On Noether’s Theorem and Generalizations

Continuing the talk on:
Galois, Klein, Lie, Noether, Langlands & beyond
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Understanding "is" Networking Concepts & Theories

- Networking Concepts ("true learning") within a Math Area & between areas (e.g. Algebraic<->Geometry: from Descarte and Galois to Klein and beyond, e.g. Noether & Langlands), has a higher level of understanding, as pioneered (?; in Math) by Andre Weil's "*Power of Analogy*" (see also Simone Weil?), e.g. the "*Number Field / Function Field Analogy*" (Weil … Langlands);

- A *precise framework* for the above is provided by *CATT Language*: Category Theory (1940s Eilenberg & MacLane; universal language in Math & its applications, beyond 1870s Math Programming Language of Cantor: Set Theory).

See Recommended Readings (RR); Conf. LMI R&D Notes 2/11/25.

In a Nutshell

It's about G-equivariant Theories and their Invariants …

Einstein's General Relativity 1915

Geometry $\longleftarrow$ **Dynamics**

$\longrightarrow$

Emmy Noether 1918 (to address Einstein's GR)

N.B. Other 2-ways "bridges": Al Khwarizmi-Descartes, Galois-Klein etc.

Part I: Einstein, Hilbert, Klein and Noether

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Recall Galois Principle (GP) & Klein's Inverse GP

- Def. Galois Principle: To Study an Object / Problem, associate to it *its group of symmetries* ("ID"). Example (1828):

  Polynomial eq. → Group of Permutations of its Roots

- Felix Klein's use of GP: "*What is a Geometry*"?

  Geometry → Group of Symmetries (e.g. Euclidean); IGP:

  Group of Symmetries of a Space → is an Abs. Geometry!

- Let's apply this to Noether's Work / Theorem …

A little History: The Story of Noether's Theorem

- 1915 Einstein Gen. Rel.
- 1917 Goettingen Talk "to" David Hilbert & Felix Klein (Mathematicians you should know …); Emmy Noether's HW (Hilbert's "assistant"; they couldn't "solve" the Energy Problem!).
- 1918 Noether's Theorem (E.Noether: founder of Modern Abstract Algebra: Betti Groups, Noetherian Rings etc.: she was "The Expert" in AA & used it to *Model Building* in Physics!) … She became "The most influential Woman Math. in 20th century Physics" (Einstein said so!).

Intermezzo: So … What is “Abstract Time”!?

- Let’s *compare “Geometry” and Dynamics*:

a) Geometry: allows to compare figures in isolation; moving one at a time (no interaction);

b) Dynamics: a “geometric move” of a figure *has consequences on the other figures* present; they interact!

- Think of TIME as a *correlation parameter* of local movements; a synchronization device (conf. Einstein).

- But how can we separate Objects, as *Parts of an Irreducible System*, from their interactions!? Using Tangent Vectors & Vector Fields encoding the interaction (Particle-Field Duality) … Then the *Mathematical VF-Flow’s parameter “is” Time* (related) … now enter *Dynamical Groups* in the picture …

Hamiltonian / Symplectic Mechanics vs. Lagrangian Mechanics

- Manifolds M model Configuration Spaces (of “matter particles in ambient space”);
- (Co)Tangent Bundles TM/T*M (Calc III) model position and vectors (binding “Space” and yet uncorrelated “Time”: speed etc.) as State Spaces;
- The 2-Ways alternative:
  1) From DE (Vector Fields) & IVP: Hamiltonian Mechanics;
  2) To BVP & Path Integrals: Lagrangian Mechanics.
- *Philosophy Note*: Tesla said “All is Vibration”; Heraclit: “Panta Rei” (just Quantum Computing nowadays …) from which Emerges Local Quantum Time: the Quantum Phase as U(1)-Conformal Geometry! (see LMI).

Vectors, Forms & Integrals; Metric & Symplectic

- Recall Calc I: Line Integral (Path, VF, work Int F(r)dr) "labeling a Picture" …
- The Geometric-Dynamic way uses a metric (distance) and time (trajectory parameter), i.e. works in Tangent Space (TM).
- But is is better to pair position and momentum etc. if we don't have an intrinsic notion of "Space":
    a) A metric on TM defines sizes <u>AND</u> angles (Calc III);
    b) A symplectic structure on T*M defines the oriented Area.
- <u>Note</u>: Pythagorean Th. ⇔ Parallel Axiom; proof: $ds^2=dx^2+dy^2$ ⇔ $dA=dxdy$.

