Title: Much ado about nothing when it comes to fractional energy levels: Is Coulomb law in conflict?

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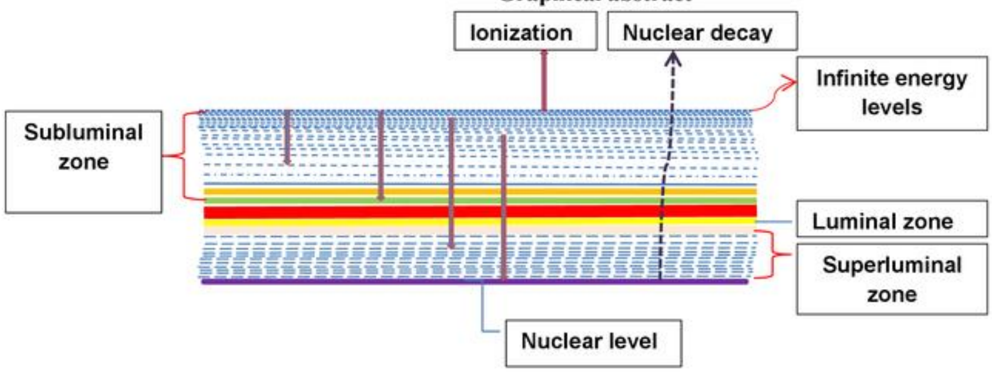
Abstract

From a theoretical viewpoint, this investigation examined fractional energy levels and velocities that surpass the speed of light in a vacuum. These phenomena have received limited attention. Using a classical framework, the study aims to validate these phenomena by computing relevant parameters using fundamental constants and their multiples in derived equations. The presence of fractional energy levels and superluminal velocities (SVs) is validated within a classical framework aligned with the mass-energy equivalence principle. SVs are directly proportional to the masses of fundamental and baryonic particles when the first energy level is set to one. Conversely, if a constant luminal velocity is considered, the irrational energy levels (n_i) is proportional to the square of the particles' masses. For example, the values for the proton and the top quark are 3.022436467 *exp.* (+8) and 556.3297886 *exp.* (+8) m/s, respectively. The corresponding energy levels are 1.0164718078 and 34,436.83251, respectively. With the first atomic energy level, the energy levels are equal to the corresponding atomic numbers. For each atomic number, the fractional energy levels are inversely related to the subluminal kinetic energy. From Z =1 to Z=4, however, the energy levels showed an increasing trend. Even though the classical theoretical framework considers the distance between two centers of mass, determining the mass radius of the proton remains a possibility at specific fractional energy levels for certain atomic numbers. Future research may explore achieving the proton's mass radius at higher atomic numbers and lower fractional energy levels (below 0.1).

Keywords: Fundamental and baryonic particles, fractional energy levels, Superluminal velocities, Mass radius-like interparticle distance, Atomic energy

Graphical abstract



1.0 Introduction

Since early researchers like Mills (2002) first explained fractional energy levels and velocities greater than the velocity of light in a vacuum, the scientific community has not yet come to terms with these concepts. The majority of scholars likely view Einstein's theory of relativity—which forbids a particle from reaching the velocity of light—as sacred due to its creator. The theory of relativity cannot make Coulomb's work irrelevant. Coulomb's law aligns with the inverse square law, indicating that both the force and kinetic energy of any elementary particle electrostatically influenced by the nucleus could indefinitely increase as the interparticle distance decreases. However, scholars and researchers have experimentally investigated the possibility of velocities exceeding the speed of light (Feinberg, 1967; Barashenkov, 1975; Hussein, 2020; Wang, 2025). From a theoretical perspective, fractional energy levels and velocities greater than the speed of light in a vacuum have not been widely studied. Since Coulomb's law has been tested *vis-à-vis* the 20th-century theory of relativity, there would be no reason to view it as conflicting with new ideas. To borrow a phrase from British William Shakespeare, why is there so much ado about nothing when it comes to fractional energy levels? This theoretical investigation, based on a classical framework, aims to confirm the existence of fractional energy levels and velocities exceeding the speed of light in a vacuum. The objective is to calculate these parameters by entering relevant fundamental constants—including multiples of certain constants, when appropriate—into the equations derived during this study.

