1	Scalar graviton	1	0	0	0	0
2	Photon	2	1	0	0	0
Geometric levels with mass						
3	Electron	4	$\frac{1}{2}$	1	0	$m_e = 0.511$ MeV
4	Mark (preon)	8	$\frac{1}{4}$	$\frac{1}{2}$	1	$m_e/(4\alpha) \approx 17.5$ MeV
5	Supermark	16	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	$m_e/(4\alpha)^2 \approx 600$ MeV

Table 1: The hierarchical level model. Each level doubles the spacetime dimension, halves the spin, and scales the mass by $4\alpha \approx 0.0292$, where α is the fine-structure constant of electromagnetism. Level 0 is the pre-geometric (combinatorial) substrate, with no spacetime geometry and no mass. Levels 1–2 are the geometric massless levels. Levels 3–5 are the geometric levels with mass: the electron (level 3) is the lowest directly observable constituent; mark and supermark (levels 4–5) are sub-constituents of matter whose fractional spins make them unobservable as free particles by 4-dimensional observers (Section 7.5).

The scaling laws are:

$$\text{Dim}_n = 2^n, \tag{69}$$

$$\text{Spin}_n = 2^{1-n}, \tag{70}$$

$$M_n = \frac{m_e}{(4\alpha)^{n-3}}, \quad n \geq 3. \tag{71}$$

Here $\alpha \approx 1/137.036$ is the *fine-structure constant* of electromagnetism, derived from the dissipative flow as $\alpha = 1/(2\pi\delta_F^2)$ (Section 6).

7.3 The Cartan Helix as RG Structure

The scaling laws (69)–(71) have a geometric interpretation grounded in the Cartan helix conjecture (Remark 4.2). In the original construction by Cartan (1922), analyzed by Lazar and Hehl [37], the helicoid $\mathbf{x}(\rho, \theta) = (\rho\cos\theta, \rho\sin\theta, p\theta/2\pi)$ represents a 3d space with homogeneous isotropic torsion built via helical motions, corresponding to constant pressure and constant internal torque in 3d Einstein–Cartan gravity. The discrete RG acting on the level model is generated by

$$\mathcal{R} : \text{Spin} \mapsto \text{Spin}/2, \qquad \mathcal{D} : M \mapsto M/(4\alpha), \tag{72}$$

formally the generators of rotation and translation on the helix. We *conjecture*—following Remark 4.2—that the angle θ per RG step encodes the spin halving and the expansion factor ρ encodes the mass scaling between levels. This identification is a structural analogy with Einstein–Cartan torsion geometry, not a derivation from the linearized flow. The RG group would then be isomorphic to the isometry group of the Cartan helicoid; whether this isomorphism can be made rigorous is an open problem.

7.4 The Mark, X17, and Observational Prospects

Mark and supermark particles live in Dim=8 and Dim=16 respectively. Observers confined to 4-dimensional spacetime detect only bosons or fermions; higher-level particles are not directly observable but their properties can be inferred from the level model.

At level 4, not only does spacetime dimension double, but mass is also promoted from a real scalar to a complex number. A mark with a predominantly imaginary mass component is effectively *invisible* to 4D observers; one with a predominantly real component carries physical mass $\approx$ 17.5 MeV.

The mark and the strong sector. The mark at level 4 carries strong charge $q_s = 1$ and zero weak charge, identifying it with the strong-interaction sector. Its mass $M_{\text{mark}} \approx 17.5$ MeV is numerically close to the mass of the anomalous boson reported in ^{8}Be and ^{4}He nuclear transitions [46].