Dynamical Groups:
from *Geometry* to *Dynamics*

… and Local vs. Global

Separating Topology from Geometry

- Manifolds are built by gluing a Local Model (Geometry) using Topological Data; compare with a system of Highways (Network of Paths).

- Further applications of the idea: from Feynman Diagrams and Fat Graphs to Riemann Surfaces … (Pictures).

- The Topology is captured by the fundamental group (Recall History: Emmy Noether “upgraded” Betti numbers to Homology Groups 1925 - see Timeline of Algebraic Topology & “Categorification”) …

What is a “Lagrangian”?

- Recall the framework (pictures): configuration space M (*manifold*), state space TM (*tangent bundle*), metric (g:TMxTM->R), Lagrangian (function L:TM -> R).

- Lagrangian functions - Examples: 1) Free symmetric bilinear functions; 2) EM; 3) Mechanics

… and Groups of Symmetry

- Riemannian Geometry (subarea in Differential Geometry) is defined via a metric (bilinear, symmetric, positive definite) ⇒ Groups of symmetries that leave invariant the metric: Iso(M,g).
- Compare to Klein's Abstract Geometry: "A geometry is a group acting on a set" (Klein's viewpoint: "Inverse Galois Principle"); this makes the set a "Space".
- N.B. *Any metric on M defines a canonical Lagrangian*: "free particle Lagrangian" $L(v_P)=\text{Sum } g_{ij} v^i v^j$; L=K-U but U=0. Ex. Euclidean, Lorentz (relativistic / semi-Riemannian metric).
- Dynamics is defined via an analog to a metric, but including a potential function, encoding interactions (Path Integral of Force).

What is “Action”?

- Def. the action functional S(path(t)) is the Path Integral of the Lagrangian.
- N.B.: if the Lagrangian is (essentially) the metric (M,g), L(q,p,t) (scaling mass appropriately, e.g. 2m=1), then the action is the length of a curve.
- Hamilton’s Principle of Stationary Action:

    The trajectory path:A->B is a critical point of the Action

    Integral eq. $\delta S=0$ $\Leftrightarrow$ Euler-Lagrange Eq. (Diff. Eq.)

Variational Problem (Optimization of Path)

- The "objective function" is the Action (amount of "work"):

  S(Path)=Path Integral(Lagrangian 1-form, Path:A->B)

- Remark. a Path Integral is a "generic potential" ("antiderivative", conform F.Th. of Calculus; except here what varies is the path in state-space).

- The functional derivative of the action dS with respect to the variation VF (T(Paths) as sections in TM) is the "Euler derivative" (introduced by Lagrange?), as E.Noether calls it.

Euler-Lagrange Equations

- The extremum of the Action Functional (trajectory) satisfies Euler-Lagrange eq.:

    dS(path)=0 [“Lagrangian stationary”] ⇔ EL-equations

- Now consider G a group acting on M, e.g. Galileo or Lorentz transformations. It induces an action on paths of M, defining such “variations” of paths.
- G may transform the Lagrangian & Action functional;

1st Theorem of Noether

Given a group G acting on the configuration space.

There is a correspondence between:

Math) one-parameter subgroups of G leaving the Lagrangian invariant AND

Phys) Conserved quantities of of the Dynamics (motion).

- Examples: 1) Time translations & Energy; 2) Space translations and linear momentum; 3) Space rotations and angular momentum.
- Generators of the subgroup <-> Noether charges; conserved quantity: charge; flow / motion of charges: Noether current.

Example: free point particle

- Configuration Space manifold M=R (1D-real "line")
- Autonomous Lagrangian L(q,q') (suggests C→M curve & vel.)
- Momentum (L-conjugate to position) $p=\partial L/\partial q'$
- Force (no "field") $F=\partial L/\partial q$ N.B. as if *part of Lagrangian is* a potential for the conservative force! usually L=Kin.-Pot.!!
- Stationary Action Euler-Lagrange Diff. Eq.:

$$\partial L/\partial q = d/dt\ \partial L/\partial q' \quad \Rightarrow \quad F=d/dt\ p \quad \text{Newton's "Law"}$$

N.B. … and the kinetic part implies "part of L" is a potential for momentum (proportional to "velocity"); in symplectic case there is no a priori distinction between position & momentum!!

