On the Nature of the Superfluid Phase in Liquid He4

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Abstract

A hypothesis on the physical nature of the superfluid phase in liquid helium is presented. It is shown that atomic delocalization is possible only along one coordinate, rather than along three, as is currently accepted in classical concepts of helium's transition to a superfluid state. Filamentous coherent states of atoms form the basis of the superfluid component of liquid helium. The hypothesis is supported by the justification of the critical velocity during rotation of cylindrical vessels and thermodynamic calculations.

Keywords: liquid helium, superfluid phase, order parameter for a second-order phase transition.

Introduction

To date, most authors studying liquid helium lean toward the hypothesis that the transition of liquid helium to the superfluid state is associated with Bose condensation in gases [1-2]. Furthermore, many authors suggest that the superfluid phase is itself a Bose condensate. Although Landau, the creator of the mathematical theory of superfluidity [3], explicitly wrote that the ideas of L. Tisza [4], the founder of the physical theory of superfluidity, regarding the connection between the superfluid phase and the Bose condensate were invalid, he did not provide a detailed description of the superfluid phase, but merely stated that it is a quantum coherent state within a normal liquid and proposed a possible spectrum of excitations. Authors [3,5-6] very rigorously, literally from first principles, described the excitation spectrum of a system of interacting bosons, but the ground state itself remained beyond physical description. It should be noted that a Bose-Einstein condensate in liquid helium has not yet been experimentally detected.

It would seem that the true physical nature of the superfluid phase has no bearing on the development of the theory and practice of liquid He4 at low temperatures. All studies use only the fact that liquid helium at temperatures below the λ-transition is a two-component liquid, and the superfluid phase is a quantum coherent state of the liquid with the lowest energy and a known spectrum of energy excitations. It is important that this state is unique, and therefore has zero entropy. The coherence hypothesis allows us to describe the superfluid phase using an ψ-function similar to the ψ-function in the Schrödinger equation [2-3]. It turns out that the nature of the superfluid phase is not a very relevant issue. It is worth noting that the modern theory of superfluidity with great difficulty and with many assumptions explains the critical velocities during rotation and during the movement of superfluid helium through pipes and capillaries, which differ by many orders of magnitude from the critical velocity [2], obtained from the spectrum of energy excitations. The fact of the irrotational flow of the superfluid component has still not been explained. Some researchers consider the fact of irrotational flow of the superfluid phase more interesting than the phenomenon of superfluidity itself. The only explanation was given by L.D. Landau more than 80 years ago: superfluidity is a quantum phenomenon, and in all quantum phenomena, the angular momentum is quantized, and since it is quantized, there is an energy gap when exciting rotational degrees of freedom. It should be noted that the authors [7] point out that the spectrum of energy excitations predicted by L.D. Landau is in no way related to the phenomenon of superfluidity, since a similar spectrum of excitations is observed in other liquids.

