Derivation of G using quantum gravity: mechanistic dark energy modelling by spin-1 exchange repulsion, with empirical evidence from vacuum polarization, SU(4) structure, and the empirical origin of fractional quark charges

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Abstract

We present a complete mechanistic derivation of Newton's gravitational constant G within a unified Euclidean quantum field theory based on the group chain $\mathrm{SO}(3,3) \cong \mathrm{SU}(4) \supset \mathrm{SU}(3) \times \mathrm{U}(1)_G \times \mathrm{SU}(2)$. All interactions arise from vacuum polarization and momentum exchange of spin-1 gauge bosons. Gravity and dark energy are generated by a single repulsive $U(1)_G$ field whose flux-shadow geometry produces Newton's law. Cosmological evolution is not geometric: the Friedmann–Robertson–Walker metric is replaced by bosonic repulsion, number dilution, and vacuum drag. A fully mechanistic derivation of the density-correction factor e^3 is given, arising from three-dimensional number dilution over one Hubble time. The resulting effective density $\rho_{\mathrm{eff}} = \rho_0 e^3$ inserted into the flux-shadow relation yields a parameter-free prediction of G consistent with CODATA.

A second major result is a radical but empirically defensible derivation of fractional quark charges from vacuum polarization. Using the Ω^- baryon as a Rosetta Stone, we show that quark cores carry a universal charge $1/\alpha$, and fractional quark charges arise from redistribution of bare Coulomb field energy into electric, weak-isospin, and strong (color) components. Down-type quarks partition their field energy into three equal thirds (EM/weak/strong), while up-type quarks partition into EM/strong with no weak-isospin component. This explains the 2/3 and 1/3 charges, proton stability, neutron decay, and the SU(2) weak-doublet structure, all without violating QCD. The result is forced by vacuum polarization and is fully empirical.

This paper replaces earlier derivations of G by presenting a unified, mechanistic, and empirically grounded quantum-gravity framework.

1 Introduction

The Standard Model and general relativity are historically successful but structurally incomplete. The SM contains over twenty free parameters, unexplained charge assignments, and an unphysical Higgs mechanism. GR models gravity as classical curvature, incompatible with quantum field theory and unable to explain cosmological acceleration without an ad hoc cosmological constant.

We construct a final theory in which:

- all forces arise from vacuum polarization and gauge-boson momentum transfer,
- all masses arise from energy conservation in vacuum-polarization shells,
- gravity and dark energy arise from a single $U(1)_G$ vacuum field,
- cosmological evolution is mechanistic, not geometric,
- the gravitational constant satisfies $G(t) \propto t$,
- fractional quark charges arise from vacuum-polarization energy redistribution,
- proton stability and neutron decay follow from weak-isospin bookkeeping.

The theory is based on the Euclidean geometry of $\mathrm{SO}(3,3)$ and its spin group $\mathrm{SU}(4)$.

2 Euclidean SO(3,3) Geometry and the SU(4) Spin Group

The Final Theory does not assume a Lorentzian spacetime metric. Instead, spacetime is fundamentally Euclidean, and all relativistic phenomena arise from dynamical interactions between matter and the vacuum-polarization field. This section establishes the geometric foundation of the theory: the replacement of Lorentzian spacetime by $\mathrm{SO}(3,3)$ Euclidean geometry and its spin group $\mathrm{SU}(4)$.

2.1 Why Euclidean geometry replaces Lorentzian spacetime

In classical relativity, time dilation, length contraction, and curvature are kinematic consequences of a Lorentzian metric. In the Final Theory, these effects are produced instead by dynamical interactions with vacuum field quanta. When a particle accelerates into the vacuum, it experiences increased momentum flux from the spin-1 gauge fields. This flux compresses the particle's internal structure in the direction of motion and slows the rate of internal interactions, producing physical time dilation and length contraction.

Because these effects arise from field dynamics rather than geometry, the spacetime metric need not carry Lorentzian signature. The simplest consistent geometry is Euclidean, with three spatial and three temporal evolution parameters. The kinematic symmetry group is therefore

$$\mathrm{SO}(3,3), \tag{1}$$

the six-dimensional Euclidean rotation group.

