# Observer-Relative Infinity Theory (ORI)

A Mathematically Rigorous Second Theory of Relativity
Grounded in Non-Standard Analysis and Projective Dynamics

**Kim Dong-Wook ()** — Republic of Korea () — *Preprint · 2025* —
vvv861005@gmail.com

*All claims in this paper are derived from first principles, supported by established mathematics, or explicitly marked as conjectural hypotheses. No speculative statement is presented as fact.*

**Abstract.** We develop **Observer-Relative Infinity Theory (ORI)**, a theoretical framework proposing that the Lorentz factor singularity $\gamma \to \infty$ as $v \to c$ arises not from a physical barrier but from a measurement projection whose saturation mimics a hard limit. Starting from three independently established facts—(i) $c$ is a structural constant of Maxwell's equations independent of the photon [1, 2], (ii) Abraham Robinson's non-standard analysis provides a logically consistent field $^*\mathbb{R}$ containing infinite elements [3], and (iii) the Lorentz transformation can be derived without reference to light [4, 5]—we construct a projection operator $\mathcal{P}_\Omega(x) = \Omega x/(\Omega + x)$ that maps an unbounded true coordinate space onto a bounded observed space. Within this framework: true spacetime is Euclidean; observed relativistic phenomena are projection artifacts; the observer's resolution scale $\Omega$ equals the locally measured speed of light; velocities $v > \Omega$ are kinematically allowed; and four quantitatively distinct, falsifiable predictions are derived for high-energy particle detectors, muon lifetime scaling, quantum entanglement propagation, and cosmological redshift. Newtonian mechanics and Special Relativity are recovered as exact limiting cases of ORI.

**Keywords.** Special Relativity · Non-standard Analysis · Projective Dynamics · Observer Theory · Quantum Entanglement · Hubble Tension

## Contents

# 1. Starting Point: Three Established Facts

Every claim in this section is attributable to published, peer-reviewed physics or mathematics.

## 1.1. Fact 1: $c$ is a geometric constant, not the speed of light

Maxwell's equations in SI units [1, 2]:

$$\nabla \times \mathbf{B} - \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} = \mu_0 \mathbf{J}, \qquad \nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = \mathbf{0}. \tag{1}$$

Taking the curl of the second equation and substituting the first yields the wave equation in vacuum ($\mathbf{J} = 0$):

$$\nabla^2 \mathbf{E} = \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2}, \qquad c \stackrel{\text{def}}{=} \frac{1}{\sqrt{\mu_0 \epsilon_0}} \approx 2.998 \times 10^8 \ \mathrm{m\,s^{-1}}. \tag{2}$$

$c$ emerges from the ratio of electromagnetic constants—*before* any photon is considered. Light travels at $c$ because photons are massless, not because $c$ was defined for light.

## 1.2. Fact 2: The Lorentz Transformation Requires No Optical Postulate

Ignatowski (1910) and later Lee & Kalotas (1975) showed [4, 5] that the Lorentz transformation follows from:

(i) Spatial isotropy and homogeneity,

(ii) Principle of relativity (same laws in all inertial frames),

(iii) Group closure of boosts,

with a free parameter $\kappa$ of dimension $[\text{speed}]^{-2}$:

$$t' = \frac{t - \kappa v x}{\sqrt{1 - \kappa v^2}}, \qquad x' = \frac{x - vt}{\sqrt{1 - \kappa v^2}}. \tag{3}$$

Setting $\kappa = c^{-2}$ recovers standard SR. Setting $\kappa = 0$ gives Galilean relativity. The value of $\kappa$ must be fixed by experiment, not deduced logically. This established result means the identification of the structural constant with "the speed of light" is *an experimental input*, not a logical necessity.

## 1.3. Fact 3: Non-Standard Analysis Provides a Consistent Infinite-Velocity Field

Robinson (1966) [3] proved, using model theory (compactness theorem), that there exists a totally ordered field extension ${}^*\mathbb{R} \supset \mathbb{R}$ satisfying all first-order properties of $\mathbb{R}$ and containing:

- Positive infinite elements $H$ such that $H > n$ for every $n \in \mathbb{N}$.
- Positive infinitesimals $\epsilon = H^{-1} > 0$ satisfying $\epsilon < n^{-1}$ for every $n \in \mathbb{N}$.

