ERRATUM TO EXERCISE A4.2 IN “AN INTRODUCTION TO THE THEORY OF THE RIEMANN ZETA FUNCTION” (1988) BY S. J. PATTERSON

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ABSTRACT. The evaluation of coefficients of the Laurent series of $\Gamma(x)$ on page 135 of Patterson's book “An introduction to the theory of the Riemann zeta function” has sign and other errors which are corrected here. A C program is listed which demonstrates the application of Taylor series to compute the Gamma and Barnes G-function. [viXra:2507.0094]

1. LAURENT SERIES OF $\Gamma(z)$

The Laurent series of Euler's Γ-function near the pole at $z=0$ (with residuum 1) defines coefficients $a_k = [x^k]\Gamma(z)$:

$$\Gamma(z) = \frac{1}{z} + a_0 + a_1 z + a_2 z^2 + a_3 z^3 + \cdots . \tag{1}$$

Obeying the functional equation [1, 6.1.15],

$$z\Gamma(z) = \Gamma(z+1) \tag{2}$$

the coefficients a_k resurface as coefficients of the Taylor series at $z=1$:

$$\Gamma(z+1) = 1 + a_0 z + a_1 z^2 + a_2 z^3 + a_3 z^4 + \cdots = \sum_{k\ge 0} a_{k-1} z^k \tag{3}$$

and its derivative

$$\Gamma'(z+1) = \sum_{k\ge 1} k a_{k-1} z^{k-1} \tag{4}$$

supplied with the definition

$$a_{-1} \equiv 1. \tag{5}$$

We may start the evaluation of the a_i with the series of the digamma function [1, 6.3.14][6, §1.17][5]

$$\psi(1+z) = -\gamma + \sum_{n\ge 2} (-)^n \zeta(n) z^{n-1} \tag{6}$$

where ζ is the Riemann zeta function and [7, A001620]

$$\gamma \approx 0.577215664\ldots \tag{7}$$

the Euler-Mascheroni constant.

Remark 1. *In numerical practice this is replaced by Chebyshev series* [19, 18, 15] *or Padé series* [5, 4, 10].

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The digamma function is the derivative of the logarithm of the Γ-function [1, 6.3.1]

$$\psi(z)\Gamma(z) = \Gamma'(z). \tag{8}$$

Transition $z \to z+1$

$$\psi(z+1)\Gamma(z+1) = \Gamma'(z+1) \tag{9}$$

and insertion of the power series of the three terms shows that a convolution of the coefficients of ψ and Γ gives Γ', a tight coupling within the coefficients of Γ:

$$\left[-\gamma - \sum_{m\ge 1} (-)^m \zeta(m+1) z^m\right] \sum_{k\ge 0} a_{k-1} z^k = \sum_{k\ge 0} (k+1) a_k z^k, \tag{10}$$

which is Jensen's Equation (49') [9]. Equating coefficients of z^0, z^1, z^2 etc. on both sides of the equation yields

$$-\gamma a_{-1} = 1a_0; \tag{11}$$

$$-\gamma a_0 + \zeta(2) a_{-1} = 2a_1; \tag{12}$$

$$-\gamma a_1 + \zeta(2) a_0 - \zeta(3) a_{-1} = 3a_2; \tag{13}$$

$$-\gamma a_2 + \zeta(2) a_1 - \zeta(3) a_0 + \zeta(4) a_{-1} = 4a_3, \tag{14}$$

so the a_i on the right hand sides are recursively computed starting from (5) [8, 8.321.1]:

Algorithm 1.

$$a_i = \frac{1}{i+1}\left[-\gamma a_{i-1} + \sum_{k=2}^{i+1} (-)^k \zeta(k) a_{i-k}\right]; \quad a_{-1} = 1. \tag{15}$$