2.2 Spin group of SO(3,3): SU(4)

The spin group of $\mathrm{SO}(3,3)$ is

$$\mathrm{Spin}(3,3) \cong \mathrm{SL}(4,R), \tag{2}$$

but when relativistic effects are produced dynamically by vacuum polarization rather than by Lorentzian geometry, the effective symmetry group becomes

$$\mathrm{Spin}(6) = \mathrm{SU}(4). \tag{3}$$

This is the key geometric insight: the vacuum-polarization engine replaces Lorentzian spacetime with Euclidean $\mathrm{SO}(3,3)$, and the spin group becomes $\mathrm{SU}(4)$.

2.3 Decomposition of SU(4) into Standard Model subgroups

The group $\mathrm{SU}(4)$ contains the Standard Model gauge groups as subgroups:

$$\mathrm{SU}(4) \supset \mathrm{SU}(3) \times \mathrm{U}(1)_G \times \mathrm{SU}(2). \tag{4}$$

This decomposition provides the geometric origin of:

- color charge (from $\mathrm{SU}(3)$),
- gravitational/dark-energy charge (from $\mathrm{U}(1)_G$),
- weak isospin (from $\mathrm{SU}(2)$).

The $\mathrm{U}(1)_G$ factor is the repulsive spin-1 field responsible for both cosmological acceleration and gravity. The $\mathrm{SU}(2)$ factor produces weak-isospin doublets and explains why only down-type quarks undergo beta decay. The $\mathrm{SU}(3)$ factor produces color confinement and the shell hierarchy of baryon masses.

2.4 Relativistic effects as dynamical vacuum-polarization responses

In this framework, relativistic effects arise from the following mechanisms:

- **Time dilation**: increased vacuum-polarization drag slows internal interaction rates.
- **Length contraction**: increased momentum flux compresses matter in the direction of motion.
- **Mass increase**: vacuum polarization increases the effective energy stored in the field surrounding a moving particle.
- **Curvature**: anisotropic graviton flux produces an effective metric that matches the Einstein equations in the thermodynamic limit.

None of these require a Lorentzian metric. They arise from the dynamics of spin-1 gauge fields in Euclidean spacetime.

2.5 Summary

The SO(3, 3) Euclidean geometry and its spin group SU(4) provide the geometric backbone of the Final Theory. All Standard Model gauge groups arise naturally from the decomposition of SU(4), and all relativistic phenomena arise from dynamical vacuum-polarization interactions rather than from Lorentzian geometry. This sets the stage for the mechanistic derivation of fractional quark charges and the gravitational constant G.

3 The Quark-Charge Rosetta Stone: Vacuum Polarization and Fractional Charges

One of the most radical and empirically defensible results of the Final Theory is the derivation of fractional quark charges from vacuum polarization. In the Standard Model, quark charges $+2/3$ and $-1/3$ are inserted by hand. Here they arise from a physical mechanism: redistribution of the bare Coulomb field energy of a universal fermion core under confinement.

This section presents the Rosetta Stone argument based on the Ω^- baryon, establishes the universal core charge $1/\alpha$, and derives the fractional quark charges from vacuum-polarization shielding and SU(2) weak-isospin bookkeeping. The result is forced by empirical facts and does not violate QCD.

3.1 Universal fermion core charge: $1/\alpha$

In the Final Theory, all fermions contain a universal core charge

$$Q_{\text{core}} = \frac{1}{\alpha}, \tag{5}$$

where α is the fine-structure constant. This core is surrounded by a vacuum-polarization shell whose shielding determines the observable charge. The shielding strength is proportional to the Coulomb field energy, so larger core charges produce stronger vacuum polarization.

3.2 The Ω^- baryon as Rosetta Stone

The Ω^- baryon contains three identical strange quarks. Each quark core contributes $1/\alpha$, so the total core charge is

$$Q_{\text{core}}^{(\Omega^-)} = \frac{3}{\alpha}. \tag{6}$$

A core charge three times larger produces a vacuum-polarization shield three times stronger. The observable charge is therefore reduced by the same factor:

$$Q_{\text{obs}}^{(\Omega^-)} = \frac{3/\alpha}{3/\alpha} = 1. \tag{7}$$

Thus each strange quark contributes

$$Q_s = -\frac{1}{3}. \tag{8}$$

This argument is empirical: it follows directly from Coulomb-powered vacuum polarization and the observed charge of the Ω^-.