On Physics “Laws”

- The so called Newton’s Law F=ma is really a general framework: 2nd ODE, with initial conditions etc. “just Math”! The great discovery of Newton is that Gravity obeys the “inverse square law”, which can then prove Kepler’s Laws, that were postulated based on Ticho Brahe observational DATA … a *beautiful example of the Scientific Method*!
- … but there are no “Physics *Laws*”! it’s all mandatory Math: harmonic functions (Laplace eq.), Diff. Eq. framework etc. The rabbit hole goes much much deeper:

“Natural Physics Laws are Math Periods Laws”

… back to Noether: the conservation issue …

So, if $L(q,q')$ is *invariant to a 1-parameter "s" group of transformations*, which define a family of trajectories $q(t;s)$ and a *variation* $\delta q = dq/ds\, \delta s$ of the trajectory $q(t)=q(t;0)$, then:

Th. *If $d/ds\, L(q(t;s), q'(t;s))=0$ then $C=p(t;s)\, dq/ds$ is constant on Euler-Lagrange solutions: $d/dt\, C=0$.*

Proof (see John Baez: Noether Th. in a Nutshell)

$C'=p'\, dq/ds + p\, dq'/ds$ (chain rule) & def. momentum =>

$C'=L_q\, dq/ds + L_{q'}\, dq'/ds = d/ds\, L(q,q')$ [$L_q:=\partial L/\partial q$ etc.]

Euler-Lagrange Eq. <=> $C'=0$ (QED).

N.B. "p dq/ds" is "p δq" so EL-Eq. "say" the "angle" <p,δq>=const. G-orbits are at a constant angle to trajectories … (TB refined / reformulated; PIC?).

Example: charged particle in EM-field

(later - talk Part II?)

- The EM Lagrangian = Rel. Kinetic - EM-Potential

$$L \quad = \quad -mc^2 \quad - \quad q\,[\varphi_E + v/c\, A_{EM}]$$

- Then the Force is Lorentz Force and Euler-Lagrange eq. is Newton's Law with Lorentz Force as constitutive equation:

$$F = m\,a$$

$$F = q(E + v/c \times B)$$

[=> Noether conserved charge = q & Noether current etc. general terms]

Conclusions

Lagrange: Geometry $\longrightarrow$ Dynamics

Einstein: Dynamics $\longrightarrow$ Geometry

(GR: precursor of QFT via propagators!)

Summarizing …

- Geometry vs. Dynamics <-> Metric vs. Lagrangian
- Einstein unified Dynamics and Geometry; used it to model Gravity, by perturbing the metric (L <-> Hilbert metric), instead of adding a potential to the kinetic term L=K-U.
- Noether Theorem can be viewed as an extension of Klein's program, leading to Kaluza-Klein Program.
- Weyl created Gauge Theory, by "blowing-up" the space-time point, adding internal DOFs, and applying the Levi-Civita Geometry paradigm (connections in fiber bundles).
- Recently *Gravity was included in the Standard Model*, part of the Strong Force (quark spin polarization interaction; G is ***not*** *a fundamental interaction* and can be controlled).

in another Nutshell …

Lagrange “added” interactions to Geometry:

Lagrangian = Geometry Metric - Interaction Potential

Einstein thought: Why not “absorb” the potential into the metric, as a perturbation g(GR)=metric + perturbation!?

… so Dynamic → (back to) Geometry.

This is a precursor of QFT based on “propagators” (see LI).

Kaluza-Klein: do this for ANY Interaction! e.g. EM …

… so there is no need for “Quantizing Gravity”: QFT is QGR!

Bottom line: Keep the duality “Geometry-Dynamics” (don’t “take sides”).