But there is one issue that distinguishes the theory of superfluidity from other theories of condensed matter, and surprisingly, the scientific community pays little attention to it. As is well known, the λ-transition to the superfluid state is a second-order transition. As one of the founders of the theory of critical phenomena, A.Z. Patashinsky, taught me, [8] The main thing in the theory of second-order phase transitions is the concept of the order parameter and its conjugate field. After determining the order parameter and its conjugate field, pure mathematics begins. In all theories of condensed matter where second-order phase transitions are observed, both the order parameter and the conjugate field are known: magnetic transitions (magnetic moment - magnetic field), liquid-vapor (near the critical point) - (density - pressure), superconductivity (superconducting current - magnetic field), ferroelectrics (dipole moment - electric field), etc.In superfluid helium, if we take the density of the superfluid condensate as the order parameter, it's unclear what the conjugate field will be. From the most general physical considerations, the angular velocity vector $\vec{\omega}$ suggests itself as the conjugate field, by analogy with superconductors, for which there is a critical magnetic field value at which vortices arise. In this case, it's unclear how to organize the Hamiltonian (a scalar) from the vector $\vec{\omega}$ and the density of the superfluid condensate (a scalar).There is another almost philosophical question. It is generally accepted that a superfluid condensate is a kind of coherent quantum state of helium atoms and its motion should be described by a wave function □ by analogy with the wave function in the Schrödinger equation [2 p. 336]. As the author points out [9 p. 130], the physical phenomena described by quantum theory are related to the processes described by classical mechanics. In very brief: the fundamental level of a quantum system is a cyclic motion in a classical formulation with the same set of interaction potentials between particles. For example, phonons in quantum mechanics are oscillations of a harmonic chain in classical mechanics, a hydrogen atom is a quantization of the rotation of two charged particles in classical mechanics, etc. One can pose the following question: are there trajectories of particles of a classical liquid that can describe the coherent state of a superfluid condensate? In the absence of an understanding of the physics of a superfluid condensate, there is no understanding of the mechanism by which the condensate is involved in rotation [2]. By analogy with superconductors, it is believed that there is a vortex core around which the superfluid phase rotates in a circular motion. In superconductors, the circular motion of superconducting electrons is ensured by the Lorentz force, while in the case of the superfluid component of liquid helium, long-range forces are absent.

At this point, we make the following statement: the coherent state of the superfluid phase is associated with the delocalization of helium atoms. Atoms in a localized state cannot move collectively in either a solid or a liquid. The only collective motions are rotations of three or four atoms, and in liquids, string-like diffusion (collective movement along rectilinear threads) is also possible. Any other motions are energetically improbable [10].

Physical concept of the superfluid component of liquid He4.

What physical facts do we have so far?

1. The phase transition to the superfluid state is a second-order transition, meaning that the spatial regions occupied by the new state have macroscopic, rather than atomic, dimensions already near the transition point.

2. The superfluid state is quantum coherent and moves easily, without friction, within the normal helium component.

3. The flow of a superfluid liquid at low velocities is irrotational.

4. The movement of three-dimensional clusters containing a large number of atoms within a normal liquid is impossible.

The only option that satisfies all of the above facts is one-dimensional rectilinear filaments that can move only along their own axis. Yes, this is a condensate (a delocalized state of a group of atoms), but only along one axis. The order parameter is the concentration of macroscopically large filaments; since a filament has a direction (a vector), the conjugate field is the angular velocity vector $\vec{\omega}$. We emphasize that, as L.D. Landau pointed out about the superfluid phase, this is not a set of specific atoms, but a potential well of a specific structure in the multidimensional configuration space of liquid helium. As computer modeling of the potential relief of the liquid state shows, there exist states with reduced potential energy, much lower than the energy of ordinary random arrangements of atoms [11].

A one-dimensional filament in a liquid medium has two types of excitations: the phonon spectrum (which at high frequencies can exhibit features such as a roton minimum) and excitations associated with the filament's spatial motion. Note that phonons and rotons do not cause mass displacement. Due to the absence of single-particle excitations with an energy spectrum:

$$\varepsilon = \frac{p^2}{2*m}$$

where **ε** is the energy, **p** is the momentum, and **m** is the atomic mass, the filamentary state cannot exchange energy with slowly moving (less than the speed of sound in the filament) atoms of the normal helium phase. The most suitable mathematical model describing the phonon (roton) spectrum, in our opinion, should be something like the Frenkel-Kontorova crowdion model [12].

The most pressing problem is the excitations associated with the spatial motion of a coherent filamentary state, which are responsible for the phenomenon of superfluidity. Because it has been shown experimentally that all critical velocities in superfluid helium are in no way connected with the velocities obtained from the spectrum of phonons (rotons) [2]!!!