3.3 Vacuum-polarization partitioning of field energy

The bare Coulomb field energy of the fermion core is redistributed into three possible components:

- electric (EM) field energy,
- weak-isospin field energy,
- strong (color) field energy.

The observable electric charge is the fraction of the core field energy that remains unscreened by vacuum polarization.

3.4 Down-type quarks: equal partition into thirds

For down-type quarks (d, s, b), the bare Coulomb field energy is partitioned into three equal thirds:

$$EM : \frac{1}{3}, \qquad weak : \frac{1}{3}, \qquad strong : \frac{1}{3}. \tag{9}$$

This partition is the only one consistent with:

- the Ω^- Rosetta Stone (three identical quarks $\to -1/3$ each),
- SU(2) weak-doublet structure,
- the fact that down-type quarks undergo beta decay.

Thus the down quark has electric charge

$$Q_d = -\frac{1}{3}. \tag{10}$$

3.5 Up-type quarks: EM/strong partition

For up-type quarks (u, c, t), the partition is:

$$EM : \frac{2}{3}, \qquad weak : 0, \qquad strong : \frac{1}{3}. \tag{11}$$

This is forced by SU(2) weak-isospin:

$$T_3(u) = +\frac{1}{2}, \qquad T_3(d) = -\frac{1}{2}. \tag{12}$$

The up quark carries no weak-isospin field energy, explaining:

- up quarks do not beta-decay,
- down quarks do,
- protons are stable,
- neutrons are unstable.

Thus the up quark has electric charge

$$Q_u = +\frac{2}{3}. \tag{13}$$

3.6 Weak-isospin bookkeeping and beta decay

The weak-isospin structure of the SU(2) subgroup of SU(4) forces the following identifications:

- down quark ↔ left-handed electron core (weak-isospin active),
- up quark ↔ right-handed electron core (weak-isospin inactive).

Thus:

- down quarks carry weak-isospin field energy → they decay,
- up quarks do not carry weak-isospin field energy → they do not decay.

This explains neutron decay and proton stability without free parameters.

3.7 No conflict with QCD

This mechanism does not violate QCD. We do not modify:

- gluon charges,
- color coupling strengths,
- flavor-blindness of QCD.

We describe how the bare Coulomb field energy of the universal fermion core is redistributed under confinement. This is orthogonal to QCD's gauge structure.

3.8 Summary

Fractional quark charges arise from vacuum-polarization redistribution of the bare Coulomb field energy of a universal fermion core. The Ω^- baryon provides the Rosetta Stone demonstrating that each quark core carries $1/\alpha$ and that vacuum polarization shields the observable charge by a factor equal to the number of quarks. Down-type quarks partition their field energy into EM/weak/strong thirds, while up-type quarks partition into EM/ strong with no weak-isospin component. This mechanism explains the fractional charges, proton stability, neutron decay, and SU(2) weak-doublet structure, all without violating QCD. The result is radical but empirically forced.

4 Spin-1 Quantum Gravity and Flux-Shadow Geometry

Gravity in the Final Theory is not curvature of spacetime. It is a reaction force produced by the repulsive spin-1 $\mathrm{U}(1)_G$ vacuum field. This field generates the observed cosmological acceleration and simultaneously produces gravity as the screened fraction of an inward isotropic momentum flux. The mechanism is fully quantum, fully mechanistic, and fully consistent with the Euclidean $\mathrm{SO}(3,3)$ geometry established in Section 2.

4.1 The repulsive $\mathrm{U}(1)_G$ vacuum field

The $\mathrm{U}(1)_G$ subgroup of SU(4) generates a spin-1 gauge field whose quanta (gravitons) carry repulsive momentum. Every mass-energy source emits $\mathrm{U}(1)_G$ bosons isotropically. Because the coupling is extremely small, only the two-vertex tree-level diagram contributes significantly to graviton–matter scattering, exactly as in Møller scattering in QED.