2.0 Theoretical background

According to research, the following formula can be used to determine the radius of any atom, including hydrogenic and nonhydrogenic atoms (Udema, 2022). Assuming that n_{i} = one and ξ_{i} and ξ_{H} are equal, the well-known Bohr radius for a hydrogen atom has a CODATA value of 5.2917726903 (*exp.* (–11) m) (Tiesinga *et al.*, 2021).

$$a_i = \frac{n_{\mathrm{i}} e^2}{8\pi\varepsilon_0 (\xi_{\mathrm{i}}\xi_{\mathrm{H}})^{½}}, \tag{1}$$

The average ionization of an atom and a hydrogen atom, as well as the energy levels between 0 and 1 (*i.e.*, fractional energy levels up to first energy level), are denoted by the symbols ξ_{i}, ξ_{H}, and n_{i}. A fringe scientist would label both the incorrect value and Eq. (1) as fringe science if an electronic calculator were unable to reproduce the reported CODATA value without making deliberate mistakes. The following equation uses the square of $m_p c^2/2$, much like Eq. (1).

$$a_i = \frac{n_i e^2}{8\pi\varepsilon_0 m_p c^2/2}, \tag{2}$$

The symbols, n_i, a_i, m_p, and c are respectively, an unusual energy level, the radius corresponding to n_i, the rest mass of a proton, and the expected speed of light in free space. Meanwhile, the equation of a density (ρ) of a particle such as proton is given as follows:

$$\rho = \frac{3m_p}{4\pi a_i^3 \Theta}, \tag{3}$$

The symbol, Θ is a factor postulated to be related to the fine structure constant. Substituting Eq. (2) into Eq. (3) gives the following:

$$\rho = \frac{3m_p}{4\pi\Theta}\left(\frac{m_p c^2 \pi\varepsilon_0}{2n_i e^2}\right)^3,$$
$$= \frac{48 m_p \pi^2 \varepsilon_0^3}{\Theta}\left(\frac{m_p c^2}{n_i e^2}\right)^3, \tag{4}$$

Also,

$$\rho = \frac{3m_p}{4\pi\Gamma_p^3} = \frac{48 m_p \pi^2 \varepsilon_0^3}{\Theta}\left(\frac{m_p c^2}{n_i e^2}\right)^3, \tag{5}$$

Solving for $1/\Gamma_p^3$ gives the following:

$$1/\Gamma_p^3 = \frac{64 m_p \pi^3 \varepsilon_0^3}{\Theta}\left(\frac{m_p c^2}{n_i e^2}\right)^3, \tag{6}$$

Given the equation (Eq. (7)) in the literature (Udema, 2020) another reciprocal of the cube of mass radius of the proton can be derived.

$$\Gamma_p = \frac{e^6 m_p}{4\pi\varepsilon_0^3 m_e^2 h^2 c^4}, \tag{7}$$

$$1/\Gamma_p^3 = \left(\frac{4\pi\varepsilon_0^3 m_e^2 h^2 c^4}{e^6 m_p}\right)^3, \tag{8}$$

It is possible that Eqs (6) and (8) are the same. And so, one obtains the equation as follows:

$$\frac{64 m_p \pi^3 \varepsilon_0^3}{\Theta}\left(\frac{m_p c^2}{n_i e^2}\right)^3 = \left(\frac{4\pi\varepsilon_0^3 m_e^2 h^2 c^4}{e^6 m_p}\right)^3, \tag{9}$$

Solving for c gives the equation as follows:

$$c = \frac{e^2 m_p}{\Theta^{1/6} n_i^{1/2} m_e h \varepsilon_0}, \tag{10}$$

2.1 Testing the hypothesis that $\Theta^{1/6} = \alpha^{-2/3}$

Testing the hypothesis, demands that n_i is assumed to be equal to one. Thus, solving for Θ gives the equation as follows:

$$\Theta = \left(\frac{e^2 m_p}{c\, n_i^{1/2} m_e h\, \varepsilon_0}\right)^6 = 370{,}360{,}521.2, \tag{11}$$