Example 1. *The first examples are* [7, A090998,A385960,A385961,A385962]

$$a_0 = -\gamma \approx -0.5772156649015329; \tag{16}$$

$$a_1 = \frac{1}{2}[\gamma^2 + \zeta(2)] \approx 0.9890559953279726; \tag{17}$$

$$a_2 = \frac{1}{3!}[-\gamma^3 - 3\gamma\zeta(2) - 2\zeta(3)] \approx -0.9074790760808863; \tag{18}$$

$$a_3 = \frac{1}{4!}[\gamma^4 + 6\gamma^2\zeta(2) + 8\gamma\zeta(3) + 3\zeta(2)^2 + 6\zeta(4)] \approx 0.9817280868344002; \tag{19}$$

$$a_4 = \frac{1}{5!}[-\gamma^5 - 10\gamma^3\zeta(2) - 20\gamma^2\zeta(3) - (15\zeta(2)^2 + 30\zeta(4))\gamma - 20\zeta(2)\zeta(3) - 24\zeta(5)] \approx -0.9819950689031452; \tag{20}$$

2. Corrigendum

The main result: On page 135 of Patterson's book [14], there is

- a sign error in front of $\zeta(2)$ in (17),
- a sign error in front of $3\gamma\zeta(2)$ and a missing factor in front of $\zeta(3)$ in (18),
- two sign errors and a wrong factor for $\gamma^2\zeta(2)$ in (19),
- two sign errors and a missing mixed term $\zeta(2)\zeta(3)$ in (20).

3. Reciprocal Γ-function

The expansion coefficients c_k of [1, 6.1.34]

$$\frac{1}{\Gamma(z)} = \sum_{k\ge 1} c_k z^k = \frac{1}{\frac{1}{z} + a_0 + a_1 z + a_2 z^2 + \cdots} = \frac{z}{1 + a_0 z + a_1 z^2 z + a_2 z^3 + a_3 z^4 + \cdots} \quad (21)$$

are obtained by series reversion [1, 3.6.25], which means comparison of like powers of z of both sides of

$$\left[\sum_{k\ge 1} c_k z^k\right]\left[\sum_{l\ge 0} a_{l-1} z^l\right] = z. \quad (22)$$

Resummation $\sum_{k\ge 1}\sum_{l\ge 0} = \sum_{m=1}^{\infty}\sum_{k=1}^{m}$ at $k+l=m$ and solving for c_m yields the recurrence

Algorithm 2.

$$c_m = -\sum_{k=1}^{m-1} c_k a_{m-k-1}, \quad m > 1. \quad (23)$$

Algorithm 3. *This can be simplified to* [12, 5.7.2][8, 8.321.2]

$$c_m = \frac{1}{m-1}\left[\gamma c_{m-1} - \sum_{i=2}^{m-1} (-)^i \zeta(i) c_{m-i}\right], \quad m > 3. \quad (24)$$

Example 2. [7, A001620,A385965,A385966]

$$c_1 = 1; \quad (25)$$

$$c_2 = \gamma \approx 0.577215664901533; \quad (26)$$

$$c_3 = \frac{1}{12}[-\pi^2 + 6\gamma^2] \approx -0.655878071520254; \quad (27)$$

$$c_4 = \frac{1}{12}[4\zeta(3) - \pi^2\gamma + 2\gamma^3] \approx -0.042002635034095; \quad (28)$$

$$c_5 = \frac{1}{1440}[\pi^4 - 60\pi^2\gamma^2 + 60\gamma^4 + 480\gamma\zeta(3)] \approx 0.166538611382291; \quad (29)$$

$$c_6 = \frac{1}{1440}[288\zeta(5) - 20\pi^2\gamma^3 + \pi^4\gamma + 12\gamma^5 -40\pi^2\zeta(3) + 240\zeta(3)\gamma^2] \approx -0.042197734555544; \quad (30)$$

Numerical values have been tabulated up to $k = 40$ by Wrench [20, Table 5], reproduced in Table 1.