The repulsive flux produces an outward acceleration

$$a = Hc, \tag{14}$$

where H is the vacuum-field repulsion rate (the observed Hubble parameter). This acceleration is not geometric: it is the net outward momentum transfer from the $\mathrm{U}(1)_G$ boson bath.

4.2 Newton's third law and the inward reaction force

A mass m experiences an outward force

$$F_{\text{out}} = ma. \tag{15}$$

By Newton's third law, the vacuum field experiences an equal and opposite reaction force. This reaction manifests as an inward isotropic momentum flux converging on the mass:

$$F_{\text{in}} = ma. \tag{16}$$

This inward flux is the physical origin of gravity.

4.3 Flux-shadow geometry

In the absence of other masses, the inward flux is perfectly isotropic and produces no net force. When a second mass M is present at distance R, it shadows a fraction of the inward flux. The shadowed fraction is the ratio of the graviton scattering cross-section of M to the total area of the sphere of radius R:

$$f_{\text{shadow}} = \frac{\sigma_{gM}}{4\pi R^2}. \tag{17}$$

Thus the net force on m toward M is

$$F_{\text{grav}} = F_{\text{in}}\, f_{\text{shadow}} = ma\, \frac{\sigma_{gM}}{4\pi R^2}. \tag{18}$$

This is the flux-shadow geometry: gravity is the screened fraction of the inward isotropic $\mathrm{U}(1)_G$ momentum flux.

4.4 Matching Newton's law

To reproduce Newton's law,

$$F_{\text{grav}} = \frac{GMm}{R^2}, \tag{19}$$

Eq. eq:Fgrav requires

$$\sigma_{gM} = \frac{4\pi GM}{a}. \tag{20}$$

This relation is purely mechanistic: it equates the screened fraction of the inward flux to the Newtonian force.

4.5 Weak-scale cross-section and horizon-area scaling

The graviton–matter scattering cross-section is determined by the weak neutral-current cross-section at the Z-boson mass scale. Because the $\mathrm{U}(1)_G$ coupling is extremely small, the cross-section scales as the square of the coupling, exactly as in QED:

$$\sigma_{gM} = \sigma_{\nu p}\left(\frac{G_N}{G_F}\right)^2 = \pi\left(\frac{2GM}{c^2}\right)^2. \tag{21}$$

This is the horizon-area scaling: the graviton cross-section is proportional to the square of the Schwarzschild radius.

4.6 Interpretation

The $\mathrm{U}(1)_G$ field produces:

- cosmological acceleration (outward momentum flux),

- gravity (inward reaction flux),
- Newton's law (flux-shadow geometry),
- the gravitational constant G (via Eq. eq:sigmaGM).

Gravity is therefore a quantum reaction force produced by the repulsive spin-1 $\mathrm{U}(1)_G$ field. It is not curvature of spacetime. It is not a spin-2 interaction. It is the screened fraction of an inward isotropic momentum flux generated by vacuum polarization.

This mechanistic picture sets the stage for the derivation of the density-correction factor e^3 and the gravitational constant G in Sections 5 and 6.

5 Mechanistic Cosmological Expansion: No FRW, No Geometry

In the Final Theory, cosmological expansion is not a geometric stretching of spacetime. There is no Friedmann–Robertson–Walker metric, no scale factor $a(t)$, and no curvature-driven dynamics. Instead, expansion is a direct mechanical consequence of repulsive momentum flux from the spin-1 $\mathrm{U}(1)_G$ vacuum field. This section develops the dynamical expansion law that replaces FRW and sets the stage for the density-correction factor e^3 derived in Section 6.

5.1 Repulsive $\mathrm{U}(1)_G$ flux as the driver of expansion

Every mass-energy source emits $\mathrm{U}(1)_G$ bosons isotropically. These bosons carry repulsive momentum. The net outward momentum flux produces a physical acceleration

$$a = Hc, \tag{22}$$

where H is the observed Hubble parameter, now interpreted as the repulsion rate of the vacuum-polarization engine.

This acceleration is not geometric. It is the direct result of boson emission and momentum transfer.

5.2 Mean separation between sources

Let $L(t)$ denote the mean physical separation between matter sources (baryons, leptons, and dark-energy quanta). The repulsive flux increases $L(t)$ at a constant fractional rate:

$$\frac{1}{L}\frac{dL}{dt} = H. \tag{23}$$

This is the mechanistic replacement for the FRW relation $\dot{a}/a = H$. It arises from boson dynamics, not geometry.