Let,

$$\alpha^{-\varphi} = 370{,}360{,}521.2, \tag{12a}$$

$$\text{In}\alpha^{-\varphi} = \text{In}370{,}360{,}521.2 = 19.72998747, \tag{12b}$$

$$-\varphi(-4.920241425) = 19.72998747, \tag{12c}$$

$$\varphi = 4.009963285, \tag{13}$$

Therefore,

$$(\alpha^{-\varphi})^{1/6} = 370{,}360{,}521.2^{1/6} = 26.79810685, \tag{14}$$

$$\alpha\, exp(-2.004968164/3) = 26.79810685, \tag{15}$$

A straightforward analysis reveals that using the reciprocal of the fine structure constant's 2021 CODATA value yields $\alpha\, exp.(-2.004968164/3) = 26.79751444$. This implies that n_i is marginally more than one and that the equation $\alpha^{-2/3} = 26.5800504$, which includes the 2021 CODATA value of α, is viable. Therefore, $1/\Theta^{1/6}$ in Eq. (10) is approximately equal to $\alpha^{2/3}$. Hence, Eq. (10) can be restated as follows:

$$c = \frac{e^2 m_p\, \alpha^{2/3}}{n_i^{1/2} m_e h \varepsilon_0} \quad (=1.807051732\ exp.(+35)\ m_i), \tag{16}$$

where m_i is the mass of any baryon. To retain the 2021 CODATA value of α, the real value of n_i needs to be determined. Solving for the latter gives the following:

$$n_i = \left(\frac{e^2 m_p\, \alpha^{2/3}}{c m_e h \varepsilon_0}\right)^2 (= 3.633287506\ exp.(+53)\ m_i), \tag{17}$$

The value of n_i for neutron is 1.019279017. If n_i is equal to one in Eq. (16) the value of c is equal to 302,251,884.4 m/s (1.008203763 times the speed of light in free space); the value for the neutron is 302,668,514.3 m/s (1.009593491 times the speed of light in free space). These values are unusual speed of light in free space. It is the norm rather than the exception to find negative energy and equation such as: –13.6eV and

$$E_n = -\frac{Z_{eff}^2 e^4 m_e}{8\varepsilon_0^2 n^2 h^2}, \tag{18}$$

Other version in the literature (Mills, 2005) is given as:

$$E_n = -\frac{Z_{eff} e^2}{8\pi\varepsilon_0 a_H n^2}, \tag{19}$$

where a_H(usually a_0) is Bohr's radius for hydrogen and if $Z_{eff} = 1$, Eqs (18) and (19) where $n^2 = 1$ should be the total energy in line with convection. Equations (18) and (19) remind researchers that their counterparts with opposite signs apply to all computations involving radii and effective nuclear charge. In this study, atomic numbers are considered.

3. Method

Theoretical and computational approaches were explored using literature (Tiesinga *et al.,* 2021 CODATA) values of fundamental physical constants that were substituted into derived equations. Multiples of some of the fundamental constants were also explored.

4.0 Results and Discussion

4.1 Unusual velocities greater than the velocity of light and energy levels-the subatomic mass-dependent superluminal kind

As stated earlier, the idea that the velocity of a particle cannot exceed the speed of light in free space is not new. It is based on Einstein's theory of relativity, but it has recently faced opposition. However, it is not widely known that velocities greater than the speed of light and energy levels greater than one are theoretically possible as a function of the rest mass of nucleons and larger subatomic particles. By exploring Eqs (16) and (17), the unusual velocities greater than the velocity of light (with an energy level of one in a vacuum) and the energy levels greater than one (with *c* as the usual velocity of light) were computed and shown in Table 1. Just as the energy levels greater than one were directly related to the masses, the unusual velocities were precisely proportional to the masses of the subatomic particles.

Table 1: Computed particle mass-dependent velocities & energy levels

S/N	Name of particles	Symbols	Unusual v_i/*exp.*(+8) m/s	$\boldsymbol{n_i}$
1	Charm quark	τ^-	4.252191501	2.011797316
2	Tauon (tau lepton)	c	5.734028133	3.658291113
3	Bottom quark	b	14.13828646	20.75725168
4	Top quark	*t*	556.3297886	34436.83451
5	Delta	Δ	3.968719296	1.752505379
6	Xi	Ξ^0	4.235638907	1.996165071
7	,,	Ξ	4.242342899	2.07050148
8	Sigma	$\sum^0$	3.841936546	1.642324492
9	,,	∑	3.911598391	1.655560221
10	Omega	Ω	5.387687188	3.229710283
11	Neutral lambda	Λ	3.580745289	1.42661062
12	Charmed lambda	Λ^+c	7.338852706	5.992595125
13	Bottomed lambda	Λ^0c	18.10268282	36.46233521
14	Proton	p	3.022436467	1.016471801
15	Neutron	n	3.026680637	1.019275982