Splitting off a factor $z+1$ on the right hand side of (21),

$$\frac{1}{z\Gamma(z)} = (1+z)\sum_{k\ge 0} b_k z^k, \quad (31)$$

defines coefficients b_k that have been tabulated up to $k = 38$ by Wrench [20, Table 4]. The computation of the Γ-function with Wrench's equation (20)

$$\ln[z\Gamma(z)] = -\gamma z + \sum_{k\ge 2} (-)^k \zeta(k) z^k / k, \quad |z| < 1 \quad (32)$$

is demonstrated in the C-function `Gamma()` in the appendix.

k	c_k
1	1.00000000000000000000000000000000000e+00
2	5.77215664901532860606512090082402431e-01
3	-6.55878071520253881077019515145390481e-01
4	-4.20026350340952355290039348754298187e-02
5	1.66538611382291489501700795102105236e-01
6	-4.21977345555443367482083012891873913e-02
7	-9.62197152787697356211492167234819898e-03
8	7.21894324666309954239501034044657271e-03
9	-1.16516759185906511211397108401838867e-03
10	-2.15241674114950972815729963053647806e-04
11	1.28050282388116186153198626328164323e-04
12	-2.01348547807882386556893914210218184e-05
13	-1.25049348214267065734535947383309224e-06
14	1.13302723198169588237412962033074494e-06
15	-2.05633841697760710345015413002057284e-07
16	6.11609510448141581786249868285534287e-09
17	5.00200764446922293005566504805999130e-09
18	-1.18127457048702014458812656543650558e-09
19	1.04342671169110051049154033231225019e-10
20	7.78226343990507125404993731136077723e-12
21	-3.69680561864220570818781587808576624e-12
22	5.10037028745447597901548132286323180e-13
23	-2.05832605356650678322242954485523742e-14
24	-5.34812253942301798237001731872793995e-15
25	1.22677862823826079015889384662242243e-15
26	-1.18125930169745876951376458684229783e-16
27	1.18669225475160033257977724292867407e-18
28	1.41238065531803178155580394756670904e-18
29	-2.29874568443537020659247858063369926e-19
30	1.71440632192733743338396337026725707e-20
31	1.33735173049369311486478139512226802e-22
32	-2.05423355176667278932502535135573380e-22
33	2.73603004860799984483150990433098201e-23
34	-1.73235644591051663905742845156477980e-24
35	-2.36061902449928728734345073542753101e-26
36	1.86498294171729443071841316187866690e-26
37	-2.21809562420719720439971691362686038e-27
38	1.29778197494799366882441448633059417e-28
39	1.18069747496652840622274541550997152e-30
40	-1.12458434927708809029365467426143951e-30

TABLE 1. Coefficients c_k in (21).

4. DOUBLE GAMMA-FUNCTION

Barnes' Double-Gamma-Function G satisfies [12, §5.17][13]