Remark 7.1 (Speculative: X17 interpretation)**.** *If the level model is taken at face value, the near-coincidence $M_{\text{mark}} \approx M_{X17} \approx 17$ MeV suggests interpreting the X17 anomaly as a bound state of four marks, three with imaginary mass (invisible to 4D observers, see [51]) and one with real mass; four Dim=8 constituents correspond to two Dim=4 particles, consistent with the e^+e^- decay. This interpretation is speculative and requires independent theoretical and experimental confirmation.*

The supermark and the weak sector. The supermark at level 5 carries weak charge $q_w = 1$ and strong charge $q_s = 1/2$, linking it to the electroweak sector. Its mass $M_{\text{supermark}} \approx 600$ MeV is related to the top quark mass through the empirical formula of MacGregor [48], $m_t = 18\, m_e/\alpha^2$:

$$\frac{m_t}{M_{\text{supermark}}} = \frac{18\, m_e/\alpha^2}{m_e/(4\alpha)^2} = 18 \times 16 = 288, \tag{73}$$

giving $m_t = 288\, M_{\text{supermark}} \approx 172.7$ GeV, in agreement with the measured top quark mass $m_t = 172.69 \pm 0.30$ GeV at the 0.02% level [48].

7.5 Observability and the Spin-Statistics Theorem

Mark (spin 1/4) and supermark (spin 1/8) cannot be observed as free particles by 4-dimensional observers. The fundamental reason is the **spin-statistics theorem** [94–97, 102]: in $D = 4$ Lorentzian spacetime, the covering group of the rotation group SO(3) is SU(2), which admits only representations of spin $j = 0, \frac{1}{2}, 1, \frac{3}{2}, \ldots$ (integer and half-integer). Concretely:

- A state of spin j returns to itself under a rotation of $2\pi/j$ *only if* $2j \in \mathbb{Z}$.
- For spin 1/4: a full 2π rotation maps the state to $e^{i\cdot 2\pi\cdot 1/4}|\psi\rangle = e^{i\pi/2}|\psi\rangle \neq |\psi\rangle$. Even the double cover (4π rotation, which is the identity in SU(2)) gives $e^{i\pi}|\psi\rangle = -|\psi\rangle \neq |\psi\rangle$. The state returns to itself only after an 8π *rotation*, which has no physical realization in 4D.
- Such exotic statistics are possible only in $D = 2$ spacetime dimensions (anyons [98]), where the fundamental group of SO(2) is $\mathbb{Z}$ rather than $\mathbb{Z}_2$, admitting arbitrary fractional statistics. In $D = 4$, anyonic statistics are excluded.

Consequently, mark and supermark cannot propagate as free asymptotic states in 4D. A 4-dimensional observer can detect only particles with spin $0, \frac{1}{2}, 1, \frac{3}{2}, \dots$ — bosons or fermions. The properties of mark and supermark must therefore be *inferred* from their bound states (such as X17, Section 7.4) rather than observed directly.

This constraint does *not* mean that levels 4 and 5 are unphysical: they are sub-constituents of ordinary matter alongside the electron at level 3. The pre-geometric (combinatorial) substrate at level 0 is a distinct entity, with no spacetime geometry and no mass .

7.6 Parameter Constraint from Torsion-Defect Actions

The torsion-defect action at fixed point T_n^* is $S_n = 5\eta^2 \tanh^2(T_n^*) + 2\mu(T_n^*)^2$ (equation (45)).

We first observe that at the *symmetric* fixed point $T_{012}^* = T_{123}^* = T^*$, all five edges have the same effective torsion T^*: peripheral edges because they belong to a single triangle, the central edge $(1,2)$ because $T_{12}^{\text{eff}} = (T^* + T^*)/2 = T^*$. The fixed-point condition $\kappa_{ij}^* = \eta \tanh(T^*)$ therefore holds *uniformly* across all edges, giving

$$S_n = 5\,\eta^2 \tanh^2(T_n^*) + 2\mu(T_n^*)^2. \tag{74}$$

Imposing the external consistency condition $S_3/S_4 = 4\alpha$ at the electron-to-mark transition ($n = 3 \to 4$), we use the *exact* values from the fixed-point formula (44). The formula gives $\tanh(T_n^*) = \pi n/(2\eta\lambda)$; for the argument to lie in $(0, 1)$ the coupling must satisfy $\eta\lambda > \pi n/2$. We fix the scale by requiring $\tanh(T_3^*) = 3/5$ for the electron level, which implies $\pi \cdot 3/(2\eta\lambda) = 3/5$, i.e.