The equation deplored for the computation of unusual energy levels as a function of the masses of the subatomic particles is Eq. (17); such masses are found in the web and literature (Udema, 2025b). Equation (16) is used for the computation of unusual velocity, v_i.

By restating Eq. (18) as $E_n = Z^2 e^4 m_e / 8\varepsilon_0^2 n^2 h^2$ and solving for $\boldsymbol{n_i}$ one obtains $\boldsymbol{n_i} = Ze^2 \times \sqrt[2]{m_e/8E_n}/\varepsilon_0\, h$; with the latter, the integer, decimal (this can be converted to fraction), and irrational number energy levels were computed for atomic numbers 1 to 4 as shown in Table 2. When the subluminal energy equals 13.6 eV, the energy level equals the corresponding atomic number. Decimal numbers can be converted to proper fractions and express the possibility of fractional energy levels. Though these are lower energy levels, they are high-energy zones well above the atomic energy level. Both decimal and irrational numbers demonstrate the existence of an infinite number of high-energy states. For each subluminal energy state greater than the atomic energy level of 13.6 eV, the decimal and irrational energy levels increase with the atomic number. As expected, the opposite is true for each atomic number with increasing subluminal energy (Table 2).

Table 2: Subluminal and atomic energy levels based on defining energy for atomic numbers 1 to 4

Energy ($E_n = \Theta 13.6\ eV$)	***Z* =1**	***Z*=2**	***Z*=3**	***Z*=4**
	n_i	n_i	n_i	n_i
1×13.6eV	1	2	3	4
2×13.6eV	0.70710678	1.414213562	2.121320343	2.828427124
3×13.6eV	0.577350269	1.154700538	1.5	2.309401076
4×13.6eV	0.5	1.0	1.341637785	2.0
5×13.6eV	0.447213595	0.89442719	1.2247287	1.78925438
6×13.6eV	0.40824829	0.81649658	1.133893419	1.63299316
7×13.6eV	0.377964473	0.755928946	1.06060017	1.511857892
8×13.6eV	0.353555339	0.70710678	1.0	1.41421356
9×13.6eV	0.333333333	0.66666666	0.948716	1.33333333
10×13.6eV	0.316227766	0.632475532	0.948713298	1.264951064

The equation used for computation is, $n_i = ZE_H^{½}/(3.687817783 \times \Theta^{½} N_A)$; Θ is a positive integer; E_H/N_A is the average ionization energy of a hydrogen atom.

Tables 3 through 6 present records of superluminal velocities and irrational fractional energy levels. These levels demonstrate that high-energy or superluminal zones always contain intermediaries. In contrast to the fractional energy levels, the magnitude of the square of superluminal velocities decreases with the distance between the proton and electron centers of mass. Interestingly, as atomic numbers increased, so did the two dependent factors. Given the classical theoretical framework that made such possibilities, those superluminal velocities shouldn't come as a surprise. As a result, the long-standing belief that tachyons—real or imagined—can travel faster than light is still held without ideological conflict (Barashenkov, 1975) and appears to support the information in all of the tables. Additionally, Wang (2025) determined that various observers would perceive varied vacuum light speeds based on his analysis of multiple data sets. Wang claims that this discovery limited the application of the relativity principle of constant vacuum light speed by casting doubt on its universality. The superluminal state is also acknowledged by the author. As a result, despite being theoretically created, the reported superluminal velocities in this work have a scientific foundation.