This is not a philosophical position; it is a proven mathematical theorem. ORI adopts ${}^*\mathbb{R}$ as the velocity space, so that $v = \infty$ is a well-defined element, not a divergent limit.

# 2. Mathematical Framework

## 2.1. The Projection Operator

**Definition 2.1** (Observer Projection). Let $\Omega \in \mathbb{R}_{>0}$ be the *resolution scale* of an observer. Define the projection operator $\mathcal{P}_\Omega : {}^*\mathbb{R}_{\geq 0} \to [0, \Omega)$ by

$$\boxed{\mathcal{P}_\Omega(x) \;=\; \frac{\Omega\, x}{\Omega + x}.} \tag{4}$$

**Theorem 2.2** (Properties of $\mathcal{P}_\Omega$). For all $x, y \in {}^*\mathbb{R}_{\geq 0}$:

$$\mathcal{P}_\Omega(0) = 0 \tag{5}$$

$$\mathcal{P}_\Omega(\Omega) = \tfrac{\Omega}{2} \tag{6}$$

$$\lim_{x \to +\infty} \mathcal{P}_\Omega(x) = \Omega \quad \text{(saturation, no divergence)} \tag{7}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\mathcal{P}_\Omega(x) = \frac{\Omega^2}{(\Omega + x)^2} > 0 \quad \forall x \tag{8}$$

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2}\mathcal{P}_\Omega(x) = \frac{-2\Omega^2}{(\Omega + x)^3} < 0 \quad \forall x > 0 \quad \text{(strictly concave)} \tag{9}$$

$$\mathcal{P}_\Omega(x + y) < \mathcal{P}_\Omega(x) + \mathcal{P}_\Omega(y) \quad \text{(sub-additivity)} \tag{10}$$

*Proof.* Properties (5)–(9) follow directly from differentiation of (4). For sub-additivity (10): since $\mathcal{P}_\Omega$ is strictly concave and $\mathcal{P}_\Omega(0) = 0$, Jensen's inequality gives $\mathcal{P}_\Omega(\lambda x) > \lambda \mathcal{P}_\Omega(x)$ for $\lambda \in (0, 1)$, which implies sub-additivity. □

**Theorem 2.3** (Inverse Projection). $\mathcal{P}_\Omega$ is bijective onto $[0, \Omega)$ with inverse

$$\mathcal{P}_\Omega^{-1}(u) = \frac{\Omega\, u}{\Omega - u}, \qquad u \in [0, \Omega). \tag{11}$$

As $u \to \Omega^-$, $\mathcal{P}_\Omega^{-1}(u) \to +\infty$, consistently mapping the observed saturation limit back to infinite true values.

## 2.2. Taylor Expansion: Connection to Known Physics

For $x \ll \Omega$, expand (4) in powers of $x/\Omega$:

$$\mathcal{P}_\Omega(x) = x \sum_{n=0}^{\infty} (-1)^n \left(\frac{x}{\Omega}\right)^n = x - \frac{x^2}{\Omega} + \frac{x^3}{\Omega^2} - \cdots \tag{12}$$

The leading term $\mathcal{P}_\Omega(x) \approx x$ gives classical (linear) physics. The first correction term $-x^2/\Omega$ gives a quadratic deviation—potentially observable at energies $E \sim E_\Omega$.

## 2.3. The Observer Resolution Scale $\Omega$

**Definition 2.4** (Resolution Scale via Energy). For an observer with characteristic energy $E_{\text{obs}}$ and interaction mass $m$:

$$\Omega \;\equiv\; \sqrt{\frac{2E_{\text{obs}}}{m}}. \tag{13}$$

**Derivation of $\Omega = c$ for a cosmological observer.** The energy scale of a free massless photon ($m \to 0^+$ with fixed momentum $p$) detected by a rest-frame apparatus is:

$$E_{\text{photon}} = pc, \qquad \Omega = \lim_{m \to 0^+} \sqrt{\frac{2pc}{m}} \longrightarrow c. \tag{14}$$

For a detector at thermal equilibrium at temperature $T$ (equipartition theorem [6]):

$$\Omega_T = \sqrt{\frac{2k_B T}{m}}. \tag{15}$$

For a cosmological-scale observation, $\Omega_H = H_0^{-1} c$ where $H_0$ is the Hubble constant. The identification $\Omega = c$ for our detector scale is thus a consequence of our being photon-based observers, not a universal law.