We propose an explanation for the critical angular velocity $\vec{\omega}$ during rotation in a cylindrical vessel. The maximum length of the filament is equal to the vessel's diameter **D**, so the number of atoms **N** in this filamentary structure and its mass **M** are equal to:

$$N = D/a \text{ и } M = m_0 * N$$

where **a** is the interatomic distance and $m_0$is the mass of a helium atom. An elementary act of mass transfer is the displacement of the filament by one interatomic distance. The filament is, as it were, located in a pit of width **a**. This approach clarifies the mechanism of angular momentum transfer to the superfluid phase: upon collision with a wall, elementary excitations (such as crow-dions) arise in the filament, which move along the filament and can be elastically reflected from the vessel walls. Upon collision with a wall moving with a velocity **ν=ω*D/2**, the filament can ac-quire momentum $P=M*\nu= m_0*N*D*\nu/(2*\mathbf{a})$. Since the filament is a certain quantum state, the mini-mum momentum can be estimated from the Heisenberg uncertainty principle

$P \geq \hbar/(\mathbf{2} * \boldsymbol{a})$substituting the values yields:

$$m_0*N*D*\nu/2 \geq \frac{\hbar}{2*a} \text{ или } m_0*D^2*\omega \geq \hbar \qquad (1)$$

thus, the critical angular velocity ω_{cr} at which excitations associated with the movement of a coherent filamentary state in space appear is equal to:

$$\omega_{cr} = \frac{\hbar}{D^2 * m_0} \qquad (2)$$

Thus, there is no circular rotation of the superfluid component, but it transfers angular momentum through rectilinear motion perpendicular to the cylinder axis, but near the axis, see Fig. 1a. With such motion, the tangential velocity V_θ of the superfluid component is indeed described by the formula:

$$V_\theta = \frac{\hbar}{m_0 * r} \qquad (3)$$

An illusion of annular rotation is created, and rings with different radii have the same angular momentum. Thus, the minimum critical rotation speed is associated with the occurrence of excitations in chains of length D. With an increase in the rotation speed, excitations with a shorter length may arise. It appears that stable excitations are inscribed regular polygons, see Fig. 1b: a triangle, a quadrangle, etc. Translational movements along these trajectories give the impression that the centers of rotation of the vortices are shifted from the center of the cylinder. The reasoning given in deriving (1) for a triangle gives a critical speed value $2/3^{1/2}$greater than the value (2), for a quadrangle the coefficient will be equal to $2^{1/2}$. Very similar values of critical rotation speeds were obtained in separate experiments (not all) in the work [13].

Next, we consider the motion of superfluid helium in narrow channels of width **D**. Let's assume a chain located at an angle α to the channel wall. Then the chain length will be L = D/sin(α). The chain can exchange a perpendicular component of momentum with the wall: $P_{\perp}$=M*v_S*sin(α), where $\mathbf{v_S}$ is the velocity of the superfluid component. In this case, we obtain the expression for the critical velocity:

$$\mathbf{V_{cr}}= \frac{\hbar}{D \ * m_0} \qquad (4)$$

This simple formula, as stated in [2], describes the experimental data surprisingly well.

The question arises as to how to account for the numerous experiments in which vortex structures have been observed in large numbers. We noted above that there is no physical force that would cause a filamentary superfluid structure to bend into a ring. But this statement is valid as long as the superfluid component is at rest! Once the superfluid component begins to rotate due to the movement of filamentary structures and the velocity field (3) arises, the newly created filamentary structures are subject to acceleration associated with the Coriolis force:

$$\vec{a}_k = 2*(\vec{\omega}(r) * \vec{v})$$

where $\vec{\boldsymbol{v}}$ is the velocity of mass movement along the filamentary structure. The tangential component of the velocity is described by formula (3), which means that the radial component of the Coriolis force is maximum near the center of rotation. Thus, ring structures with a small radius near the center of rotation and a large radius far from it arise. It turns out that the centrifugal acceleration F_c = v^2/racting on the superfluid ring is compensated by the Coriolis acceleration and taking into account the fact that the angular velocity is related to the tangential linear velocity $\omega(r)=V_\theta/r$. These arguments are confirmed by the experiment [14], which discovered a range of rotational velocities in which there is a velocity distribution according to law (3), but there are no vortices yet. In addition, the existence of two relaxation times when stopping a rotating vessel with superfluid helium was shown. The author associated one short time (several seconds) with the disappearance of the vortices, but the mechanism of another relaxation time (tens of minutes) remained unclear.