$$\Gamma(x)G(x) = G(x+1), \quad G(1) = 1. \tag{33}$$

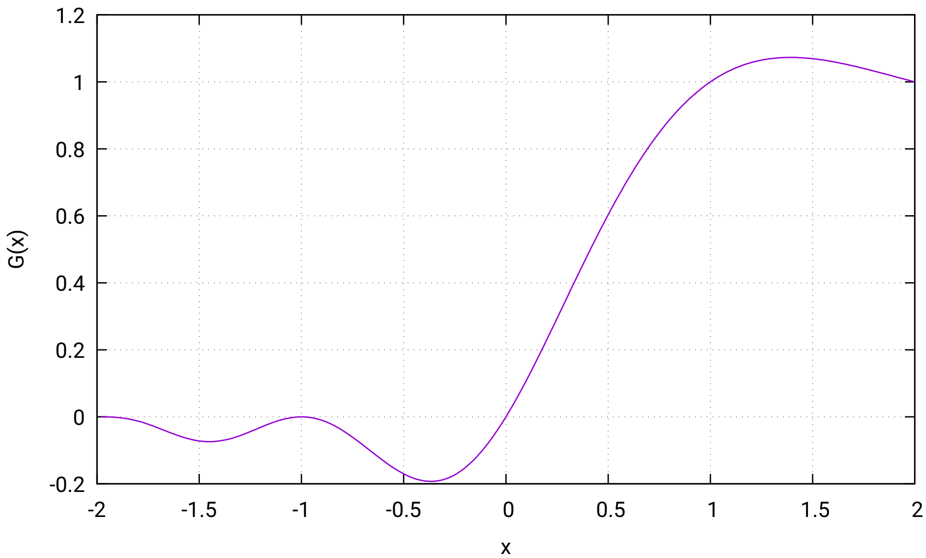

FIGURE 1. Barnes $G(x)$ on the real axis near the origin

It defines a logarithmic derivative $\varphi(x) = (d/dx)\log G(x)$

$$\varphi(x)G(x) = G'(x) \tag{34}$$

satisfying [3, (2.17)][2, §12][11, (29)]

$$\varphi(x) = (x-1)[\psi(x) - 1] + \varphi(1), \quad \varphi(1) = \frac{\ln(2\pi) - 1}{2}. \tag{35}$$

Remark 2. *Special values are* $G(1/2) \approx 0.603244$ [7, A087014]*, a local maximum* $\varphi(x) = 0$, $G(x) \approx 1.0730$ *at* $x \approx 1.39147$ [7, A245081,A245082]*, and a local minimum* $\varphi(x) = 0$ *at* $x \approx 2.5576$ [7, A245083].

A followup of (6) is the Taylor expansion

$$\varphi(x) = \varphi(1) - (\gamma+1)(x-1) + \zeta(2)(x-1)^2 - \zeta(3)(x-1)^3 + \zeta(4)(x-1)^4 - \cdots . \tag{36}$$

Taylor coefficients g_k of G are fixed as

$$G(1+x) = 1 + \sum_{k\geq 1} g_k x^k. \tag{37}$$

Insertion into (34) yields the analog of (10):

$$\begin{aligned}&[\varphi(1) - (\gamma+1)x + \zeta(2)x^2 - \zeta(3)x^3 + \zeta(4)x^4 - \cdots](1 + g_1 x + g_2 x^2 + g_3 x^3 + \cdots) \\ &\qquad = g_1 + 2g_2 x + 3g_3 x^2 + \cdots .\end{aligned} \tag{38}$$

Equating equal powers of x on both sides, the convolution of the right hand side has an obvious pattern:

$$\varphi(1) = g_1; \tag{39}$$

$$\varphi(1)g_1 - (\gamma+1) = 2g_2; \tag{40}$$

$$\varphi(1)g_2 - (\gamma+1)g_1 + \zeta(2) = 3g_3; \tag{41}$$

$$\varphi(1)g_3 - (\gamma+1)g_2 + \zeta(2)g_1 - \zeta(3) = 4g_4; \tag{42}$$

Algorithm 4.

$$g_k = \frac{1}{k}\left[\varphi(1)g_{k-1} - (\gamma+1)g_{k-2} + \sum_{l=2}^{k-1}(-)^l\zeta(l)g_{k-l-1}\right], \quad g_0 \equiv 1. \tag{43}$$

Example 3. [7, A122914]

$$g_1 = \varphi(1) \approx 0.4189385332046727417803297364; \tag{44}$$

$$g_2 = \frac{1}{8}[\ln(2\pi) - 1]^2 - \frac{1}{4}(\gamma+1) \approx -0.7008530851489250875957714314; \tag{45}$$

$$g_3 = \frac{1}{48}[\ln(2\pi)-1]^3 - \frac{1}{4}(1+\gamma)[\ln(2\pi)-1] + \frac{1}{18}\pi^2 \approx 0.2301877620542284435458148723 6. \tag{46}$$

The computation of the G-function via (37) for positive real values based on the first 40 of these g_k coefficients is illustrated in the C-function `BarnesG()` in the program in the appendix.

