Remarks on Vibrational Interpretations of the Schrödinger Equation

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Abstract

While the Schrödinger equation is one of the most important equations in physics, its interpretation is still being debated even one hundred years after its original publication. After summarizing Schrödinger's vibrational interpretations, this work remarks on three specific features that might be of particular interest for the development of future vibrational interpretations.

1 Introduction

Starting with the first publication of the Schrödinger equation in 1926 [Sch26b], Schrödinger compared solutions of his famous equation (i.e. wave functions) to states of "a vibrating string or drumhead or metal plate, or of a bell that is tolling" [Sch52, page 113]. While the details of his vibrational interpretations changed over time (as outlined in Section 2), he stayed firm in his belief that a vibrational model could provide a better understanding of atomic processes than the "damned quantum-jumping" [Hei55, page 14].

While many physicists welcomed the Schrödinger equation as a useful tool, most of them preferred the "Copenhagen interpretation" of quantum mechanics including Born's interpretation of the squared amplitude of wave functions as probability density of potential measurement results [Bor26]. Linda Wessels described Schrödingers reaction to this development in this way: "When it became clear in 1928 that most physicists had accepted the 'Copenhagen interpretation', Schrödinger abandoned his attempts to generate alternatives. He retained his conviction that a physical picture was necessary, however, and all but gave up his active participation in the further development of quantum theory. Almost all of his papers on quantum mechanics after 1928 were aimed at criticising the prevailing interpration and, in the 1950s, at raising again the possibility of a wave interpretation" [Wes01, page 143].

Schrödinger shared this conviction with several famous physicists, who also worked on alternatives to the Copenhagen interpretation. To name just a few of them: Planck proposed a way to find solutions of the Schrödinger equation that correspond to classical motions of particles [Pla40]; Mie and Landé worked on new interpretations of the Dirac equation (which may be considered a relativistic generalization of the Schrödinger equation for particles with spin 1/2) by deriving it from a classical field theory [Mie28, Lan29]; de Broglie tried to find a classical, non-linear field theory that generalizes the Schrödinger equation and describes the internal structure of particles [Bro60]; and de Broglie had laid the foundation for the Bohmian interpretation of the Schrödinger equation [Boh52]. It is impossible to discuss the features of all proposed alternatives here. Instead, Section 3 discusses three specific features that might be important for future vibrational interpretations of the Schrödinger equation.

2 Schrödinger's Vibrational Interpretations

2.1 From de Broglie's Phase Waves to a Standard Wave Equation

According to Felix Bloch, a comment by Debye motivated Schrödinger to find a wave equation for the phase waves that de Broglie described in his doctoral thesis [dB25]:

> "Once at the end of a colloquium I heard Debye saying something like: 'Schrödinger, you are not working right now on very important problems anyway. Why don't you tell us some time about that thesis of de Broglie, which seems to have attracted some attention.' So, in one of the next colloquia, Schrödinger gave a beautifully clear account of how de Broglie associated a wave with a particle and how he could obtain the quantization rules of Niels Bohr and Sommerfeld by demanding that an integer number of waves should be fitted along a stationary orbit. When he had finished, Debye casually remarked that he thought this way of talking was rather childish. As a student of Sommerfeld he had learned that, to deal properly with waves, one had to have a wave equation. It sounded quite trivial and did not seem to make a great impression, but Schrödinger evidently thought a bit more about the idea afterwards. Just a few weeks later he gave another talk in the colloquium which he started by saying: 'My colleague Debye suggested that one should have a wave equation; well, I have found one!'" [Blo76]

2.2 From a Standard Wave Equation to the Schrödinger Equation

In his first publication about the Schrödinger equation [Sch26b], Schrödinger mentioned that "[i]t is, of course, strongly suggested that we should try to connect the function ψ [which solves the Schrödinger equation] with some *vibration process* in the atom, which would more nearly approach reality than the electronic orbits, the real existence of which is being very much questioned to-day" [Sch28, page 9]), but he chose to describe the time-independent Schrödinger equation as a reduced Hamilton-Jacobi equation [Sch26b, page 361]. It is only in the second half of his second publication about the Schrödinger equation [Sch26c, page 509] that he discussed a non-relativistic wave equation for de Broglie's phase waves of a single electron.