In [15], the change in the energy of a superfluid thread during rotation is described, which makes it possible to write an interaction Hamiltonian that includes the angular velocity and concentration of the superfluid phase ρ_s.

The superthermal conductivity of He4 is below the transition point to the superfluid state.

It is known [2] that boiling ceases in liquid helium below the transition point, and this effect is associated with the infinitely high thermal conductivity of the superfluid phase, which instantly equalizes the temperature throughout the entire volume of the liquid. Let us consider how this phenomenon can be explained under the assumption that the superfluid phase is something related (similar) to a three-dimensional Bose condensate. Near the transition point, the concentration of the superfluid phase is close to zero, which leads to two variants of the arrangement of atoms included in the superfluid component in space (Fig. 2). Variant 1: the atoms of the superfluid phase are distributed uniformly in the space between the atoms of the normal phase, but at a distance much greater than the interatomic distance in liquid helium. In this case, it is impossible to physically explain the coherent motion of the superfluid component, due to the absence of long-range interactions. Variant 2: the atoms of the superfluid phase are compactly located in space, but these regions occupy a small fraction of the volume of the entire liquid. In this case, the infinite rate of heat (mass) transfer throughout the fluid, which is essentially in its normal state, is also inexplicable. Infinite thermal conductivity requires coherent (collective) movement of superfluid atoms, even at low concentrations, which is only possible in the presence of macroscopically large one-dimensional filaments.

Irrotational flow of the superfluid component of liquid helium.

Let the superfluid component be concentrated in filamentary structures that permeate the entire thickness of the liquid in various directions. Along each filament, the superfluid phase can displace atoms. Even at rest, atoms of the superfluid phase can move (oscillate) along the filament direction at a velocity V_k, as observed in the ground state of a conventional quantum oscillator. The concentration of filaments per unit volume and their spatial structure depend only on the density of the liquid, but the degree of filling of the filamentary states, effectively the density of the superfluid phase $\rho_s(T,P)$, may depend on temperature T, pressure P, and other parameters. Consider a small but macroscopic volume of helium in the superfluid state (Fig. 3). We have two points located adjacent along the x-axis. Then the resulting flux from point 1 to point 2 is equal to:

$$\Delta I_x = \rho_{s1}(T,P) * V_{kx} - \rho_{s2}(T,P) * V_{kx} = V_{kx} * \Delta x * \frac{\partial \rho_s(T,P)}{\partial x} \qquad (5)$$

Where $\rho_{s1}(T,P)$ and$\rho_{s2}(T,P)$ are the concentrations of the superfluid component at points 1 and 2, respectively, V_{kx} is the projection of the velocity V_k onto the x-axis averaged over all filaments, and Δx is the distance between points 1 and 2 along the x-axis. Averaging at point 1 is performed over the right hemisphere, and at point 2 over the left hemisphere. Clearly, the velocity V_{kx} averaged over the entire sphere is zero. A similar expression can be written for the resulting flow along the y- and z-axes:

$$\Delta I_y = V_{ky} * \Delta y * \frac{\partial \rho_s(T,P)}{\partial y} \; ; \; \Delta I_z = V_{kz} * \Delta z * \frac{\partial \rho_s(T,P)}{\partial z}$$