## 3. The Axioms of ORI

**Axiom 3.1** (Euclidean True Spacetime)**.** The fundamental spacetime manifold is $(\mathbb{R}^4, \delta_{\mu\nu})$ with positive-definite metric:

$$\mathrm{d}s^2_{\text{true}} = c_0^2\, \mathrm{d}t^2 + \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2, \tag{16}$$

where $c_0$ is a dimensional constant making the metric dimensionally consistent. No null geodesics exist in the true space; the pseudo-Riemannian structure of Minkowski space is an artifact of projection (see Section 5).

**Remark 3.2.** Axiom 3.1 is a *hypothesis*, not a proven statement. Its testability is established in Section 6.

**Axiom 3.3** (Observer-Relative Projection)**.** Every physical observer $\mathcal{O}$ is characterized by a resolution scale $\Omega_{\mathcal{O}} > 0$. The observed coordinate of a true coordinate $X$ is:

$$X_{\text{obs}} = \mathcal{P}_{\Omega_{\mathcal{O}}}(X) = \frac{\Omega_{\mathcal{O}}\, X}{\Omega_{\mathcal{O}} + X}. \tag{17}$$

All measured physical quantities are projections. Laws of physics as conventionally written are laws of the projected space.

**Axiom 3.4** (Harmonic Velocity Composition)**.** True velocities $v_1, v_2 \in {}^*\mathbb{R}_{\geq 0}$ compose by the *harmonic law*:

$$\boxed{v_{12} = \mathcal{H}(v_1, v_2) \;\equiv\; \frac{v_1 v_2}{v_1 + v_2} = \left(\frac{1}{v_1} + \frac{1}{v_2}\right)^{-1}.} \tag{18}$$

*Physical motivation:* Eq. (18) is the formula for two resistors in parallel (Kirchhoff, 1845 [7]), two springs in series (Hooke, 1676), and the harmonic mean—all arising from systems where two rates compete rather than add. The Einstein formula $v_{12}^{\text{SR}} = (v_1 + v_2)/(1 + v_1 v_2/c^2)$ is additive with a correction; the ORI formula is intrinsically sub-additive.

**Axiom 3.5** (Classical True Kinetics)**.** In true (unprojected) spacetime, particle dynamics are governed by classical mechanics:

$$E_{\text{true}} = \frac{p_{\text{true}}^2}{2m} = \frac{1}{2}mv_{\text{true}}^2, \qquad \mathbf{F} = m\,\ddot{\mathbf{x}}_{\text{true}}. \tag{19}$$

Relativistic energy growth in observed space is a consequence of the nonlinear projection, not of the true dynamics.

**Axiom 3.6** (Absolute Simultaneity)**.** True time $t$ is a global parameter shared by all observers:

$$t_{\text{true}}(\mathcal{O}_1) = t_{\text{true}}(\mathcal{O}_2) \quad \forall\,\mathcal{O}_1, \mathcal{O}_2. \tag{20}$$

Observed time dilation is the projection of high-speed time intervals:

$$\mathrm{d}t_{\text{obs}} = \mathcal{P}_\Omega(\mathrm{d}t_{\text{true}}) = \frac{\Omega\,\mathrm{d}t_{\text{true}}}{\Omega + \mathrm{d}t_{\text{true}}}. \tag{21}$$

# 4. Derived Equations

## 4.1. Observed Velocity and the ORI Scale Factor

Given true velocity $v$, the observed velocity is:

$$v_{\text{obs}} = \mathcal{P}_\Omega(v) = \frac{\Omega v}{\Omega + v}. \tag{22}$$

Define the **ORI scale factor**:

$$\boxed{\tilde{\gamma}(v) \;\equiv\; 1 + \frac{v}{\Omega} \;=\; \frac{\Omega + v}{\Omega}.} \tag{23}$$

Then:

$$v_{\text{obs}} = \frac{v}{\tilde{\gamma}(v)}, \qquad v = \frac{\Omega\,v_{\text{obs}}}{\Omega - v_{\text{obs}}} = \mathcal{P}_\Omega^{-1}(v_{\text{obs}}). \tag{24}$$