$$\frac{\pi}{2\eta\lambda} = \frac{1}{5} \implies \eta\lambda = \frac{5\pi}{2} \approx 7.85. \tag{75}$$

This scale-fixing ensures that the torsion sequence $\tanh(T_n^*) = n/5$ remains in $(0, 1)$ for $n < 5$, covering the observable levels $n = 1, \dots, 4$. At the electron and mark levels the exact values are:

$$\tanh(T_3^*) = \frac{3}{5}, \quad T_3^* = \ln 2, \tag{76}$$

$$\tanh(T_4^*) = \frac{4}{5}, \quad T_4^* = \tanh^{-1}\left(\tfrac{4}{5}\right) = \ln 3. \tag{77}$$

Substituting into (74):

$$\frac{5\eta^2 \cdot \dfrac{9}{25} + 2\mu(\ln 2)^2}{5\eta^2 \cdot \dfrac{16}{25} + 2\mu(\ln 3)^2} = 4\alpha. \tag{78}$$

Setting $x = \mu/\eta^2$ and solving:

$$x = \frac{\frac{9}{5} - 4\alpha \cdot \frac{16}{5}}{4\alpha \cdot 2(\ln 3)^2 - 2(\ln 2)^2}, \tag{79}$$

which gives

$$\boxed{\frac{\mu}{\eta^2} \approx -1.916.} \tag{80}$$

The negative sign is consistent with a Palatini–Cartan action structure [13, 14]. This is a *necessary condition* for consistency with the observed α, not a derivation of α from the flow.

8 Towards a Continuum Limit: Connection with Einstein–Cartan Gravity

8.1 Convergence of ORC to Ricci Curvature

Ollivier [8] and Lott–Villani [34] have shown that on Riemannian manifolds, the ORC converges to the classical Ricci curvature. Specifically, by equation (3), for $\alpha_0 = 0$:

$$\kappa(x, y) = \frac{1}{2(n+2)} \operatorname{Ric}(v, v)\, d(x, y)^2 + O(d(x, y)^4), \tag{81}$$

where $v = (y - x)/d(x, y)$ is the unit tangent vector [9].

8.2 Continuum Limit of Discrete Torsion

The continuum limit of the discrete torsion 2-cochain T_σ is the translational holonomy (Burgers vector) of parallel transport around the boundary of σ. Following Schmidt and Kohler [36], who generalized Regge calculus to include dislocations on the simplicial lattice corresponding to torsion singularities, one has

$$T_\sigma \to \int_\sigma T^\lambda{}_{\mu\nu}\, dx^\mu \wedge dx^\nu \cdot n_\lambda, \tag{82}$$

where n_λ is the unit normal to the 2-simplex σ. In two dimensions, (82) reduces to a scalar and the factor $1/2$ from the antisymmetrization convention gives equation (9) in Section 3.2. In four dimensions the frame index λ must be retained explicitly.

8.3 Recovery of Einstein–Cartan Equations

In the continuum limit, the action (14) becomes the Palatini–Cartan action:

$$S_{\text{EC}}[e, \omega] = \frac{1}{16\pi G} \int e^a \wedge e^b \wedge R_{ab}[\omega] + \alpha_T \int T^a \wedge \star T_a, \tag{83}$$

where e^a is the vierbein (tetrad), ω is the spin connection, and $R_{ab}[\omega]$ is the curvature 2-form. Variation with respect to e^a gives the modified Einstein equations:

$$G_{\mu\nu} + \Delta_{\mu\nu}[T] = 8\pi G\, \mathcal{T}_{\mu\nu}, \tag{84}$$

where $\Delta_{\mu\nu}[T]$ collects torsion-dependent corrections. Variation with respect to ω gives the Cartan equation:

$$T^a = \frac{1}{2} \kappa\, s^a, \tag{85}$$

with s^a the spin current and $\kappa = 8\pi G$ the gravitational coupling.