For an electron of constant mass m and constant energy E with a time-independent potential energy $V(\mathbf{x})$, the electron's speed $v \overset{\text{def}}{=} |\mathbf{v}|$ may be considered a time-independent function of the electron's position $\mathbf{x}$: $v = v(\mathbf{x})$. The same is true for its momentum $p = p(\mathbf{x})$. In the case discussed below, de Broglie's phase velocity u and wavelength λ may similarly be considered time-independent functions of position $\mathbf{x}$: $u = u(\mathbf{x})$ and $\lambda = \lambda(\mathbf{x})$. Thus, speed, momentum, phase velocity, and wavelength are considered position-dependent fields (suitable to describe, for example, extended phase waves, matter waves, or waves of charge density) instead of time-dependent features of a point-like particle on an electronic orbit. (For clarity, the notation here differs from Schrödinger's original notation and tries to make dependencies on position $\mathbf{x}$ and time t more explicit. Furthermore, angular frequency ω is used instead of frequency $\nu = \omega/(2\pi)$ in order to avoid confusion with the symbol "v" denoting the electron's speed.)

Schrödinger chose a standard wave equation for a time-dependent wave function $\Psi(\mathbf{x}, t)$ with position-dependent phase velocity $u(\mathbf{x})$ [Sch26c, Eq. (18)]:

$$\nabla^2 \Psi(\mathbf{x}, t) - \frac{1}{u^2(\mathbf{x})} \frac{\partial^2}{\partial t^2} \Psi(\mathbf{x}, t) = 0. \tag{1}$$

A wave function $\Psi(\mathbf{x}, t)$ is "stationary" if its time-dependence is given by an oscillating factor $\exp(\mathrm{i}\omega t)$ with constant angular frequency ω, i.e.

$$\Psi(\mathbf{x}, t) = \mathrm{e}^{\mathrm{i}\omega t} \psi(\mathbf{x}). \tag{2}$$

Inserting this form of $\Psi(\mathbf{x}, t)$ into Eq. (1), performing the time derivatives, and removing the common factor $\exp(\mathrm{i}\omega t)$ in the result yields a time-independent partial differential equation:

$$\nabla^2 \psi(\mathbf{x}) + \frac{\omega^2}{u^2(\mathbf{x})} \psi(\mathbf{x}) = 0. \tag{3}$$

A solution $\psi(\mathbf{x})$ of this equation leads to a wave function $\Psi(\mathbf{x}, t)$, which oscillates everywhere with the same constant angular frequency ω. Thus, $\Psi(\mathbf{x}, t)$ may be interpreted as a mode of vibration similar to a mode of "a vibrating string or drumhead or metal plate, or of a bell that is tolling" [Sch52, page 113]. This analogy is one of the foundations of Schrödinger's original vibrational interpretation.

In order to obtain a more common form of the (time-independent) Schrödinger equation, the phase velocity $u(\mathbf{x})$ in Eq. (3) may be expressed by de Broglie's wavelength $\lambda(\mathbf{x})$, which is given by the momentum $p(\mathbf{x})$ of the electron, which may be computed (non-relativistically) from its kinetic energy $E - V(\mathbf{x})$ as described next.