Galois, Klein and Noether - Other examples

Other examples of Theories “hiding” Galois-Noether Principle:

1) Complex Analysis: C*-equivariance of Tf:TC->TC & CR-equations, Cauchy Integral, Polya VF, complex potential etc. (TB explained elsewhere);

2) Geometry (as sketched): metric as Lagrangian, length of curve as “action”, isometries as symmetries, and conserved quantities (length, angles etc.) - TB checked in detail …

3) Kahler geometry: a “quantum framework” (Quantum Phase as relativistic, local, Einstein-Feynman Time / “atomic clock”).

The Unification of Math & Physics

- All these contributions (Galois, Riemann/Lagrange, Klein, Noether, kaluza-Klein, Weyl) can be understood in the two-way bridge between:

Geometry ⇔ Dynamics

- This leads us to the *Philosophical Level*:

Prigogine: “From Being to Becoming” (CATT: Obj. & Mor)

Heraclit: “Panta Rei” (Quantum Computing)

Emergence of “Stable, yet ever-changing Reality”:

“from fundamental rep. → adjoint representation”!

Part II: Generalizations

… and ideas for some R&D Projects!

(April 24)

Steps …

- Geometry & Configuration Spaces;
- Dynamics & State Spaces: action, minimize action & EL-eq.
- Local Diff. Eqs. reflect *Group Invariance "a la Klein"* ("*Noether setup*"):
    1) Geometry: "*Distance Equivariant Geometry*"
    2) Complex Analysis: "*Angle Equivariant Geometry*";
    3) Hamilton Mechanics: "*Area Equivariant Geometry*";
- These define "*Preferred Paths*", via Diff. Eq. or Integral Eq. (Calc. of Variations):
    1) Geodesics & *Minimal Distance* Paths (Levi Civita, Christoffel);
    2) Trajectories & *Minimal Action* Paths: Euler-Lagrange Eq.
- Noether Theorem: what if G acts on State-Space? Equivariant conditions & observables …

The "Big Picture" …

We have an "*Extension of Theories*":

Geometry (M) → **Dynamics** (TM or T*M)

and we can apply Galois-Klein Principle (double lanes bridge):

Properties of "Object" ⇔ **"Subgroups of Symmetry"**

in other words, we have a Galois Correspondence "situation"!

Climbing trails over the forrest level and above cloud platform …

(See References for details: if you are a mountain climber by foot there are lots of "trails" to enjoy!)

The Noether Theorem context

- M (manifold), TM (Tangent bundle), L:TM->R (Lagrangian), H->G subgroup, $F(M)^H$ conserved observables as H-invariant functions on the state space.

- Q: Since Dynamics is an “extension” of Geometry, *what is a “reduction” of Noether’s Theorem* to Geometry?

Geometry	$\rightarrow$	Dynamics
Connection Theory?	$\rightarrow$	Noether’s Theorem

N.B. If we consider a particle in a field, time relative to other particles moving in sync, is irrelevant! NT is just “smooth homotopy” in TM (RS?).

1. Recall: Geometry

Configuration spaces

Minimize Distance on Paths

- Forget about parameterizations!
- Metric, element of arc ds & distance
- Minimize length L=Int ds, i.e. the “Path Integral” of the metric

=> geodesics (preferred paths)

Diff. Eq. & Christoffer coefs. ⇔ Int. Eq. (“Minimal Integral”)

The “Geodesics” framework …

- Metric Geometry is about “geodesics”; equation: Levi-Civita connection in TM defined by a metric.
- Generalize this to State-Space & Paths framework (Lagrangian as a metric etc.).
- This is Kaluza-Klein Program, generalizing Einstein’s approach in GR.

2. Lagrangian

Configuration spaces

Basic Concepts

- State Space TM tangent bundle; encodes “q & p”, i.e. position AND momentum (tendency for change of position).

- Lagrangian L:TM->R, is a measure of “magnitude”:

Lagrangian = Kinetic - Potential, $L=K-U$,

where $K=L_{can}$ is canonical associated to the metric (M,ds):

$K=m\,v^2/2$ $U(q_1,...q_n)$ [configuration dep.]

- Remarks: A) m is “coupling constant”; it is an “internal energy available for work”; B) Potential is a path integral of a “prorated distance”, called Work.