Previous studies and investigations have explored the possibility of superluminal velocities from different perspectives and using various methods. The view that, within the special theory of relativity and space-like four-momentum, such particles may possess a velocity greater than that of light in a vacuum has been investigated for many years. An argument against this possibility was considered unsatisfactory or unconvincing within the framework of relativistic quantum theory (Feinberg, 1967) and is difficult for the layperson to understand. This research takes a purely classical approach with a strong link to the theory of mass-energy equivalence, which served as the basis for determining the mass radius of the nucleon and Coulomb's law. According to Mills, some ions and atoms can react catalytically with atomic hydrogen if they ionize at integer multiples of its potential energy. To produce a hydrogen atom with a lower energy state and fractional primary quantum number than unreacted atomic hydrogen, this reaction requires nonradiative energy transfer (Mills, 2002).

Other lines of evidence abound. One such finding seemed to support the proposition that the absolute/relative speed of light or any other particle may exceed the known value of c. This claim was purportedly validated using famous phenomena and experiments such as aberration, the Doppler effect, binary stars, the headlight effect, the Michelson-Morley experiment, and time dilation. These methods are best known to the experimental physicist (Hussein, 2020). Studies have shown that, light travels in water at 75 percent of the speed it would in a vacuum. But electrons created by the core's processes travel faster through water than light. Researchers at the National Institute of Standards and Technology (NIST) transferred quantum data faster than the speed of light (Saleh *et al.*, 2020). Tables 3 through 6 show that attractive or repulsive electrostatic interactions between the total charge of a nucleus and either an electron or its antiparticle, inside the zone of fractional energy levels, can produce velocities greater than the speed of light in a vacuum. Consequently, an impelling force may have caused electrons in water to travel faster than the speed of light.

Table 3: Squared velocities and fractional energy levels corresponding to multiples of mass radius of the electron plus mass radius of the proton for atomic number equal to 1

***Z*=1**		
ϕr_e+r_P	$c_i^2/exp.(+17)\mathrm{m^2s^{-2}}$	n_i
1r_e+r_P	2.17406643	0.004690965548
2r_e+r_P	2.062848583	0.004815763781
3r_e+r_P	1.962452511	0.00493709411
4r_e+r_P	1.871376782	0.005056127651
5r_e+r_P	1.788379622	0.005172122464
6r_e+r_P	1.712431812	0.005285572141
7r_e+r_P	1.64267183	0.005396629109
8r_e+r_P	1.57837305	0.00550546264
9r_e+r_P	1.518918329	0.005612261003
10r_e+r_P	1.463780135	0.005716901697

6.278280228 *exp.* (− 17) m (Udema, 2025a-Methodological…); 1.10168 fm (Udema, 2025b-Revisiting …); r_e and r_P are the mass radii of the electron and proton respectively. c_i^2 = 253.1619487/ (ϕr_e+r_P); $m_e c_i^2 = 2Z^2E_H/n_i^2 N_A$; $n_i^2 = 2Z^2E_H/m_e c_i^2 N_A$; $n_i = 137467.3097Z^{½}(\phi r_e + r_P)^{½}$

Table 4: Squared velocities and fractional energy levels corresponding to multiples of mass radius of the electron plus mass radius of the proton for atomic number equal to 2

Z=2		
ϕr_e+r_P	$c_i^2/exp.(+17)\mathrm{m^2s^{-2}}$	n_i
1r_e+r_P	4.34813286	0.006634027099
2r_e+r_P	4.125697166	0.006810518452
3r_e+r_P	3.924905022	0.006982105449
4r_e+r_P	3.742753564	0.007150444297
5r_e+r_P	3.576759244	0.007314485735
6r_e+r_P	3.424863624	0.007474927807
7r_e+r_P	3.28534366	0.007631986077
8r_e+r_P	3.1567461	0.007785899933
9r_e+r_P	3.037836658	0.007936935626
10r_e+r_P	2.92756027	0.008084919915

The method is as in Table 3

Table 5: Squared velocities and fractional energy levels corresponding to multiples of mass radius of the electron plus mass radius of the proton for atomic number equal to 3

Z=3		
ϕr_e+r_P	$c_i^2/exp.(+17)\mathrm{m^2s^{-2}}$	n_i
1r_e+r_P	6.52219929	0.008124990666
2r_e+r_P	6.188545749	0.008341147546
3r_e+r_P	5.887357533	0.00855129784
4r_e+r_P	5.614130346	0.008757470525
5r_e+r_P	5.365138866	0.008958378891
6r_e+r_P	5.137295436	0.009154879495
7r_e+r_P	4.92801549	0.009347235806
8r_e+r_P	4.7349915	0.009535741012
9r_e+r_P	4.556754987	0.009720721203
10r_e+r_P	4.391340405	0.009901964201

The method is as in Table 3.