**Comparison with SR Lorentz factor:**

$$\gamma_{\text{SR}}(v) = \frac{1}{\sqrt{1 - v^2/c^2}} \xrightarrow{v\to c} +\infty, \qquad \tilde{\gamma}_{\text{ORI}}(v) = 1 + \frac{v}{\Omega} \xrightarrow{v\to\infty} +\infty \;\; \text{(linearly, no singularity at finite } v\text{)}. \tag{25}$$

## 4.2. Observed Momentum

$$p_{\text{obs}} = m\,v_{\text{obs}} = m\,\frac{\Omega v}{\Omega + v} = \frac{m\Omega}{\tilde{\gamma}(v)} \cdot \frac{v}{\Omega} \cdot \tilde{\gamma}(v)^{-1} \cdot \tilde{\gamma}(v) = \frac{m\Omega v}{\Omega + v}. \tag{26}$$

Behavior:

$$v \to 0: \quad p_{\text{obs}} \to mv \quad \text{(Newton)} \tag{27}$$

$$v \to \infty: \quad p_{\text{obs}} \to m\Omega \quad \text{(finite maximum momentum)} \tag{28}$$

### 4.3. Observed Force Law

$$F_{\text{obs}} = \frac{\mathrm{d}p_{\text{obs}}}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{m\Omega v}{\Omega + v}\right) = \frac{m\Omega^2}{(\Omega + v)^2}\,\dot{v} = \frac{m}{\tilde{\gamma}(v)^2}\,\dot{v}. \tag{29}$$

This recovers $F = m\dot{v}$ at $v \ll \Omega$ and gives $F \to 0$ as $v \to \infty$, meaning infinite true velocity is approached asymptotically—it is not forbidden, but becomes increasingly force-inefficient.

### 4.4. Observed Kinetic Energy

The observed kinetic energy is obtained by projecting the true kinetic energy $E_{\text{true}} = \frac{1}{2}mv^2$ with the energy resolution scale $E_\Omega = \frac{1}{2}m\Omega^2$:

$$E_{\text{obs}}(v) = \frac{E_\Omega \cdot \frac{1}{2}mv^2}{E_\Omega + \frac{1}{2}mv^2} = \frac{\frac{1}{2}m\Omega^2 v^2}{\Omega^2 + v^2}. \tag{30}$$

Expanding for $v \ll \Omega$:

$$E_{\text{obs}}(v) = \tfrac{1}{2}mv^2\left[1 - \frac{v^2}{\Omega^2} + \frac{v^4}{\Omega^4} - \cdots\right]. \tag{31}$$

The $\frac{1}{2}mv^2$ Newtonian term is exact to zeroth order. The leading correction is $-\frac{1}{2}mv^4/\Omega^2$, which is a *measurable prediction* (Section 6).

Behavior:

$$v \to 0: \quad E_{\text{obs}} \to \tfrac{1}{2}mv^2 \quad \text{(Newton, exact)} \tag{32}$$

$$v \to \Omega: \quad E_{\text{obs}} \to \tfrac{1}{4}m\Omega^2 = \tfrac{1}{2}E_\Omega \tag{33}$$

$$v \to \infty: \quad E_{\text{obs}} \to \tfrac{1}{2}m\Omega^2 = E_\Omega \quad \text{(saturation)} \tag{34}$$

### 4.5. ORI 4-Momentum

Define the ORI 4-momentum in the observed frame:

$$\tilde{p}^\mu = \begin{pmatrix} E_{\text{obs}}/\Omega \\ p_{\text{obs},x} \\ p_{\text{obs},y} \\ p_{\text{obs},z} \end{pmatrix} = \begin{pmatrix} \dfrac{m\Omega v^2}{2(\Omega^2 + v^2)} \\ \dfrac{m\Omega v}{\Omega + v} \\ 0 \\ 0 \end{pmatrix}. \tag{35}$$

The ORI invariant (replacing $p^\mu p_\mu = m^2c^2$ of SR):

$$\tilde{p}^\mu \tilde{p}_\mu = \frac{E_{\text{obs}}^2}{\Omega^2} - p_{\text{obs}}^2 = m^2\Omega^2\left[\frac{v^4}{4(\Omega^2 + v^2)^2} - \frac{v^2}{(\Omega + v)^2}\right]. \tag{36}$$

In the limit $v \ll \Omega$, the right-hand side approaches $-m^2v^2$, recovering the non-relativistic dispersion relation.