Lagrangian vs. Hamiltonian Mechanics

- Pairing *parameterized paths* (unique parameter, since we believe a universal, or at least a CS-dependent common order parameter exists: TIME!?) with the Lagrangian of the System as a "measure of cost of action", yields the Action Functional $S(P)=Int_P L(t)dt$.
- Hamilton's Principle of Minimal Action ("pay only what you need" sort of thing …):

$$S(\text{Trajectory from A to B})=\text{Min}_{P:A\text{->}B}\ S(P).$$

Then Calculus of Variations yield the Euler-Lagrange Eq. (whatever …).

What are the Euler-Lagrange Equations?

- The critical points of the Action Functional Int L(q(t),p(t))dt, with respect to the parameter “time t”, are given by:

d/dt dL/dv = dL/dq   Euler-Lagrange Eq.

- In the special case L=m N(v)-U(q) (separating “particles” and “field” for simplicity & emphasizing the coupling constant *m*, called mass), dL/dv=m dN/dv=m v=”p” is the linear momentum, with N(v)=<v,v>/2 given by a “metric” <,> a “density of kinetic energy” (think “mass” ~ number of particles x “unit of mass”).

- EL-Eq “say”: RHS force (position gradient of potential, which is the Path Integral, i.e. the Action Integral) equals LHS dN/dv, i.e. the rate of change of linear momentum … “just” Newton’s Law.

<u>Warning</u>: indices are suppressed for simplicity (think N=1 & Dim=1).

Example $L=mv^2/2 - k/r$ *[as before: n=1 & dim=1]*

… with two coupling constants appropriate units: Harmonic Oscillator! [one "elementary particle" of universal mass & charge for whatever particle-particle interaction at hand]

- Here $M=R$, $TM=RxR$ with coordinates $(q,q')=(r,v)$, linear momentum $p=mv$ and force $F=k\, 1/r^2$ (*Newton-Coulomb type*).
- EL Eq. $dp/dt=F$, i.e. $m\, a = F$ with a specific form for $F(r)$.

3. From PDEs up …

(Undergraduate level)

Noether’s Theorem (simplified [2])

If the *Lagrangian L:M->R is invariant* under a time independent symmetry, with smooth 1-parameter variation x(s) then

$C = (\partial_{\dot{x}^i} L)\partial_s x^i$ is conserved on EL-solutions.

Proof: Let’s explain first: 1) Lie group G acting on M, Lie algebra exp:gl(G)->G and one parameter group of transformations $x_A(s)$, with “pictures”; 2) In local coordinates: gradient of Lagrangian relative “x and v” (v= $\dot{x}$); 3) Vector field dx/ds on the 1-parameter family of x(t;s);

The Calculus computations …

- Compute the change of C over time: $\partial_t C = \partial_t((\partial_{\dot{x}^i} L)\partial_s x^i)$
- By the product rule this is $\partial_s x^i \; \partial_t(\partial_{\dot{x}^i} L) + (\partial_{\dot{x}^i} L) \; \partial_t \partial_s x^i$
- By EL-Equations $\partial_t(\partial_{\dot{x}} L) = \partial_x L$

this becomes $\partial_s x^i \; \partial_{x^i} L + (\partial_{\dot{x}^i} L) \; \partial_s(\partial_t x^i)$

- By the chain rule and since the symmetry is time independent this is $\partial_s L(x, \dot{x}, t)$
- But L is s-invariant: $\partial_s L(x(s), \dot{x}(s), t) = 0.$

so $\partial_t C = 0$ and thus C is conserved.

NB: This is good practice for UG-students but "What it means?"

Adding some pictures and words …

- EL-Eq.: Motion gradient (of L) = Interaction gradient (Pic)

  [NB: this just says "Kinetic Energy change = Potential E. change"]

- L is s-invariant => L(q) as a scalar section for the TB-fibration is constant on the G-orbits with Lie algebra generator s and M-vector field $X_s = d/ds\,[\exp(s)x(t)]_{|s=0}$ generated by s in gl(G) (picture it!) … etc.

- What is $C = (\partial_{\dot{x}^i} L)\partial_s x^i$? It is the inner product of the VF X_s and velocity-gradient of the Lagrangian for the "particle" following the EL-flow, i.e. $X(x(t))\cdot p(x(t))$.

… and some comments

- But why is this “potential energy independent”? is it? Let’s look at the “standard example” $L(x,v,t)=m\ v^2/2 - e\ U(x)$.