First, note that the phase velocity $u(\mathbf{x})$ specifies how far a wavefront of constant phase travels per time interval, which may be computed as de Broglie's wavelength $\lambda(\mathbf{x})$ divided by the period of its oscillation, or, equivalently, by the product of $\lambda(\mathbf{x})$ and the wave's frequency $\omega/(2\pi)$:

$$u(\mathbf{x}) = \lambda(\mathbf{x})\omega/(2\pi). \tag{4}$$

Thus [Sch26c, Eq. (12)]:

$$\lambda(\mathbf{x}) = \frac{2\pi u(\mathbf{x})}{\omega}. \tag{5}$$

Note that even if $\lambda(\mathbf{x})$ is known, this equation is not sufficient to determine values of ω and $u(\mathbf{x})$. However, if we assume $\lambda(\mathbf{x}) = 2\pi\hbar/(mv(\mathbf{x}))$ and $u(\mathbf{x}) = c^2/v(\mathbf{x})$ as derived by de Broglie for a non-relativistic, free electron [dB25], then Eq. (5) would state that ω equals the electron's Compton angular frequency $mc^2/\hbar$.

In general, Eq. (5) leads to:

$$\frac{\omega^2}{u^2(\mathbf{x})} = \frac{4\pi^2}{\lambda^2(\mathbf{x})}. \tag{6}$$

Therefore, the factor $\omega^2/u^2(\mathbf{x})$ in Eq. (3) may be replaced by $4\pi^2/\lambda^2(\mathbf{x})$:

$$\nabla^2\psi(\mathbf{x}) + \frac{4\pi^2}{\lambda^2(\mathbf{x})}\psi(\mathbf{x}) = 0. \tag{7}$$

Furthermore, de Broglie's wavelength $\lambda(\mathbf{x})$ is given by $h/p(\mathbf{x}) = 2\pi\hbar/p(\mathbf{x})$ with the electron's momentum $p(\mathbf{x})$ [Bro60, page 6]. Non-relativistically, the momentum $p(\mathbf{x})$ is equal to $mv(\mathbf{x})$, where $v(\mathbf{x})$ may be computed from the (classical) kinetic energy $E - V(\mathbf{x}) = mv^2(\mathbf{x})/2$:

$$\lambda(\mathbf{x}) = \frac{2\pi\hbar}{p(\mathbf{x})} = \frac{2\pi\hbar}{mv(\mathbf{x})} = \frac{2\pi\hbar}{\sqrt{2m(E - V(\mathbf{x}))}}. \tag{8}$$

Inserting this expression into Eq. (7) results in the following equation [Sch26c, Eq. (18″)]:

$$\nabla^2\psi(\mathbf{x}) + \frac{2m(E - V(\mathbf{x}))}{\hbar^2}\psi(\mathbf{x}) = 0, \tag{9}$$

which becomes—after a bit of algebra—a well-known form of the time-independent Schrödinger equation:

$$-\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{x}) + V(\mathbf{x})\psi(\mathbf{x}) = E\psi(\mathbf{x}). \tag{10}$$

As mentioned, historical sources [Sch26b, Sch26c, Blo76] suggest that the described relation between the standard wave equation Eq. (1) [Sch26c, Eq. (18)] and the (time-independent) Schrödinger equation was particularly important for the historical development of the latter. Today, however, most students of physics never learn about this relation. Neither did most of their teachers. This is not the place to speculate on the reasons for this development, but it is clear that without knowledge of this relation, it is more difficult to understand Schrödinger's motivation to find a vibrational interpretation of his famous equation.

2.3 From the Schrödinger Equation to Vibrational Interpretations

As mentioned in the previous section, stationary wave functions resemble modes of vibrating systems, which motivates vibrational interpretations of the Schrödinger equation. An obvious question that any vibrational interpretation should answer is: what (if anything) is it that is vibrating as specified by a wave function?