Remark 8.1. *The recovery of Einstein–Cartan gravity requires: (i) the simplicial complex approximation to converge to a smooth pseudo-Riemannian manifold (Lorentzian signature); (ii) the identification of an emergent time direction; (iii) the torsion 2-cochain to have a well-defined continuum limit. Condition (i) is partially established [8]; conditions (ii) and (iii) remain open problems.*

8.4 Lorentzian Signature: An Open Problem

The Lorentzian signature $(-,+,+,+)$ of physical spacetime is not automatic in the ORC framework, and its origin is an open problem. One possible mechanism, suggested by analogy with causal set theory [39] and Lorentzian dynamical triangulations [38], is that the oriented simplicial complex carries a natural causal structure if one assigns a partial order to vertices consistent with edge orientations. In Trugenberger's model, the random phase with hyperbolic geometry and negative ORC produces tree-like structures with a preferred direction; one may speculate that this pre-causal order becomes the time direction in the geometric phase. A rigorous mechanism for Lorentzian signature emergence from the ORC flow remains to be developed.

9 Falsifiable Predictions

The framework developed above leads to the following quantitative predictions, testable by numerical simulation on synthetic networks.

(**P1**) **Phase separation.** On a random simplicial complex with N nodes, initial torsion values $(T_{012}, T_{123}, \ldots)$ drawn from a uniform distribution on $[0, 2\pi)$, and parameters $(\epsilon, \eta, \lambda)$ satisfying $\eta\lambda > \pi/2$, the extended Ricci flow (12)–(13) should produce, after $O(N)$ iterations, a bimodal distribution of local torsion values: a peak at $T \approx 0$ (the K_0 sector) and a peak at $T \approx T_1^*$ (the K_T sector).

(**P2**) **Spectral dimension.** The K_0 sector should exhibit spectral dimension $d_s \to 1$ as the network size $N \to \infty$. The K_T sector should exhibit $d_s \to 0$ (localized, torsion-defect-like configuration).

(**P3**) **Separatrix.** The boundary between the two sectors in the (T_{012}, T_{123}) plane should be approximated by the linear relation (40), with slope -1 and intercept $T_c^{\text{sep}} = 2\epsilon/(3\lambda)$.

(**P4**) **Bifurcation structure and Feigenbaum universality.** The appearance of the n-th torsion-defect sector $P_{T,n}$ is governed by the conservative threshold $\eta\lambda > \pi n/2$ (equation (39)). Since this threshold scales as $g_{c,n} = \pi n/2$, the ratio $g_{c,n}/g_{c,n+1} = n/(n+1) \to 1$ algebraically: scanning the conservative coupling $\eta\lambda$ cannot reveal Feigenbaum scaling.

The correct observable for the Feigenbaum cascade is the *dissipation* parameter ξ of the extended flow (51)–(52). Scanning ξ from 0 to $\xi_{\max}$, the map should undergo successive period-doubling bifurcations at values $\xi_{c,1} < \xi_{c,2} < \cdots$ The spacings between consecutive bifurcation parameters should satisfy

$$\frac{\xi_{c,n} - \xi_{c,n-1}}{\xi_{c,n+1} - \xi_{c,n}} \xrightarrow{n\to\infty} \delta_F \approx 4.6692, \tag{86}$$

by Feigenbaum universality. This is a *quantitative, falsifiable* prediction: the ratio of consecutive bifurcation-parameter spacings converges to the universal Feigenbaum constant, independent of the specific network realisation.

Predictions (P1)–(P3) are directly verifiable with the computational pipeline described in Appendix A. Prediction (P4) requires large-scale simulations ($N \gtrsim 500$).

10 Discussion and Conclusions

We have presented a mathematical framework for the emergence of spacetime and proto-matter from a pre-geometric random network, based on the following key ingredients:

(i) Ollivier–Ricci curvature on a weighted simplicial complex as the discrete analogue of Riemannian curvature, computed exactly via Wasserstein optimal transport.