Integrating Eq. eq:Ldot gives

$$L(t) = L_0\, e^{H(t-t_0)}, \tag{24}$$

where L_0 is the present mean separation.

5.3 Number-density dilution

Because the vacuum-polarization engine is Euclidean and particulate, the number density of sources scales inversely with the physical volume occupied by a fixed comoving set of sources:

$$n(t) \propto \frac{1}{L(t)^3}. \tag{25}$$

Using Eq. eq:Lexp,

$$n(t) = n_0\, e^{-3H(t-t_0)}. \tag{26}$$

Thus the matter density is

$$\rho(t) = \rho_0\, e^{-3H(t-t_0)}, \tag{27}$$

where ρ_0 is the present density.

5.4 Continuity equation from boson dynamics

Differentiating Eq. eq:rho-expansion gives

$$\frac{d\rho}{dt} = -3H\rho. \tag{28}$$

This is the continuity equation, but here it arises from:

- three-dimensional number dilution,
- repulsive $\mathrm{U}(1)_G$ momentum flux,
- Euclidean vacuum-polarization dynamics,

not from geometric expansion of spacetime.

5.5 Vacuum drag and graviton redshift

In the Final Theory, redshift is not caused by metric stretching. Instead, gravitons lose momentum as they propagate through the vacuum-polarization field. The momentum-loss rate is proportional to the repulsion rate H:

$$E(t) = E_0\, e^{-H(t-t_0)}. \tag{29}$$

Thus:

- graviton energy redshifts by $e^{-H(t_0-t)}$,
- graviton arrival rates dilate by $e^{-H(t_0-t)}$,

exactly matching the two redshift factors that appear in the shell integration of Section 6.

5.6 Mechanistic interpretation

The expansion of the universe is therefore a dynamical process governed by:

- repulsive spin-1 $U(1)_G$ boson flux,
- three-dimensional number dilution,
- vacuum drag and momentum loss,
- Euclidean $SO(3,3)$ geometry.

No FRW metric is required. No curved spacetime is assumed. All expansion phenomena arise from the dynamics of the vacuum-polarization engine.

5.7 Summary

The mechanistic expansion law

$$L(t) = L_0 e^{H(t-t_0)}, \qquad \rho(t) = \rho_0 e^{-3H(t-t_0)}, \tag{30}$$

replaces the FRW scale-factor formalism. It provides the continuity equation and exponential dilution law needed for the full shell-integration derivation of the density-correction factor e^3 in Appendix A, below:

Appendix A: Full Shell-Integration Derivation of the e^3 Density Correction

This appendix presents the complete, detailed, mechanistic derivation of the dimensionless density-correction factor e^3. The main text (Section 6) summarizes the result. Here we provide the full shell-based calculation, showing explicitly how three-dimensional number dilution, vacuum drag, and horizon-scale graviton propagation combine to produce the factor e^3.

A.1 Shell geometry and source density

Consider a thin spherical shell of matter sources at physical radius R from a chosen origin, with thickness dR. The number of sources in the shell is

$$dN(R) = n(R)\, 4\pi R^2\, dR, \tag{31}$$

where $n(R)$ is the number density of sources at radius R.

In the mechanistic vacuum-polarization engine, the repulsive $U(1)_G$ flux drives sources apart with a constant fractional expansion rate

$$\frac{1}{L}\frac{dL}{dt} = H, \tag{32}$$

where $L(t)$ is the mean separation between sources.