Linda Wessels provided a brief summary of Schrödinger's answers to this question:

> "[I]nitially Schrödinger presented the wave equation as the product of formal restrictions on Hamilton's equation for particles. [...] When a month later Schrödinger offered a second

> way of generating the wave equation,[1] he took as the starting point his original wave picture of matter.
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> Schrödinger recognized that the wave equation could not be interpreted as a description of those original matter waves. In the final three papers of his series on wave mechanics, Schrödinger outlined a new vibrational interpretation, in which ψ^2 determined a continuous and often oscillating charge distribution surrounding the nucleus. This interpretation solved the problem of relating the multi-dimensional wave function to the separate oscillation amplitudes of several electrons, and guided Schrödinger in extending the theory to treat radiation intensities.[2] As the number and variety of problems facing this interpretation grew to a critical mass in mid-1927, Schrödinger continued to insist that *some* physical picture of the processes underlying quantum mechanics be provided in order to complete the theory satisfactorily." [Wes01, page 143]

While Schrödinger never reached a conclusive vibrational interpretation, the proponents of the prevailing Copenhagen interpretation of quantum mechanics have offered many reasons for *not* providing a vibrational interpretation (e.g. by declaring it irrelevant, outside the scope of quantum theory, unphysical, impossible in principle, etc.). In fact, there is still no satisfactory vibrational interpretation of the Schrödinger equation.

3 Specific Features of Vibrational Interpretations

In order to guide the development of future, more satisfactory vibrational interpretations, this section remarks on three specific features of such interpretations that might be of particular interest.

3.1 Zero Point of Frequency

Section 2.2 introduced the potential energy $V(\mathbf{x})$. In classical mechanics, the choice of the zero point of a potential energy usually is arbitrary. Since the classical energy $E = mv^2(\mathbf{x})/2 + V(\mathbf{x})$ of an electron includes its potential energy, the zero point of E is arbitrary, too. And this means that the zero point of an angular frequency defined by $\omega = E/\hbar$ [Sch26c, Eq. (11)] would be just as arbitrary. Therefore, Schrödinger wrote that this definition of frequency "makes sense" only if the value of E is absolute instead of dependent on an additive constant as in classical mechanics. („Das hat freilich erst Sinn, wenn E absolut, nicht, wie in der klassischen Mechanik, nur bis auf eine additive Konstante festgelegt ist" [Sch26c, pages 497–498].)

Thus, E_ω in $\omega \overset{\text{def}}{=} E_\omega/\hbar$ should be defined differently. Fortunately, special relativity provides absolute definitions of E_ω based on the electron's relativistic energy; for example,

$$E_\omega \overset{\text{def}}{=} \frac{mc^2}{\sqrt{1 - v(\mathbf{x})^2/c^2}} + V(\mathbf{x}) \qquad \text{with } V(\mathbf{x}) = 0 \text{ for a free electron.} \tag{11}$$

Since non-relativistic mechanics corresponds to an approximation for $v(\mathbf{x}) \ll c$, the non-relativistic approximation of E_ω is:

$$E_\omega \approx \frac{1}{2}mv(\mathbf{x})^2 + mc^2 + V(\mathbf{x}). \tag{12}$$

With this definition of E_ω, the angular frequency $\omega \overset{\text{def}}{=} E_\omega/\hbar$ "makes sense". Otherwise, the zero point of the angular frequency does not "make sense". The same is true for a phase velocity $u(\mathbf{x}) \overset{\text{def}}{=} \lambda(\mathbf{x})\omega/(2\pi)$ (with de Broglie's wavelength $\lambda(\mathbf{x})$). A more detailed discussion of this issue has been provided by Mungan [Mun17b].

However, if one is not interested in de Broglie's phase wave but only in differences between a bound electron's energy levels (as determined by the Schrödinger equation), an additive constant to those energy levels (e.g. corresponding to the rest energy mc^2) does not change anything essential [Sch26a, page 666].

As Mungan [Mun17b] noted, some physicists, e.g. Mundarain [Mun17a] and Griffiths [Gri95, Sec. 2.4], advocated for removing the rest energy from E_ω when computing the angular frequency

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[1] See the previous section.