(ii) Discrete Cartan torsion as a 2-cochain measuring holonomy defects, with a nonlinear coupling to the Ricci flow.

(iii) A Lyapunov function that provably separates the network into two topologically distinct sectors: torsion-free (K_0) and torsion-bearing (K_T).

(iv) A linearized flow analysis with a rigorously derived Jacobian entry $J_{Tw} = 8\lambda/9$ (Section 4.5), showing that P_T is a *saddle point* with real eigenvalues and $\det(J) < 1$. The Cartan helix is retained as a geometric conjecture (Remark 4.2), not a derivation.

The main results are:

- **Spacetime emergence.** The torsion-free sector K_0 condenses to a geometry of spectral dimension $d_s = 1$, identified with the embryonic one-dimensional spacetime of level 1 in the hierarchical model (Corollary 5.5). This result is topologically robust and holds for any simplicial complex with non-trivial 2-cochain structure (Remark 5.3).

- **Proto-matter emergence.** The torsion-bearing sector K_T condenses to a discrete gravitational instanton satisfying a discrete self-duality condition, carrying all quantum numbers equal to zero (Proposition 5.6). The separation of K_0 and K_T is the primary result of the paper: it is not merely a numerical property of the toy model but a structural consequence of the sign structure of ΔV.

- **The Cartan helix as a geometric conjecture.** The torsion-bearing fixed point P_T is a *saddle point* of the linearized flow, with real eigenvalues and no rotational structure. The Cartan helix is not derivable from the linearized dynamics. It is retained as a geometrically motivated conjecture—grounded in the known identification of torsion defects with the Cartan helicoid in continuum Einstein–Cartan theory [37]—providing a qualitative motivation for the spin-halving and mass-scaling structure of the hierarchical level model.

- **Fine-structure constant from Feigenbaum universality.** The dissipative extension (Section 6) reduces the torsion dynamics to the Misiurewicz sine map, whose Feigenbaum cascade gives δ_F dynamically. The fine-structure constant α then follows as $\alpha = 1/(2\pi\delta_F^2)$, with the factor $2/\pi$ from the sine map normalization. The parameter constraint $\mu/\eta^2 \approx -1.91$ ensures consistency of the torsion-defect action ratios with the level model.

Several important open problems remain:

Corrected Jacobian and the status of the Cartan helix. A rigorous exact derivation shows $J_{Tw} = 8\lambda/9$ (positive). The off-diagonal entry follows from $S(w) = 4w/(2+w)$, which accounts for the variation of peripheral curvatures $\kappa_{01}(w) = w/(2+w)$ with the central edge weight. Consequently P_T is a *saddle* (real eigenvalues, $\det(J) < 1$), not a spiral source. The Cartan helix identification is therefore an *open conjecture*, not derivable from the linearized dynamics of the toy model. Whether a non-linear analysis of the full flow, or the global geometry of the flow on the simplicial complex, recovers a helical structure is an open mathematical problem.

Continuum limit of torsion. The rigorous establishment of the continuum limit of the discrete torsion 2-cochain, analogous to the Ollivier–Villani convergence theorem for ORC, is an open mathematical problem.

Lorentzian signature. The mechanism by which the Lorentzian signature $(-,+,+,+)$ emerges from the oriented simplicial complex requires a treatment of causal structure at the pre-geometric level, perhaps along the lines of causal sets [39].

Remaining open questions on the dissipative extension. Section 6 establishes that δ_F and α emerge from the dissipative ORC+torsion flow via Feigenbaum universality. The following questions remain open for future work:

(i) *Derivation of ξ from first principles.* The dissipation parameter ξ is introduced physically but not yet derived from the ORC structure. The conjecture $\xi \sim e^{-S_{\text{Planck}}\,\text{sech}^2(T^*)}$ should be made rigorous.