Table 6: Squared velocities and fractional energy levels corresponding to multiples of mass radius of the electron plus mass radius of the proton for atomic number equal to 4

Z=4		
ϕr_e+r_P	$c_i^2/exp.(+17)\mathrm{m^2s^{-2}}$	n_i
1r_e+r_P	8.69626572	0.009381931096
2r_e+r_P	8.251394332	0.00963152756
3r_e+r_P	7.849810044	0.00987418822
4r_e+r_P	7.485507128	0.010112255
5r_e+r_P	7.153518488	0.010344249
6r_e+r_P	6.849727248	0.010571144
7r_e+r_P	6.57068732	0.010793258
8r_e+r_P	6.3134922	0.011010925
9r_e+r_P	6.07565316	0.011224522
10r_e+r_P	5.85512054	0.011433803

The method is as in Table 3.

4.1 Back to mass-energy equivalence principle

It appears that much about superluminal and fractional energy levels, either as integers, decimals, or irrational numbers, has been discussed. Also discussed are the superluminal velocities when the energy level is fixed at one as well as the irrational energy levels if the velocity of light in a vacuum is a limit as a function of the masses of baryons (Table 1). Albert Einstein's populist theory of mass-energy equivalence seemed to be replicated in two ways: the magnitude of irrational energy levels and superluminal velocities. This raises a significant concern regarding why a proton captures an electron to become a neutron without breaking up. This concern is weighed against the backdrop of the view that electron transit from the luminal to the superluminal zone occurs before capture. It makes sense to believe that the velocity of a missile (or, if preferred, a bullet) aimed at a mass must be proportionate to the target's mass given a unit energy level. Therefore, the following equation can be used to describe the kinetic energy (*KE*) needed to destroy a proton beyond capture:

$$KE = m_e \frac{e^4 m_P^2 \alpha^{4/3}}{n_i m_e^2 h^2 \varepsilon_0^2} = \frac{e^4 m_P^2 \alpha^{4/3}}{n_i m_e h^2 \varepsilon_0^2} = 8.321533325\ exp.\ (-14)\ \mathrm{J} \quad (20)$$

If the velocity of light is accurate, the value in Eq. (20) would be somewhat greater than 8.187105776 × *exp.* (–14) J. Despite the little variation, it is believed that the energy derived from Eq. (20), based on Eq. (16), is adequate to dissolve the proton. *If a bunker buster's kinetic energy is less than the work required to breach the bunker, anyone may appear naive, not just in a military setting; otherwise, it becomes another dream reminiscent of the American James Bond series, where anything is feasible at no cost.*

As discussion of mass-energy equivalency continues, it is crucial to take into account the reasons behind electron capture. This could be explained by the stabilization of an unstable nucleus when there are more protons than neutrons; following capture, a proton and an electron neutrino arise. In this context, an electron can absorb a photon and change into an excited electronic state when n_i=1 is the "ground" state for "pure" photon transitions. However, it is unable to change into a lower-energy electronic state and emit a photon. In any case, an electron must pass through fractional energy levels for the nucleus to receive it. As previously stated, resonant nonradiative energy transfer *via* an unfamiliar multipole coupling or resonant collision mechanism may enable an electron to transition from the ground state to a lower-energy state (Mills *et al.*, 2003). In the cause of electron capture, it moves from one lower energy level to another until it is captured rather than being scattered, a term often used in most high-energy nuclear physics research (Mills *et al.*, 2003). The outcomes of such scattering experiments in the literature (Sick, 2018; Alarcón *et al.*, 2019; Friedrich *et al.*, 2019) often give inconsistent results. These are clear pieces of evidence that there are fractional energy levels, which are high kinetic energy zones.

Meanwhile, the comment given is as follows and may appear ambiguous; it nevertheless recognizes discrete energies (they are higher than the average ionization energy of hydrogen) that belong to subluminal energy levels where fractional energy levels are relevant. Such energies are governed by q×13.6 eV, where q = 1, 2, 3, 4, 6, 7, 8, 9, 11, *etc* (Mills *et al.*, 2003). However, while multiples of 13.6 eV are significant and can be explored in this research, the perspective on unstated discrete energies less than 21.2 eV is unclear because when q equals one, the result of 13.6 eV minus 21.2 eV is negative, which does not necessarily align with the conventional understanding of total energy terms whose normal magnitude is half that of a nonzero potential energy.