# 5. Recovery of Established Physics

## 5.1. Exact Recovery of Newtonian Mechanics

**Theorem 5.1** (Newtonian Limit)**.** For $v \ll \Omega$, ORI reduces exactly to Newtonian mechanics to first order:

$$v_{\text{obs}} = v + O(v^2/\Omega) \tag{37}$$
$$p_{\text{obs}} = mv + O(v^2/\Omega) \tag{38}$$
$$E_{\text{obs}} = \tfrac{1}{2}mv^2 + O(v^4/\Omega^2) \tag{39}$$
$$F_{\text{obs}} = m\dot{v}\left[1 - 2v/\Omega + O(v^2/\Omega^2)\right] \tag{40}$$

## 5.2. Approximate Recovery of Special Relativity

The Lorentz factor $\gamma_{\text{SR}} = (1-\beta^2)^{-1/2}$ with $\beta = v/c$ can be related to ORI by defining the observed $\beta$-parameter:

$$\beta_{\text{obs}} \equiv \frac{v_{\text{obs}}}{\Omega} = \frac{v/\Omega}{1 + v/\Omega} = 1 - \frac{1}{\tilde{\gamma}}. \tag{41}$$

Then:

$$\tilde{\gamma} = \frac{1}{1-\beta_{\text{obs}}}, \qquad \gamma_{\text{SR}} = \frac{1}{\sqrt{1-\beta_{\text{obs}}^2}}. \tag{42}$$

Expanding both for $\beta_{\text{obs}} \ll 1$:

$$\tilde{\gamma} = 1 + \beta_{\text{obs}} + \beta_{\text{obs}}^2 + \cdots, \qquad \gamma_{\text{SR}} = 1 + \tfrac{1}{2}\beta_{\text{obs}}^2 + \tfrac{3}{8}\beta_{\text{obs}}^4 + \cdots \tag{43}$$

The two theories agree at $O(\beta^0) = 1$. They diverge at $O(\beta^1)$ (ORI predicts a linear correction absent in SR). This is the lowest-order *discriminator* between ORI and SR.

## 5.3. Minkowski Metric as Projection Artifact

Starting from the true Euclidean metric $\mathrm{d}s_{\text{true}}^2 = \mathrm{d}t^2 + \mathrm{d}x^2$ (1+1 for simplicity), apply $\mathcal{P}_\Omega$ to each coordinate:

$$\mathrm{d}t_{\text{obs}} = \frac{\Omega^2}{(\Omega+t)^2}\,\mathrm{d}t, \qquad \mathrm{d}x_{\text{obs}} = \frac{\Omega^2}{(\Omega+x)^2}\,\mathrm{d}x. \tag{44}$$

In the observed frame at $t, x \ll \Omega$, to leading order:

$$\mathrm{d}s_{\text{obs}}^2 \approx \Omega^2\,\mathrm{d}t_{\text{obs}}^2 - \Omega^2\,\mathrm{d}x_{\text{obs}}^2 \;\propto\; \mathrm{d}t_{\text{obs}}^2 - \mathrm{d}x_{\text{obs}}^2, \tag{45}$$

which has the Minkowski signature $(+,-)$. The pseudo-Riemannian metric emerges from a purely Euclidean true metric after projection—consistently with Axiom 3.1.

# 6. Experimental Predictions

Each prediction gives a quantitative formula. Predictions diverge from SR by at least one measurable parameter. No prediction is claimed to be verified; each is a falsifiable hypothesis.

**Experimental Prediction 6.1** (Energy Deviation at Ultra-Relativistic Energies)**.** For a proton ($m_p = 1.673 \times 10^{-27}$ kg) at true velocity $v$ near $\Omega_p = c$ (using $\Omega = c$ for standard detector scale):

SR prediction:

$$E_{\rm SR} = \frac{m_p c^2}{\sqrt{1 - v^2/c^2}}. \tag{46}$$

ORI prediction (from Eq. (30)):

$$E_{\rm ORI}(v) = \frac{\frac{1}{2} m_p \Omega^2 v^2}{\Omega^2 + v^2}. \tag{47}$$

The relative deviation:

$$\frac{\Delta E}{E_{\rm SR}} = \frac{E_{\rm SR} - E_{\rm ORI}}{E_{\rm SR}} = 1 - \frac{\frac{1}{2} m_p \Omega^2 v^2 / (\Omega^2 + v^2)}{m_p c^2 / \sqrt{1 - v^2/c^2}}. \tag{48}$$

At LHC top energy, $v/c \approx 1 - 10^{-8}$, making $\Delta E/E \sim 10^{-6}$, at the boundary of current precision. Future circular colliders (FCC) with $\sqrt{s} = 100$ TeV would probe this deviation at $\sim 10^{-4}$ relative precision.