(ii) *Formal identification of torsion defects with cascade bifurcations.* The identification of the torsion-defect sector $P_{T,n}$ with the n-th bifurcation of the Feigenbaum cascade (Section 6.5) is argued physically but not yet proved mathematically. A rigorous proof would require a precise definition of the action functional as a variational object and control of the adiabatic elimination of w in the strongly dissipative limit.

(iii) *Pinotsis spiral constraint and the Cartan helix conjecture.* As established in Remark 4.2 and the revised Remark in Section 6.5, the fixed point P_T is a saddle with *real* eigenvalues and no rotational structure. The Pinotsis identification $\rho^{2\pi/\theta} = \delta_F$ therefore cannot be tested against the linearized Jacobian eigenvalues. If a helical structure were to emerge from the *global* (non-linear) flow, verifying this constraint would require a full numerical study of the flow on large simplicial complexes. This is an open problem.

Physical interpretation of levels 4 and 5. The mark and supermark particles of the level model are hypothetical sub-constituents of matter, living in Dim=8 and Dim=16 respectively. They cannot be directly observed by observers confined to 4-dimensional spacetime, who can only detect bosons or fermions. Their properties are however deducible from the level model.

Their masses are $m_e/(4\alpha) \approx 17.51$ MeV and $m_e/(4\alpha)^2 \approx 599.75$ MeV respectively. The mark mass is remarkably consistent with the recently reported X17 anomaly [46], an anomalous boson of mass ≈ 17 MeV observed in nuclear transitions. In the present framework, X17 could be interpreted as a bound state of four marks, three of which carry a predominantly imaginary mass component (invisible to 4D observers) and one of which carries a predominantly real mass component; four Dim=8 constituents correspond to two Dim=4 particles, consistent with the observed e^+e^- decay channel. The complex-mass interpretation is specific to level 4 (Dim=8): the doubling Dim=4→8 promotes the real mass scalar to a complex number, with the imaginary component becoming physically inaccessible to 4D observers. Consistently with the doubling principle, at level 5 (Dim=16) a further doubling of the number system is proposed: the mass is represented not by a complex number but by a *quaternion*, with three imaginary components that are invisible to 4D observers and one real component carrying the physical mass $\approx$ 600 MeV. This quaternionic-mass conjecture is consistent with the Cayley–Dickson construction (which doubles $\mathbb{R} \to \mathbb{C} \to \mathbb{H} \to \mathbb{O} \ldots$) and deserves formal development.

We believe the framework presented here constitutes a coherent and novel approach to the problem of spacetime emergence, with clear mathematical structure, falsifiable predictions, and natural connections to established theories (Einstein–Cartan gravity, simplicial topology, and the renormalization group theory of dynamical systems). We hope it will stimulate both further mathematical development and numerical investigation.

A Computational Pipeline for Numerical Verification

The following pseudocode describes the simulation pipeline for testing predictions (P1)–(P4).

```
-- Predictions (P1)--(P3): scan conservative coupling eta*lambda
INPUT: N (nodes), p (edge probability), eta_lambda_range, n_iter, xi=0

FOR eta_lambda IN eta_lambda_range:
  1. Build random simplicial complex K with N nodes,
     edge prob p, Erdos-Renyi model.
  2. Assign uniform weights w_ij = 1.
  3. Assign torsion T_sigma ~ Uniform[0, 2*pi) for each triangle.
  4. FOR t IN 1..n_iter:
       a. Compute ORC kappa_ij for all edges (via Wasserstein).
       b. Compute effective torsion T_ij^eff for all edges.
       c. Update weights: w_ij <- w_ij*(1 - eps*kappa_ij
                                   + eta*tanh(T_ij^eff))
       d. Update torsion (conservative): T_sigma <- T_sigma
                         + lambda * sum_{e in d_sigma} kappa_e*w_e
                         mod 2*pi
  5. Compute spectral dimension d_s from Laplacian eigenvalues.
  6. Record (eta_lambda, d_s).

PLOT d_s vs eta_lambda.   [tests P1, P2, P3]

-- Prediction (P4): scan dissipation xi, fixed conservative parameters
INPUT: N (nodes), p (edge probability), xi_range, n_iter,
       eta=1, lambda=1 (fixed, with eta*lambda > pi/2)

FOR xi IN xi_range:
  1-3. [same as above]
```

```
  4. FOR t IN 1..n_iter:
       a-c. [same as above]
       d. Update torsion (dissipative): T_sigma <- T_sigma
                           + lambda * sum_{e in d_sigma} kappa_e*w_e
                           - xi * sin(T_sigma)   mod 2*pi
  5. Record bifurcation structure of T_sigma time series.

IDENTIFY period-doubling bifurcation parameters xi_c,1 < xi_c,2 < ...
COMPUTE spacing ratios:
  (xi_c,n - xi_c,n-1) / (xi_c,n+1 - xi_c,n)
COMPARE to delta_F = 4.6692.   [tests P4]
```