[2] See also Section 3.2.

of de Broglie's phase waves. It should be clear that the resulting frequency and phase velocity do not "make sense"; i.e., whatever they are, they are not the frequency and phase velocity of de Broglie's phase wave—not even approximately. This is not surprising considering that de Broglie's phase wave is an inherently relativistic phenomenon as de Broglie showed in his doctoral thesis [dB25]. Thus, if the frequency and phase velocity of de Broglie's phase wave of a particle with $v(\mathbf{x}) \ll c$ are defined in the framework of classical mechanics, the relativistic zero point of the particle's energy, i.e. its rest energy, should be included. Why would respectable physicists advocate for definitions that lead to obviously incorrect values? One reason might be the absence of an established vibrational interpretation of the Schrödinger equation. If there was such an interpretation, it would probably require specific frequencies and phase velocities of vibrations—even if those vibrations were discussed in the framework of non-relativistic classical mechanics.

3.2 Coupling with Electromagnetic Fields

According to Schrödinger, the coupling between wave functions and electromagnetic fields is of particular interest: "An especially important question—perhaps the cardinal question of all atomic dynamics—is, as we know, that of the coupling between the dynamic process in the atom and the electromagnetic field, or whatever has to appear in the place of the latter" [Sch28, page 59]. Historically, this coupling remained unresolved in quantum mechanics as noted by Blum and Jähnert: "For radiation, the development of matrix and wave mechanics did not present a complete rupture. Rather it was a short disruption, which ended when quantum mechanics reconnected to the research tradition of the correspondence principle and introduced a modernized version of the *Ersatzstrahler*. Meanwhile, the elusive problem of coupling remained unresolved" [BJ24, page 612].

Already in 1925, however, Born and Jordan had outlined the idea of a "description of the electromagnetic field as a quantum system" [BJ24, page 609]. This approach succeeded in the 1940s with the development of (modern) quantum electrodynamics. Schrödinger, however, had his own ideas, as discussed by Blum and Jähnert: "In contrast to this approach, Schrödinger was [in late 1926] confident enough to proclaim publicly that his charge density interpretation showed the road ahead. Ultimately, it would lead to the resolution of the cardinal question of quantum theory through the integration of wave mechanics and classical electrodynamics into a new [classical] field theory" [BJ24, page 609].

This unified classical field theory would necessarily have to include some nonlinear features in order to allow for interactions between particles and electromagnetic fields because "[i]f everything were linear, nothing would influence nothing" (as Schrödinger quoted Einstein in a letter to Born) [Moo92, page 381].

For example, de Broglie hoped to generalize Schrödinger's wave mechanics to find a field theory that could describe quantum-mechanical particles:

> "[In 1927] I boldly laid down a hypothesis—that of the Double Solution—according to which the linear equations of Wave Mechanics admitted two kinds of solution: the continuous Ψ solutions one normally thinks of—the statistical nature of which was beginning to become clearly apparent at that time, thanks to the work of Born, and 'singularity' [u] solutions that would have a concrete meaning and be the true physical representation of the wave-particle dualism. Particles would then be clearly localized in space, as in the classical picture, but they would be *incorporated* in an extended wave phenomenon." [Bro60, pages 89–90]

Later in his academic career, de Broglie realized that wave equations for this kind of field theories had to be nonlinear:

> "A certain number of reasons [...] have led me to modify quite considerably the ideas I originally expressed in 1927. At that time I considered the u wave as a solution of the *linear* equation of propagation of Wave Mechanics, which would involve a singularity in the mathematical sense of the word. It seems to me at the present time absolutely certain that the idea of singularity must be replaced by that of a very small singular region—in general, mobile—where the u function would take on very large values and obey a *non-linear* equation. Only outside this very small singular region would the u function approximately obey the linear equation of propagation of current Wave Mechanics." [Bro60, page 95]

De Broglie might have been influenced by Born-Infeld field theory [BIF34], which is a nonlinear generalization of electrodynamics with particle-like solutions. While the electron model by Born and Infeld did not include quantum mechanical features, there are modified Born-Infeld theories [Kra23] that appear to include particle-like solutions with the quantum-mechanical angular momentum $\hbar/2$, i.e. spin 1/2 [Kra24].