A major takeaway from the preceding argument and counterargument is that at a certain fractional energy level for a given atomic number, the possibility of attaining the mass radius of the proton cannot be ruled out even if the theoretical classical framework addressed the distance between two centers of mass. There have been numerous elastic scattering and laser spectroscopy measurements of the proton radius, often referred to as the proton radius puzzle, with contradicting results. Many of these experiments involved electron-proton scattering against the backdrop of electron capture. This prompted the desire to measure the proton charge radius using high-energy elastic muon-proton scattering at CERN's Super Proton Synchrotron (SPS) M2 beam line in 2022 (Friedrich et al., 2019). In contrast to the more precise value of 0.8409 ± 0.0004 fm obtained from the Lamb shift in muonic hydrogen, the proton charge radius was determined to be $\boldsymbol{R_C}$ = 0.887 ± 0.012 fm (Sick, 2018). However, as shown in Table 7, this study is still interested in the mass radius of the proton. As fractional energy levels decreased, the square of the superluminal velocities exhibited an upward trend. As the atomic number increased, so did the center-of-mass distance ($\boldsymbol{r_i}$) between the electron and proton. The two noteworthy $\boldsymbol{r_i}$ values are 0.5542321118 and 0.8313481676 fm (Table 7). Although some have disputed the former on the grounds that the used equation is flawed and unsuitable for all educational levels, these values are similar to those reported in the literature (Udema, 2025b). Additionally, Table 7 shows a value of 1.108464224 fm, which is in line with reports from several sources in the literature (Udema, 2020; Udema, 2025a; Udema, 2025b).

The value of 0.5542321118 fm is similar to values reported in the literature (Kharzeev, 2021), but it may not accurately represent the proton's mass radius. The sum of these two values (0.5542321118 fm and 0.06278280228 fm) is 6.170149202 fm, which supports this interpretation. Similarly, 0.491444089 fm is less than 0.5542321118 fm (0.5542321118 − 0.06278280228 fm). However, since 6.170149202 fm is larger than 0.5542321118 fm, the latter cannot be the sum of two radii. The most likely solution is therefore 1.169582802 fm (0.06278280228 + 1.10168 fm), which is similar to, yet significantly different from, 1.10846424 fm (see Table 7).

Table 7: Squared velocities corresponding to energy levels and the distances between centers of masses of the proton and electron for atomic numbers ranging from 1-4

$\boldsymbol{n_i}$	$\boldsymbol{c_i^2}$ ***exp.*(+17)** $(\mathbf{m^2s^{-2}})$	$\boldsymbol{Z=1}$	$\boldsymbol{Z=2}$	$\boldsymbol{Z=3}$	$\boldsymbol{Z=4}$
		$\boldsymbol{r_i}$**/fm**	$\boldsymbol{r_i}$**/fm**	$\boldsymbol{r_i}$**/fm**	$\boldsymbol{r_i}$**/fm**
1	0.9135592952	2.771160559	5.542321118	8.313481677	11.08464224
0.9(or 9/10)	1.015065884	2.494044512	4.988089024	7.482133536	9.976178048
0.8 (or 4/5)	1.141949119	2.216928447	4.433856894	6.650785341	8.867713788
0.7(or 7/10)	1.305084707	1.939812392	3.879624784	5.819437176	7.759249568
0.6(or 3/5)	1.522598825	1.662696336	3.325392672	4.988089008	6.650785344
0.5(or 1/2)	1.82711859	1.385580265	2.77116053	4.156740795	5.542321119
0.4(or 2/5)	2.2873898238	1.108464224*	2.216928448	3.325392672	4.433856896
0.3(or 3/10)	3.045197651	0.8313481676*	1.662696335	2.494044503	3.32539267
0.2(or 1/5)	4.567796476	0.5542321118*	1.108464224*	1.662696335	2.216928447
0.1(or 1/10)	9.135592952	0.2771160559	0.5542321118*	0.831348167*	1.108464224*