The Taylor series for $G(x)$ follows from the Taylor series for $G(1+x)$ by solving (33) for $G(x)$ with (21):

$$G(x) = \frac{G(1+x)}{\Gamma(x)} \equiv \sum_{j\ge 1} h_j x^j = [\sum_{l\ge 0} g_l x^l][\sum_{k\ge 1} c_k x^k]. \tag{47}$$

$$\therefore h_j = \sum_{k=1}^{j} g_{j-k}c_k; \quad h_0 = 0; g_0 = 1. \tag{48}$$

Example 4. *The list of* h_j *starts*

$$h_1 = 1; \tag{49}$$

$$h_2 = \varphi(1) + \gamma \approx 0.9961541981062056023868418264880; \tag{50}$$

$$h_3 = \frac{1}{2}[\varphi(1)+\gamma]^2 - \frac{1}{2}(1+\gamma) - \frac{\pi^2}{12} \approx -1.1149132726725708898650525079570; \tag{51}$$

$$h_4 = \frac{1}{6}[\varphi(1)+\gamma]^3 - \frac{1}{2}[\varphi(1)+\gamma](\gamma+1+\frac{\pi^2}{6}) + \frac{\pi^2}{18} + \frac{\zeta(3)}{3} \approx -0.4911308497661988159228588397 44. \tag{52}$$