Let us set $\Delta x = \Delta y = \Delta z = \Delta$. It is obvious that the average value of the projection of all threads on the x, y and z axes are equal to each other, that is, $V_{kx} = V_{ky} = V_{kz} = V_k /2$. Then, for the vector of the resulting flow of the superfluid phase, we can write the expression:

$$\vec{\Delta} I = \left(\frac{\partial \rho_s(T,P)}{\partial x}, \frac{\partial \rho_s(T,P)}{\partial y}, \frac{\partial \rho_s(T,P)}{\partial z}\right) * \Delta * V_0 = \text{grad}\,(\rho_s(T,P)) * \Delta * V_k/2 \qquad (6)$$

This automatically implies the irrotational nature of the flow of the superfluid component. In the three-dimensional, standard model of a superfluid condensate, such a simple derivation of irrotational flow is impossible, because it is impossible to explain the physical meaning of the parameters V_{kx}, V_{ky}, and V_{kz}, much less their equality.

Thermodynamics of He4 below the superfluid transition point.

For the atoms of the superfluid component to move coherently relative to the normal phase, they must be delocalized (see point 4 of the second section). In the normal phase of the liquid, the atoms are located in potential wells, meaning they possess nonzero energy [16-17]. During delocalization, this energy disappears, and this should manifest itself in a change in enthalpy and heat capacity during the transition from the normal phase to the superfluid phase. We will assume that the wells are harmonic and calculate the zero-point energy:

$$E=\frac{\hbar*\omega_0}{2}$$

The oscillation frequency ω_0 can be estimated from the speed of sound **c**, which is expressed by the formula:

$$\mathbf{c} = \boldsymbol{a} * \boldsymbol{\omega_0}$$

in helium just above the transition point at a temperature of 2.2 K **c**=220 m/sec, the value of the interatomic distance **a** is found from the density of helium [18] a = $3.6*10^{-10}$ m, from which it follows that the zero-point energy is equal to:

$$E= 2.2 \text{ K/atom}= 4.58 \text{ j/gramm} \qquad (7)$$

We found the delocalization energy for one coordinate (note the surprising value of this figure); for delocalization along all axes, it should be three times greater.

Below we will use experimental data from source [19] at a pressure of 2.5 atm for the heat capacity of helium below the λ-point. Note that the results of heat capacity measurements in work [18], obtained at saturated vapor pressure, give similar figures. Numerical integration of heat capacity values from a temperature of 2.15 K to 0.8 K (in this range, almost 95% of the normal phase transforms into a superfluid phase) gives a change in enthalpy equal to:

$$\Delta H=2.7\pm0.2 \text{ j/gramm} \qquad (8)$$

This is significantly less than the energy released during delocalization along just one coordinate (7). Considering that the speed of sound below the λ-point increases from 220 m/sec to 240 m/sec, this means that the harmonic well becomes harder along the other two coordinates, and the increase in zero-point energy must be taken into account:

$$\hbar * \Delta\omega_0=0.83 \text{ j/gramm}$$

Thus, the change in enthalpy associated with delocalization along one coordinate and an increase in the rigidity of the harmonic well along the other two coordinates is equal to:

$$\Delta H = 4.58 - 0.83 = 3.75 \text{ j/gramm} \qquad (9)$$

This is already close to the experimental value (7). On the other hand, these calculations demonstrate that delocalization of the superfluid phase atoms along all three coordinates should lead to heat capacity values several times greater than the experimental values. Some delocalization must occur, otherwise the superfluid phase cannot move relative to the normal helium component.

Conclusion

Why could such an unusual interpretation of the physical nature of the superfluid phase of liquid helium only emerge in recent years? The fact is that real proof of the collective motions of atoms in liquids only emerged with the development of computer experiments using molecular dynamics. Only these computer experiments could provide any real understanding of the potential relief of a multiparticle, strongly interacting system. It became clear what kind of atomic-level motions are possible in a classical liquid. The dynamics of atomic motion in liquids with significant quantum effects can only be based on classical analogs, since the quantum dynamics of atoms lies beyond our experience.