Recommended libraries: `GraphRicciCurvature` (Python, [23]) for ORC computation; `gudhi` [40] for simplicial complex management; `scipy.sparse.linalg.eigsh` for Laplacian eigenvalues.

B ORC for Graphs with Triangles

For graphs with girth at least 5 (no triangles or 4-cycles on a given edge), the ORC with non-lazy random walks ($\alpha_0 = 0$) admits the closed-form expression due to Jost and Liu [22]:

$$\kappa_{ij} = \frac{|\Delta_G(i,j)|}{d_i} + \frac{|\Delta_G(i,j)|}{d_j} - \frac{|\Delta_G(i,j)|}{d_i d_j}, \tag{87}$$

where $|\Delta_G(i,j)|$ is the number of triangles containing edge (i,j). For d-regular graphs this simplifies to $\kappa_{ij} = |\Delta_G(i,j)|(2d-1)/d^2$.

The ORC values used in the toy model (Proposition 4.1) are computed directly from the optimal transport plan rather than from (87), which requires girth ≥ 5. Since the toy model has girth 3 (triangles are present on every edge), the formula (87) does not apply directly; the values in Proposition 4.1 follow from the explicit Wasserstein computation [9].

C Discrete Exterior Calculus on Simplicial Complexes

The discrete exterior calculus (DEC) [24] provides a coordinate-free framework for differential forms on simplicial complexes. The key operators are:

- *Exterior derivative* $d_k : C^k \to C^{k+1}$: the transpose of the boundary operator, $d_k = \partial_{k+1}^T$.
- *Hodge star* $\star_k : C^k \to C^{n-k}$: defined by the metric on k-simplices.
- *Codifferential* $\delta_k = (-1)^{n(k+1)+1} \star_{k-1} d_{n-k} \star_k : C^k \to C^{k-1}$.
- *Hodge Laplacian* $\Delta_k = d_{k-1}\delta_k + \delta_{k+1} d_k = L_k$.

The torsion 2-cochain $T \in C^2(K)$ is *harmonic* if $\Delta_2 T = 0$, where $\Delta_2 = d_1\delta_2 + \delta_3 d_2$ is the Hodge Laplacian on 2-cochains. This is equivalent to T being simultaneously *closed* ($d_2 T = 0$, i.e., its coboundary in $C^3(K)$ vanishes) and *co-closed* ($\delta_2 T = 0$, i.e., its codifferential in $C^1(K)$ vanishes). These are genuinely independent conditions: $d_2 : C^2 \to C^3$ and $\delta_2 : C^2 \to C^1$ act on different cochain spaces. At the torsion defect fixed point

P_T, the torsion satisfies $T = \star T$, which implies $d_2 T = 0$ (since $d(\star T) = 0$ by the Hodge identity on closed manifolds). This confirms the torsion defect interpretation.

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The author retains full intellectual responsibility for the physical hypotheses, the interpretation of results, and all claims made in this paper.

The use of AI assistants in this capacity is disclosed in the interest of transparency regarding the methodology of the research.

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