Appendix A. C Implementation of $\Gamma(x)$ and $G(x)$

```
#include <math.h>
#include <stdio.h>
#include <stdlib.h>

#ifndef M_PIl
/* Pi https://oeis.org/A000796
*/
# define M_PIl          3.141592653589793238462643383279502884L
#endif

/* G(1/2) https://oeis.org/A087014
*/
#define M_G1_2 0.603244281209446206191429224534 7L

/** Evaluation of Euler's Gamma-function.
* The argument x is reduced to the interval [0,1/2] with
* the aid of the functional equation and reflection formula, and
* then evaluated with the Taylor series of Wrench, Math. Comp. 22 (1968) 617-626, eq (20).
* @param x real-valued argument
* @return Gamma(x)
* @author Richard J. Mathar
* @since 2025-07-19
*/
long double Gamma(long double x)
{
        /* list of -gamma =zeta[1] and zeta[k] = (-1)^k*RiemannZeta(k)/k for k>=2
        */
        static long double zeta[] =
        {
                0.,
                -5.7721566490153286060651209008240243 1e-01L, /* https://oeis.org/A001620 */
                8.2246703342411321823620758332301259 2e-01L, /* https://oeis.org/A072691 */
                -4.0068563438653142846657938717048333 0e-01L, /* https://oeis.org/A386403 */
                2.7058080842778454787900092413529197 4e-01L, /* https://oeis.org/A098198 */
                -2.0738555102867398526627309729140683 4e-01L, /* https://oeis.org/A386404 */
                1.6955717699740818995241965496515342 0e-01L, /* https://oeis.org/A259928 */
                -1.4404989676884611811997107854997096 6e-01L,
                1.2550966952474304242233565481358155 7e-01L,
                -1.1133426586956469049087252991471245 1e-01L,
                1.0009945751278180853371459589003190 0e-01L,
                -9.0954017145829042232609298411497267 3e-02L,
                8.3353840546109004024886499837311637 9e-02L,
                -7.6932516411352191472827064348181338 5e-02L,
                7.1432946295361336059232753221795379 6e-02L,
                -6.6668705882420468032903448567376337 4e-02L,
                6.2500955141213040741983285717977293 8e-02L,
                -5.8823978658684582338957270605503707 6e-02L,
                5.5555767627403611102214247869145662 0e-02L,
                -5.2631679379616660733627666155673426 4e-02L,
                5.0000047698101693639805657601934171 1e-02L,
                -4.7619070330142227990783957939028779 5e-02L,
                4.5454556293204669442408636529463032 9e-02L,
                -4.3478266053040259361351002947335603 5e-02L,
                4.1666669150341210469144983851675329 3e-02L,
                -4.0000001192140140586091207442548202 8e-02L,
                3.8461539034675185706347739794557946 0e-02L,
                -3.7037037312989325549460351554852006 3e-02L,
                3.5714285847333358028159180529257285 0e-02L,
                -3.4482758684919300810794793324272756 9e-02L,
                3.3333333364377581080655606095725489 8e-02L,
                -3.2258064531150416338818658299965268 7e-02L,
                3.1250000072759744802390796625045486e-02L,
                -3.0303030306558045506878945386645375 1e-02L,
                2.9411764707594344731736088496836376 9e-02L,
                -2.8571428572260110012713457053408407 1e-02L,
                2.7777777778181997830306721784331377 4e-02L,
                -2.7027027027223674590136688676068131 6e-02L,
                2.6315789473779946830194175031322031 2e-02L,
                -2.5641025641072281785905309383271405 9e-02L,
                2.5000000000022737369600659723206331 4e-02L
        } ;
```

```
        if ( x <= 0. && round(x) == x)
                /* negative integer or zero: pole there
                */
                return nanl("") ;
        else if ( x == 1. || x== 2.)
                return 1.L ;
        else if ( x == 3. )
                return 2.L ;
        else if ( x > 20.)
        {
                /* duplication formula
                */
                return M_1_SQRT2PIl*powl(2.L,x-0.5)*Gamma(x/2.L)*Gamma((x+1.)/2.) ;
        }
        else if ( x > 1.)
        {
                /* reduce argument to region x<1
                * (x-1)(x-2)(x-3)..(x-n)Gamma(x-n)=Gamma(x)
                */
                const int n = x ;
                if ( x == n)
                        return (x-1.0L)*Gamma(x-1.0L) ;
                long double f = x-1.0L ;
                for (int i=2 ; i<=n ; i++)
                        f *= x-i ;
                return f*Gamma(x-n) ;
        }
        else if ( x < -1.)
        {
                /* lift negative argument to region 0<x<1
                * x(x+1)(x+2)..(x+n)Gamma(x)=Gamma(x+n+1)
                */
                const int n = -x ;
                long double f = x ;
                for (int i=1 ; i<=n ; i++)
                        f *= x+i ;
                return Gamma(x+n+1)/f ;
        }
        else if ( x > 0.5)
        {
                /* reflection formula for 1/2 < x< 1 to keep x small
                */
                return M_PIl/sin(M_PIl*x)/Gamma(1.L-x) ;
        }
        else
        {
                /* Wrench Math Comp 1968 eq (20).
                * Accumulate the powers with Horner scheme backwards
                * through the list of zeta[]