In conclusion, we make one more interesting remark. If our view of the physical nature of the superfluid phase is correct, then a state of He4 called a supersolid should exist: superfluidity in a solid, glass-like state. Computer experiments with disordered (non-crystalline) atomic states in classical systems demonstrate the presence of collective string-like motions of atoms in the solid phase [20].

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Conflict of Interest

The author declares that he/she has no conflict of interest.

Literature

1. Superfluidity and Bose Condensation: A Textbook / V. A. Kashurnikov, A. V. Krasavin, A. I. Maimistov, P. F. Kartsev (Moscow: MEPhI, 2008)

2. S.J. Patterman. Superfluid hydrodynamics (North-Holland Publishing Company- Amsterdam-London. American Elsevier Publishing Company.Inc.-New York, 1974)

3. L. D. Landau, Theory of superfluidity of helium II. Journal of Experimental and Theoretical Physics, **11** 592, (1941)

4. L. Tisza, Transport Phenomena in Helium II Nature, **141** 913, (1938)

5. S. T. Belyaev, Journal of Experimental and Theoretical Physics, **7** 289, (1958)

6. N. N. Bogolyubov, Izv. USSR Academy of Sciences, **11** 77, (1947)

7. V. B. Bobrov, S. A. Triger. Brief Communications on Physics of the Lebedev Physical Institute, No. 11 28, (2014)

8. A. Z. Patashinsky, V. L. Pokrovsky. Fluctuation Theory of Phase Transitions (Nauka, Moscow, 1982)

9. R. Rajaraman, An introduction to solitons and instantons in quantum field theory (North-Holland Publishing Company, Amsterdam-New York-Oxford, 1982)

10. V.V.Alekseenko, On the mechanism of atomic diffusion in condensed matter. Fiz. Tverd. Tela (St. Petersburg) **50(10)** 1775 (2008) [Phys. Solid State 50 (10), 1848 (2008)].

11. T. Sjostrom, G. De Lorenzi-Venneri, and D. C. Wallace, Potential energy surface of monatomic liquids, Physical Review B **98** 054201 (2018)

12. O.M.Braun, Y.S.Kivshar. Frenkel-Kontorova model, concepts, methods and applications. Springer, (2004)

13. R. E. Packard, T. M. Sanders. Observations on Single Vortex Lines in Rotating Superfluid Helium.Physical Review A **6**799 (1972)

14. P.J. Bendt, Superfluid Helium Critical Velocities in a Rotating Annulus, Phys. Rev. **127** 1441 (1962)

15.V.V. Alekseenko, The connection between collective diffusion in classic liquids and superfluidity phenomenon, viXra.org:1710.0028

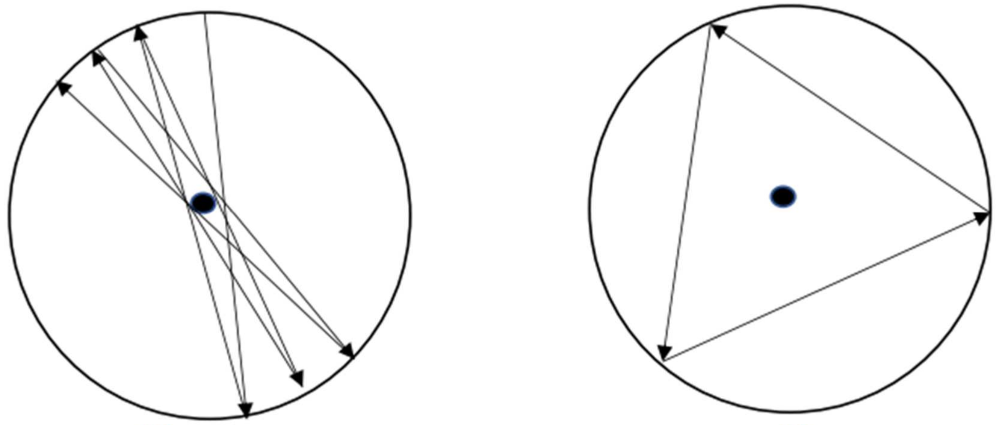
16. S. J. Yatendra, Macro-orbitals and Microscopic Theory of a System of Interacting Bosons arXiv:cond-mat/0606571v1 (2006)