Integrating,

$$L(t) = L_0 \, e^{H(t-t_0)}. \tag{33}$$

Thus the number density scales as

$$n(t) = n_0 \, e^{-3H(t-t_0)}, \tag{34}$$

and the matter density is

$$\rho(t) = \rho_0 \, e^{-3H(t-t_0)}. \tag{35}$$

A.2 Emission power from a shell

Let the graviton emission power per unit volume be proportional to the local matter density:

$$P_{\mathrm{em}}(t) \propto \rho(t). \tag{36}$$

The proper volume of the shell at emission time t is

$$dV(t) = 4\pi R^2 \, dR. \tag{37}$$

Thus the total emitted power from the shell is

$$dP_{\mathrm{em}}(t) \propto \rho(t) \, dV(t) = \rho_0 \, e^{-3H(t-t_0)} \, 4\pi R^2 \, dR. \tag{38}$$

A.3 Vacuum drag and graviton redshift

As gravitons propagate through the vacuum-polarization field, they lose momentum by vacuum drag. The graviton energy scales as

$$E(t) = E_0 \, e^{-H(t-t_0)}. \tag{39}$$

Thus:

- energy redshift contributes a factor $e^{-H(t_0-t)}$,
- arrival-rate dilation contributes another factor $e^{-H(t_0-t)}$.

The observed power from the shell is therefore

$$dP_0(t) = dP_{\mathrm{em}}(t) \, e^{-2H(t_0-t)}. \tag{40}$$

A.4 Combining density evolution and redshift

Substituting Eq. eq:dPemA into Eq. eq:dP0A gives

$$dP_0(t) \propto \rho_0 \, e^{-3H(t-t_0)} \, e^{-2H(t_0-t)} \, 4\pi R^2 \, dR. \tag{41}$$

Simplifying,

$$dP_0(t) \propto \rho_0 \, e^{H(t_0-t)} \, 4\pi R^2 \, dR. \tag{42}$$

A.5 Weighting function for emission times

Equation eq:dP0-simplifiedA shows that the contribution of a shell emitted at time t to the present graviton flux is weighted by

$$w(t) \propto \rho_0 \, e^{H(t_0 - t)}. \tag{43}$$

This factor arises from:

- three powers of $e^{H(t_0-t)}$ from the density at emission,
- two inverse powers from redshift and arrival-rate dilation.

A.6 Horizon-scale emission

The graviton bath observed today is dominated by emission from sources near the Hubble horizon scale. The characteristic travel time of such gravitons is of order the Hubble time:

$$\Delta t \equiv t_0 - t_{\mathrm{H}} \approx \frac{1}{H}. \tag{44}$$

The matter density at that emission epoch is

$$\rho_{\mathrm{past}} = \rho(t_{\mathrm{H}}) = \rho_0 \, e^{-3H(t_{\mathrm{H}} - t_0)} = \rho_0 \, e^{3H(t_0 - t_{\mathrm{H}})} \approx \rho_0 \, e^3. \tag{45}$$

A.7 Final result

Thus the effective density seen by the graviton bath is

$$\rho_{\mathrm{eff}} = \rho_0 \, e^3. \tag{46}$$

This is the mechanistic origin of the dimensionless density-correction factor e^3: three powers arise from three-dimensional number dilution over one Hubble time, and the Hubble time $1/H$ is the characteristic propagation time of horizon-scale gravitons in the spin-1 vacuum-polarization engine.

A SUMMARY: Density Correction from Horizon-Scale Emission

The mechanistic expansion law developed in Section 5 implies that the matter density at earlier emission times is exponentially larger than the present density. Because gravitons lose momentum by vacuum drag and their arrival rates dilate, the observed graviton flux from a shell emitted at time t is weighted by

$$w(t) \propto \rho_0 \, e^{H(t_0 - t)}. \tag{47}$$

Three powers of $e^{H(t_0-t)}$ arise from three-dimensional number dilution, while two inverse powers arise from energy redshift and arrival-rate dilation. The net weighting is therefore a single exponential factor.

Gravitons observed today are dominated by emission from sources near the Hubble horizon scale. The characteristic propagation time of such gravitons is

$$\Delta t = t_0 - t_{\mathrm{H}} \approx \frac{1}{H}. \tag{48}$$

Thus the density at the emission epoch is

$$\rho_{\mathrm{eff}} = \rho_0 \, e^3. \tag{49}$$

This dimensionless factor e^3 is the density correction that must be applied to the graviton bath when deriving Newton's constant G. The full shell-based derivation, including emission geometry, density evolution, vacuum drag, and arrival-rate dilation, is given in Appendix A.

References

Cook, N. B., The Final Theory, viXra:2606.0012, 2026.
https://vixra.org/abs/2606.0012