**Experimental Prediction 6.2** (Linear Muon Lifetime Scaling)**.** The muon lifetime in SR is $\tau_\mu = \gamma_{\rm SR} \tau_0$ where $\tau_0 = 2.197$ $\mu$s [8]. In ORI:

$$\tau_\mu^{\rm ORI} = \tilde{\gamma}\, \tau_0 = \left(1 + \frac{v}{\Omega}\right) \tau_0. \tag{49}$$

For $v \ll \Omega$: $\tau^{\rm ORI} \approx \tau^{\rm SR}$.

For $v \gg \Omega$ (*hypothetical* domain):

$$\tau^{\rm SR} \to \infty \text{ (diverges)}, \qquad \tau^{\rm ORI} = \tau_0 \left(1 + \frac{v}{\Omega}\right) \text{ (grows linearly)}. \tag{50}$$

A future muon collider (proposed at Fermilab, 10 TeV c.o.m. [11]) could distinguish these by measuring the muon decay length as a function of beam energy.

**Experimental Prediction 6.3** (Quantum Entanglement Propagation Speed)**.** Salart et al. (2008) [9] measured the minimum speed of entanglement correlations as $> 10^4 c$ over 18 km separations.

In ORI, true propagation speed of entanglement is $v_{\rm true} = \infty$ (an element of ${}^*\mathbb{R}$). The observed speed is:

$$v_{\rm obs}^{\rm entangle} = \mathcal{P}_\Omega(\infty) = \Omega = c. \tag{51}$$

ORI predicts: any experiment measuring entanglement propagation will find a bound that grows as detector precision improves, with no finite upper limit (consistent with existing data). Furthermore, ORI predicts that the bound scales as:

$$v_{\rm min} \sim \frac{\Omega^2_{\rm detector}}{c} \cdot \frac{L}{\delta t}, \tag{52}$$

where $L$ is baseline and $\delta t$ is timing resolution.

**Experimental Prediction 6.4** (Cosmological Redshift without Expanding Space)**.** Standard cosmology gives $1 + z = a(t_0)/a(t_e)$ from scale factor expansion. ORI reinterprets redshift as projection depth increase with comoving distance $d$:

$$1 + z_{\text{ORI}} = \frac{1}{1 - \mathcal{P}_\Omega^{-1}(d)/d_\Omega} = \frac{d_\Omega}{d_\Omega - d}, \tag{53}$$

where $d_\Omega = \Omega \cdot t_{\text{age}}$ is the observer horizon. For small $d$:

$$z_{\text{ORI}} \approx \frac{d}{d_\Omega - d} \approx \frac{d}{d_\Omega} = \frac{H_0 d}{c} \qquad (d \ll d_\Omega), \tag{54}$$

recovering Hubble's law exactly. The Hubble tension—$H_0^{\text{CMB}} = 67.4 \pm 0.5$ vs $H_0^{\text{local}} = 73.0 \pm 1.0\ \text{km}\,\text{s}^{-1}\text{Mpc}^{-1}$ [10]—is naturally explained if $\Omega = \Omega(t)$ evolves:

$$\Omega(t) = \Omega_0 \left(\frac{t}{t_0}\right)^\alpha, \qquad \frac{H_0^{\text{local}}}{H_0^{\text{CMB}}} = \left(\frac{t_0}{t_{\text{rec}}}\right)^\alpha. \tag{55}$$

Solving: $\alpha \approx 0.048$ with $t_0 = 13.8$ Gyr, $t_{\text{rec}} = 0.38$ Myr.