17. S. J. Yatendra, Microscopic theories of superfluidity/ Superconductivity/normal fluidity of a SIB/SIF developed by using PPB and their Unification, A BRIEF REPORT. Oct 24, 2024

18. J. S. Brooks, R. J. Donnelly, The calculated thermodynamic properties of superfluid helium-4.Journal of Physical and Chemical Reference Data 6, 51 (1977); https://doi.org/10.1063/1.555549 Published Online: 15 October 2009

19. V. D. Arp, R. D. McCarty, D. G. Friend, Thermophysical Properties of Helium-4 from 0.8 to 1500 K with Pressures to 2000 MPa, Natl. Inst. Stand. Technol., Tech. Note 1334 (revised), 152 pages (September 1998)

20. V. Teboul, A. Monteil, L.C. Fai, A. Kerrache and S. Maabou An investigation of string-like cooperative motion in a strong network glass-former . Eur. Phys. J. B 40, 49–54 (2004) DOI: 10.1140/epjb/e2004-00237-3



A) Б)

Fig. 1. Movement along filamentary structures at a constant speed andreflection from the walls allows angular momentum to be transferred to the superfluidcomponent of liquid helium, leading to a dependence of the tangential velocity on the radius according to formula (2). A) trajectories with maximum chain length and minimum critical rotational velocity. B)Trajectory of an inscribed triangle, motion along which occurs withincreasing rotational velocity.

A) B)

Fig. 2. Spatial arrangement of atoms belonging to the normal phase (spheres without filling) and the superfluid phase (spheres with filling) at a low concentration of the superfluid phase. A) Atoms belonging to the superfluid phase are uniformly distributed in space. B) Atoms of the superfluid phase are concentrated in certain three-dimensional regions.

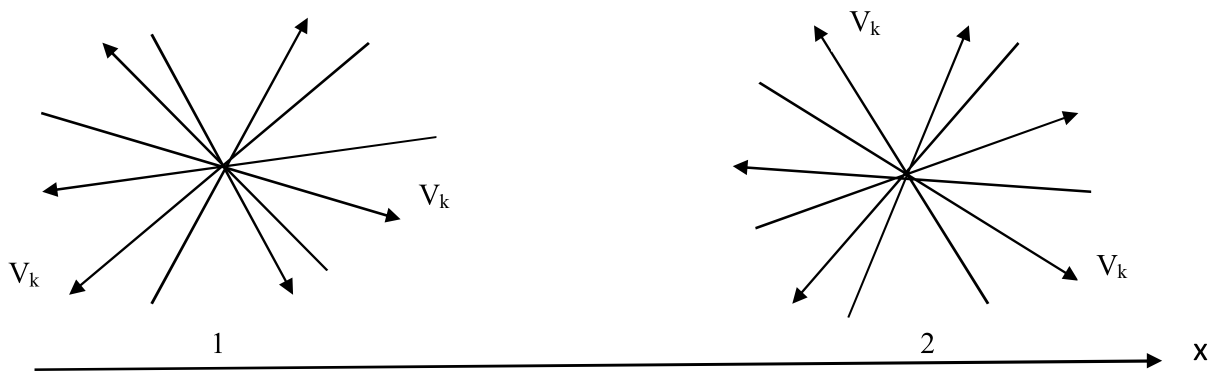


Fig. 3 Graphical representation of the flow of the superfluid phase between points 1 and 2 along the x-axis. V_k is the velocity of movement of helium atoms belonging to the superfluid phase along the direction of the thread.