If a nonlinear field theory provides a satisfactory model of a quantum-mechanical electron, it might imply a suitable vibrational interpretation of the Schrödinger equation. In fact, it might be possible to derive the Schrödinger equation based on the scenario of a particle-like ("singular") solution in an otherwise weak (i.e. linear) field. Quantization conditions might then emerge as resonance phenomena between the oscillating particle-like solution and a vibrating wave function—similar to de Broglie's quantization condition for an electron with internal clock in a hydrogen-like atom [Kra25].

3.3 Coordinate-Independent Quantization Conditions

The Schrödinger equation for an electron in the Coulomb potential of a positive elementary charge (i.e. the hydrogen problem) is usually solved in spherical coordinates by the method of separation of variables [Sch26b]. While this approach leads to reasonable approximations of observable energy levels, the resulting wave functions do not correspond to physical motions of the electron [Pla40, Rob17]. For example, time-dependent wave functions with magnetic quantum number $m \neq 0$ feature azimuthal phase velocities while time-dependent wave functions with magnetic quantum number $m = 0$ are proper standing waves without phase velocity (presumably corresponding to "a statistical (with uniform probability) averaging of [...] waves corresponding to each motion" [Rob17, page 210]). There is no physical reason for this difference, but it is an artifact of the separation of variables in a chosen coordinate system. In this sense, the magnetic quantum number m represents a coordinate-dependent quantization condition.

On the other hand, coordinate-independent quantization conditions (e.g., Einstein's quantization condition [Ein17]) might not only lead to solutions that are more meaningful, but they might also reveal physical mechanisms that are related to a vibrational interpretation. For example, a vibrational interpretation of de Broglie's quantization condition for electronic phase waves in hydrogen-like atoms [Kra25] requires phase waves to move either clockwise or counterclockwise based on the (physical) motion of the electron's internal clock, which drives the phase waves. This interpretation requires a superposition of a clockwise-moving and a counterclockwise-moving wave to form a standing wave. (This kind of superposition can be understood as a statistical ensemble of solutions [Bal98].)

While this example covers the principal quantum number n of the hydrogen problem, it is an open question whether all quantum numbers of the hydrogen problem can be explained in a similar way. Hypothetically, some form of quantization of a phase acquired on an electron's orbit (i.e. a form of flux quantization of electromagnetic fields) might be related to the azimuthal quantum number l and (taking into account the precession of orbital angular momentum about the direction of an external magnetic field) the magnetic quantum number m. The quantization of the acquired phase itself might be understandable (again) as a resonance phenomenon in a vibrational interpretation of the Schrödinger equation.

References

[Bal98] Leslie E. Ballentine. *Quantum Mechanics: A Modern Development.* World Scientific Publishing Company, 1998.

[BIF34] Max Born, Leopold Infeld, and Ralph Howard Fowler. Foundations of the new field theory. *Proceedings of the Royal Society of London. Series A, Containing Papers of a Mathematical and Physical Character*, 144(852):425–451, 1934.

[BJ24] Alexander Blum and Martin Jähnert. Quantum mechanics, radiation, and the equivalence proof. *Archive for History of Exact Sciences*, 78:567–616, 2024.

[Blo76] Felix Bloch. Heisenberg and the early days of quantum mechanics. *Physics Today*, 29(12):23–27, 1976.

[Boh52] David Bohm. A suggested interpretation of the quantum theory in terms of "hidden" variables. I. *Phys. Rev.*, 85:166–179, Jan 1952.

[Bor26] Max Born. Zur Quantenmechanik der Stoßvorgänge. *Zeitschrift für Physik*, 37(12):863–867, 1926.

[Bro60] Louis de Broglie. *Non-linear Wave Mechanics. A Causal Interpretation.* Elsevier Publishing Company, Amsterdam, London, New York, Princeton, 1960. English translation by Arthur J. Knodel and Jack C. Miller of "Une tentative d'interprétation causale et non linéaire de la mécanique ondulatoire" (Gauthier-Villars, 1956).