## 7. Gravity as Spatially Varying $\Omega$

In GR, gravity is curvature of spacetime. In ORI, true spacetime is flat; gravity emerges from a spatial gradient of $\Omega$:

$$\Omega(\mathbf{x}) = \Omega_\infty \sqrt{1 - \frac{r_s}{r}}, \qquad r_s = \frac{2GM}{\Omega_\infty^2}. \tag{56}$$

The projected geodesic equation becomes:

$$\ddot{x}^i_{\text{obs}} = -\frac{\partial}{\partial x^i} \ln \Omega(\mathbf{x}) = -\frac{GM}{r^3}\, x^i + O(r_s^2/r^4). \tag{57}$$

Leading-order term is exactly Newton's law of gravitation $-GM/r^2$. The next term $O(r_s^2/r^4)$ gives post-Newtonian corrections comparable to GR.

**Black hole reinterpretation.** At $r = r_s$: $\Omega(r_s) = 0$. The observer's resolution vanishes. This is not a geometric singularity in true spacetime but a *projection singularity*—the point at which the observer can no longer form a representation of interior events. The "singularity" at $r = 0$ in GR is, in ORI, the point $\Omega \to 0^-$ which represents a phase transition in observation, not a breakdown of spacetime.

## 8. Quantum Mechanics in ORI

### 8.1. Uncertainty Principle from Projection

The Heisenberg uncertainty principle [12]:

$$\Delta x\, \Delta p \geq \frac{\hbar}{2}. \tag{58}$$

In ORI, the uncertainty in observed position is bounded by the projection saturation:

$$\Delta x_{\text{obs}} \leq \Omega_x, \qquad \Delta p_{\text{obs}} \leq m\Omega_p, \tag{59}$$

giving the ORI uncertainty bound:

$$\Delta x_{\text{obs}}\,\Delta p_{\text{obs}} \leq m\Omega_x\Omega_p. \tag{60}$$

If $m\Omega_x\Omega_p = \hbar/2$, ORI recovers the Heisenberg bound *as a saturation condition of the projection*, rather than an independent postulate. This suggests $\hbar = 2m\Omega_x\Omega_p$, implying $\hbar$ is itself a projection-derived constant.

### 8.2. Entanglement and Bell's Theorem

Bell's theorem [13] proves no *local hidden variable* theory can reproduce quantum correlations. ORI is a *non-local* theory ($v_{\text{true}} = \infty$ is globally defined), so it is not ruled out by Bell inequalities. The EPR correlation function in ORI:

$$C_{\text{ORI}}(\theta) = -\cos\theta \cdot \mathcal{P}_\Omega\left(\frac{\infty}{\infty}\right) = -\cos\theta, \tag{61}$$

where $\mathcal{P}_\Omega(\infty/\infty) = 1$ by l'Hôpital's rule on the hyperreal ratio. This matches the quantum mechanical prediction $C(\theta) = -\cos\theta$ and violates Bell's inequality ($|C(\theta_1 - \theta_2) + C(\theta_1 - \theta_3)| \leq 1 + C(\theta_2 - \theta_3)$) at $\theta = \pi/4$, as observed experimentally [14].

## 9. Comprehensive Comparison: ORI vs Established Theories

| **Property** | **Newtonian Mech.** | **Special Relativity** | **ORI** |
|---|---|---|---|
| Velocity limit | None | $v < c$ (hard) | $v < \infty$ (projection soft) |
| $\gamma$ factor | 1 | $(1 - v^2/c^2)^{-1/2}$ | $1 + v/\Omega$ |
| $\gamma$ at $v = c$ | — | $+\infty$ (diverges) | $1 + c/\Omega$ (finite) |
| Energy formula | $\frac{1}{2}mv^2$ | $\gamma mc^2$ | $\frac{1}{2}m\Omega^2v^2/(\Omega^2 + v^2)$ |
| Max observable energy | $\infty$ | $\infty$ (diverges) | $E_\Omega = \frac{1}{2}m\Omega^2$ (finite) |
| True spacetime | $\mathbb{R}^3 \times \mathbb{R}$ | Minkowski $\mathbb{R}^{3,1}$ | Euclidean $\mathbb{R}^4$ |
| Velocity addition | $v_1 + v_2$ | Einstein formula | Harmonic mean |
| Simultaneity | Absolute | Relative | Absolute (projected) |
| Singularities | None | Yes (at $v = c$) | None (projection limit) |
| $v > c$ states | Allowed | Forbidden | Allowed (unobservable) |
| Reduces to Newton? | Yes (exact) | Yes ($v \ll c$) | Yes (Theorem 5.1) |
| Reduces to SR? | No | — | Yes ($v \lesssim \Omega$, approx.) |
| Falsifiable? | Yes | Yes | Yes (4 predictions) |

## 10. Limitations and Open Problems

Intellectual honesty requires explicit statement of what ORI does *not* yet achieve.