                */
                int i = sizeof(zeta)/sizeof(long double)-1 ;
                long double val = zeta[i] ;
                for(; i > 0 ; i--)
                        val = val*x+zeta[i-1] ;
                return expl(val)/x ;
        }

} /* Gamma */

/** Evaluation of Barnes G (double-gamma) function.
* The argument x is reduced to the interval [0.5,1.4] with the
* aid of the doubling formula and then evaluated using the roughly 40
* first coefficients of the Taylor series around x=1.
* @param x real-valued positive argument
* @return G(x)
* @author Richard J. Mathar
* @since 2025-07-19
*/
long double BarnesG(long double x)
{
        /* list of expansion coefficients G(1+x)= sum_{k >=0} g[k]*x^k
```

```
*/
static long double g[] =
{
        1.L,
        4.1893853320467274178032973640561
76e-01L, /* https://oeis.org/A122914 */
        -7.00853085148925087595771431401216
3e-01L,
        2.30187762054228443545814872367019
3e-01L,
        1.72225087528078998916308640029319
5e-01L,
        -1.73005171274947443483880847215520
8e-01L,
        4.89155824817262038548913866287253
0e-02L,
        1.77633865129910160874773641282287
2e-02L,
        -2.09485748951365136346311586069672
5e-02L,
        7.55335681623434196813070629505721
7e-03L,
        3.15535561921693926890904112262925
0e-05L,
        -1.34264308694232620701089637922556
7e-03L,
        6.73087694604740210788948227863640
7e-04L,
        -1.29179036195976318066329617465073
2e-04L,
        -3.31368766254740596666635998018281
9e-05L,
        3.31688034900566307971192371981249
1e-05L,
        -1.13639809149594245673571740102150
1e-05L,
        1.25480892232823212800200032688669
5e-06L,
        7.30786719399949489866734383153401
7e-07L,
        -4.63920030881648104622260497678990
8e-07L,
        1.28194578803224021391215742600872
2e-07L,
        -9.30014507815918160203910142536101
1e-09L,
        -8.32209423950211250143828250553827
7e-09L,
        4.37644145928718076001121893909620
2e-09L,
        -1.08588801232159474463676532196396
7e-09L,
        7.19007573210825589682339484993648
8e-11L,
        6.07396063994367591234678479931050
4e-11L,
        -3.01904585912229995115659006162404
7e-11L,
        7.28512384692180003978520297619058
0e-12L,
        -5.90842282475264458525382566124236
2e-13L,
        -3.02548288321848140514326673632474
8e-13L,
        1.58314338020816307623995965696234
1e-13L,
        -3.93864761037243811253728532885672
4e-14L,
        4.20905415596858301897642493873189
5e-15L,
        1.00608073226976852583258324415147
1e-15L,
        -6.40189549922903382251350898670659
1e-16L,
        1.71787449387445206489812099170252
7e-16L,
        -2.34464329669705473601944070173245
1e-17L,
        -1.75771086869883958540359885305871
3e-18L,
        1.98499226326453542626104328151408
1e-18L,
        -6.01620592797951999372977100076804
4e-19
} ;

if ( x == 0.)
        return 0.L ;
else if ( x == 1. || x== 2. || x == 3.)
        return 1.L ;
else if ( x < 0.)
        return nanl("") ;
else if ( x >= 1.4L)
{
        /* reduce argument to region x<1.4 by doubling formula
        */
        const long double xhalf = x/2.L ;
        long double gg = BarnesG(xhalf)*BarnesG(xhalf+0.5L)/M_G1_2 ;
        gg *= gg ;
        gg *= powl(4.0L, (xhalf-1.0L)*(xhalf-0.5L))
                *powl(M_PIl,-xhalf)*Gamma(xhalf) ;
        return gg ;
}
else if ( x < 0.6L)
{
        /* reduce argument to region x>0.6 by (inverted) doubling formula,
        * i.e., calculating the G(2x)and G(x+1/2) where 2x and x+1/2
        * are closer to 1 (the origin of the Taylor expansion).
        */
        long double gg = BarnesG(2.0L*x)/Gamma(x)
                *powl(4.0L, (x-1.0L)*(0.5L-x))
                *powl(M_PIl,x) ;
```

```
                gg = sqrtl(gg) ;
                return gg *M_G1_2 / BarnesG(x+0.5L) ;
        }
        else
        {
                /* Accumulate the powers with Horner scheme backwards
                * through the list of g[], G(1+x)= sum_k g[k]*x^k.
                * This x in the formula is one less than the x in the argument.
                */
                x -= 1.0L ;
                int i = sizeof(g)/sizeof(long double)-1 ;
                long double val = g[i] ;
                for(; i > 0 ; i--)
                        val = val*x+g[i-1] ;
                return val ;
        }

} /* BarnesG */

#ifdef TEST
/** Test function for the computation of Gamma(x) and Barnes G(x).
* To be compiled with
*    gcc -O2 -o Gamma -DTEST Gamma.c -lm
* There is a single command line argument, the value of x.
* The program prints x, Gamma(x) and G(x) to standard output.
* Note that the number of digits printed exceeds the typical precision
* on IEEE computers, so even using an argument like 1.0 on the command
* line will echo a different value in that standard output.
*/
int main(int argc, char *argv[])
{
        if (argc < 2)
        {
                printf("usage: %s x\n",argv[0]) ;
                return 1;
        }
        long double x= strtold(argv[1],NULL) ;
        printf("%.25Le %.25Le %.25Le\n",x,Gamma(x),BarnesG(x)) ;
        return 0 ;
}
#endif
```

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