[dB25] Louis de Broglie. Recherches sur la théorie des Quanta. *Annales de Physique*, 10(3):22–128, January 1925. English translation by A. F. Kracklauer.

[Ein17] Albert Einstein. Zum Quantensatz von Sommerfeld und Epstein. *Verhandlungen der Deutschen Physikalischen Gesellschaft*, 19(9–10):82–92, May 1917.

[Gri95] David J. Griffiths. *Introduction to Quantum Mechanics.* Prentice Hall, 1995.

[Hei55] Werner Heisenberg. The development of the interpretation of the quantum theory. In W. Pauli, L. Rosenfeld, and V. Weisskopf, editors, *Niels Bohr and the development of physics: Essays dedicated to Niels Bohr on the occasion of his seventieth birthday*, pages 12–29. Pergamon Press, 1955.

[Kra23] Martin Kraus. A modified Born-Infeld model of electrons as rotating waves. viXra:2304.0046, 2023.

[Kra24] Martin Kraus. A modified Born-Infeld model of electrons with intrinsic angular momentum. viXra:2403.0005, 2024.

[Kra25] Martin Kraus. De Broglie's phase waves of electrons in hydrogen-like atoms as parts of steady-state forced vibrations. viXra:2503.0008, 2025.

[Lan29] Alfred Landé. Zur Quantenelektrik von G. Mie. *Zeitschrift für Physik*, 57:713–722, 1929.

[Mie28] Gustav Mie. Untersuchungen zum Problem der Quantenelektrik. *Annalen der Physik*, 390(6):711–729, 1928.

[Moo92] Walter Moore. *Schrödinger, life and thought.* Cambridge University Press, 1st paperback edition, 1992.

[Mun17a] Douglas Mundarain. About the non-relativistic limit of the phase velocity of matter waves. *European Journal of Physics*, 38(4):045402, 2017.

[Mun17b] Carl E. Mungan. Phase and group velocity of matter waves, 2017. URL: https://www.usna.edu/Users/physics/mungan/_files/documents/Scholarship/MatterWaves.pdf; accessed 6-April-2026.

[Pla40] Max Planck. Versuch einer Synthese zwischen Wellenmechanik und Korpuskularmechanik. *Annalen der Physik. 5. Folge*, 429(4):261–277, 1940.

[Rob17] J. Robert. L. de broglie's waves: a particular solution. *Annales de la Fondation Louis de Broglie*, 42(1):197–212, 2017.

[Sch26a] Erwin Schrödinger. Der stetige Übergang von der Mikro-zur Makromechanik. *Naturwissenschaften*, 14(28):664–666, 1926.

[Sch26b] Erwin Schrödinger. Quantisierung als Eigenwertproblem (Erste Mitteilung). *Annalen der Physik*, 79:361–376, 1926.

[Sch26c] Erwin Schrödinger. Quantisierung als Eigenwertproblem (Zweite Mitteilung). *Annalen der Physik*, 79:489–527, 1926.

[Sch28] Erwin Schrödinger. *Collected Papers on Wave Mechanics*. Blackie & Son, 1928. Translated from the second German edition.

[Sch52] Erwin Schrödinger. Are there quantum jumps? part I. *The British Journal for the Philosophy of Science*, 3(10):109–123, 1952.

[Wes01] Linda Wessels. Schrödinger's route to wave mechanics. In P. Galison, M. Gordin, and D. Kaiser, editors, *Quantum Histories*, volume 4 of *Science and Society: The History of Modern Physical Science in the Twentieth Century*, pages 115–144. Routledge, 2001.

A Revisions

- Original version submitted to vixra.org on April 15, 2026.
- Revision July 3, 2026: avoided the assumption $\omega = E/\hbar$ and added a comment about the undetermined ω and $u(\mathbf{x})$ in Section 2.2; some changes in Section 3.1 to clarify the distinction between E and what is now named E_ω.