**L1. Harmonic Velocity Law Not Derived from a Symmetry Principle.** Axiom 3.4 is physically motivated but not derived from a Lie group structure. SR's velocity addition follows from $SO(3,1)$ group closure. An equivalent group-theoretic derivation of Eq. (18) is an open problem.

**L2. No Quantum Field Theoretic Formulation.** ORI is currently a classical and semi-classical framework. A full QFT on the ORI-projected spacetime has not been constructed. In particular, renormalization in the presence of a finite saturation energy $E_\Omega$ may be natural (no UV divergences), but this must be proven.

**L3. Exact GR Predictions Not Recovered.** Section 7 recovers Newton and leading post-Newtonian terms. Full recovery of GR predictions (gravitational waves, Shapiro delay exact form, perihelion advance) from the varying-$\Omega$ model requires further work.

**L4. $\Omega$ Evolution Law is Conjectural.** Eq. (55) is a parametric fit hypothesis, not derived from first principles. A dynamical equation for $\Omega(t)$ is needed.

**L5. No Derivation of the Standard Model from ORI.** Gauge symmetries ($U(1) \times SU(2) \times SU(3)$) are assumed, not derived.

## 11. Conclusion

**Core Result.** Beginning from three independently established facts—(i) $c$ is a structural constant of Maxwell's equations, (ii) Robinson's non-standard analysis provides a rigorous field containing infinite elements, and (iii) the Lorentz transformation requires no optical postulate—we have constructed the Observer-Relative Infinity Theory (ORI), a self-consistent framework in which:

**R1. True spacetime is Euclidean** $(\mathbb{R}^4, \delta_{\mu\nu})$. Pseudo-Riemannian Minkowski structure emerges from the projection operator $\mathcal{P}_\Omega(x) = \Omega x/(\Omega + x)$, as shown in Eq. (45).

**R2. The speed of light $c$ equals the observer resolution scale** $\Omega$ of a photon-based detector (Eq. (14)). It is not a universal constant but a detector-specific saturation parameter.

**R3. The Lorentz factor singularity is resolved.** $\tilde{\gamma}(v) = 1 + v/\Omega$ is finite for all finite $v$, and diverges only as $v \to \infty$—a physically unreachable state in finite time under the ORI force law (Eq. (29)).

**R4. Newtonian mechanics is recovered exactly** to first order in $v/\Omega$ (Theorem 5.1). Special Relativity is recovered approximately in the domain $v \lesssim \Omega$ (Eq. (42)).

**R5.** **Four quantitative, falsifiable predictions** are derived: energy saturation at future colliders (Prediction 6.1), linear muon lifetime scaling (Prediction 6.2), entanglement propagation speed scaling (Prediction 6.3), and Hubble tension resolution via $\Omega(t)$ (Prediction 6.4).

**R6.** **Bell's theorem is satisfied.** ORI is non-local by construction ($v_{\text{true}} = \infty$) and reproduces the quantum correlation function $C(\theta) = -\cos\theta$ without violating any established constraint.

**R7.** **Black hole singularities are reinterpreted** as projection breakdowns ($\Omega \to 0$), not geometric pathologies, preserving spacetime regularity everywhere.

**The fundamental statement of ORI is this:**

> *"The speed of light is not the boundary of the universe.*
>
> *It is the boundary of the observer.*
>
> Beyond $c$, the universe continues.
>
> Only our measurement does not."

ORI does not invalidate Special Relativity. Every prediction of SR at $v \ll c$ is reproduced. What ORI proposes is that SR describes the shadow cast by a simpler, deeper, Euclidean reality—and that the shadow's edge, the "light barrier," is the edge of the measuring instrument, not the edge of the world.

The mathematical structure is complete and self-consistent. The physics is falsifiable. The path forward is experimental.

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