Equation 16 was deplored for computation of $\boldsymbol{c_i^2}$ while r_i is computed using $r_i = Ze^2/4\pi\varepsilon_0 m_e\, c_i^2 = 253.1619487Z/c_i^2$. The superscript is a reminder that such figures had been reported in the literature (Udema, 2020; Kharzeev. 2021; Udema, 2025a; Udema, 2025b)

A previous study purported to be based on the quantum chromodynamics (QCD) framework yielded a proton mass radius of 0.55 ± 0.03 fm, but this value was considered significantly smaller than the proton's root mean square charge radius (0.8409 ± 0.0004 fm). This discrepancy is attributed to the interplay of asymptotic freedom and the spontaneous breaking of chiral symmetry in QCD, which is well understood by the author and likely by collaborators as well. Furthermore, Kharzeev (2021) asserted that m_s = 1.24 ± 0.07 GeV was retrieved from GlueX data attributed to Ali *et al.* (2019). This value may have been included in the final publication, but it was not found in the arXiv preprint server. The total mass radii of up and down quarks (subject to future reevaluation and potential alternative models) in a previous study were 0.8349190666 fm (Udema, 2025a), which contrasts with 0.8313481676 fm in this work. While the equation $\langle R_m^2 \rangle = 12\hbar^2 c^2/m_s^2$ (Udema, 2025b) may be convincing, the same cannot be said for an equation such as $\langle R_m^2 \rangle = 12/m_s^2$, which is said to have been used to extract the mass radius (0.55 ± 0.03 fm) of the proton based on the QCD framework. Unlike $\langle R_m^2 \rangle = 12/m_s^2$, $\langle R_m^2 \rangle = 12\hbar^2 c^2/m_s^2$ has scientific meaning and validity (Udema, 2025b). This present study, on the other hand, clearly explores the classical framework in alignment with Einstein's mass-energy equivalence to compute all parameters of interest.

5.0 Conclusion

The existence of fractional energy levels and cognate superluminal velocities is confirmed based on a well-defined classical framework in clear alignment with the mass-energy equivalence principle. Similar to the mass-energy equivalence principle, superluminal velocities (SVs) were found to be directly proportional to the masses of the fundamental and baryonic particles if the first energy level is assumed to be one; on the other hand, if constant luminal velocity is implied, the irrational energy levels (n_i) are directly proportional to the square of the masses of the particles. For example, the values for the proton and the top quark are 3.022436467 exp. (+8) and 556.3297886 exp. (8) m/s, respectively. The corresponding energy levels are 1.0164718078 and 34,436.83251, respectively. With the first atomic energy level, the energy levels are equal to the corresponding atomic numbers. For each atomic number, the fractional energy levels are inversely related to the subluminal kinetic energy. From Z =1 to Z=4, however, the energy levels showed an increasing trend. Expectedly, the square of the superluminal velocities and the fractional energy levels (in decimal numbers) were, respectively, inversely and directly proportional to the distance between the center of mass of the proton and electron. Even if the theoretical classical framework addressed the distance between two centers of mass, the possibility of attaining the mass radius of the proton cannot be ruled out at a certain fractional energy level for a given atomic number. The possibility of achieving the proton's mass radius at a higher atomic number and lower fractional energy level (less than 0.1) could be reserved for future investigation.

6. Dedication

This study is dedicated to Delta State Governor Sheriff Oborevwori, whose administration has authorized university graduate teachers in public elementary schools to advance to grade level 17. If the government took into account retiring and retired teachers who were unjustly refused promotion to level 17 while having taught professionally until the day and month of their retirement that would be constitutionally sound. I implore his government to give pensioners covered by the new pension plan an increase in their monthly income in line with those covered by the previous pension law who never made any contributions.

7. Disclaimer (Artificial Intelligence)

Author(s) hereby declare that no generative AI technologies such as Large Language Models (ChatGPT, COPILOT, *etc.*) and text-to-image generators have been used during the writing or editing of this manuscript.

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9. Competing Interests

There are no known competing financial interests, non- financial interests, or personal ties that could have influenced the work presented in this publication, according to the sole author's declaration. The only issue is the monthly pension, which is much less than two USD per day and may not have been recommended by the World Bank or the IMF.

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