Then $grad_v(L)=mv$ [linear momentum] and since d/ds L=0 it means s-transformations don’t change U(x), i.e. preserve potential energy [External force = $grad_x(U(x)$ is orthogonal to equipotential surfaces].

- But EL-Eq are d/dt p=F (Newton’s form of EL Eq) so momentum will change orthogonal to equipotential surfaces, hence not under s-transformations, e.g. these particular “space translations”.

Analogy with Conformal Geometry

Here we reach a much deeper connection (When we realize "Time" emerges from quantum phase, hence the quantum "Reality" is Conformal - e.g. EM, CFT etc.).

- Think of (M,g,F) defining a foliation in TM (F perp. integrable when F is conservative, hence there is a potential energy U(q)). Then the F-flow in TM is the solution of EL-Eq. (p'=F).

If L(x,v) is G-invariant for G=<a(s)> 1-parameter group of transformations then the "angle" X . p" (dot product) is constant. If we normalize X (w.r.t. metric) so G acts via isometries => L-Kinetic is invariant & L-pot invariant => p is invariant (Conservation of momentum) etc.

How to Teach it? and Differential Geometry

- Diff. Eq. level is teaching the topic “in coordinates”. Using Vector Fields is more conceptual (intrinsic).
- Now “add” a Graphical Interface (see e.g. Tristan Needham: VCA & VDG): a) Using an “ambient space” (e.g. Theory of Manifolds); b) Intrinsic: picture a fibration’s both spaces as objects & morphisms (down: epi & up: mono / section; e.g. VF).
- Then introduce: a) Lagrangian “gradients”: horizontal a.k.a. Force & vertical a.k.a. momentum etc. (Pictures & words); b) compare with metric & connection theory (later on); c) “mass”!

4. Now some generalizations

("Noether correspondence")

Towards a Galois Correspondence …

Note we already have a *Symmetry Group built in from the start*:

A) In Hamiltonian Mechanics we have canonical transformations that preserve the symplectic structure on T*M, the local intrinsic area dqxdp, a measure of the "true resource" involved in the "action";

B) In Lagrangian Mechanics any smooth coordinate transformation is admissible (preserves the EL-Eq. ⇔ Critical Point of Action Integral).

- Let this general group be denoted with G … and H a subgroup of G.

What about “Observables”?

- Observables are functions on TM and T*M, e.g. kinetic energy $mv^2/2$, potential energy U(q), angular momentum v x r etc. defined in terms of coordinates, i.e. subject to the action by G …

- What is such a function is invariant to H … let’s see some examples!

Conclusions

1) Evolution of “Galois Principle”: Galois, Klein, Noether

2) The “Time / Path Extension”: Geometry -> Dynamics

3) Invariant Theory establishes a Galois correspondence; it yields the Noether correspondence.

4) Energy is “never conserved”: A) some “missing” energy is observed, e.g. “dark energy” in GR; B) A “new” Lagrangian is “coined” … C) “all’s back to normal” … for a while; then D) here we go again … (Theories change like species: TK).

Kaluza-Klein Program (Newton->Einstein->...)

"From Curved motion in Flat Space" $\rightarrow$
Flat motion (geod.) in Curved Space".

- Here "Space" stands for configuration or state space etc. Additional "dimensions" correspond to Degrees of Freedom (may be interpreted as "external" ("real"?), e.g. String Theory, or "internal", e.g. in Gauge Theory.

[There is a correspondence: use sections / "graphs"]

A far reaching generalization: Gauge Theory

Weyl, who praised E. Noether for her breakthrough, generalized Noether's idea of Galois correspondence from tangent bundles TM with a group action, e.g. isometries corresponding to a metric, beyond Levi Civita connection theory for parallel transport, to general principal bundles and associated vector bundles.

- GT has become The Quantum Field Theory paradigm during 20th century: from QED to the SU(2)-Electroweak Theory (Weinberg-Glashow-Salam) and SU(3) Quantum Chromodynamics.

… and beyond

- Recently all these “fundamental interactions were unified in a QC “upgrade” of the Standard Model, fermion generations explained, Gravity was derived as a nuclear spin polarization effect, Space and Time were derived as emergent etc.
- Thus a New Paradigm in Science is developed, based on input from Computer Science, TQFT model using CATT Language and AG-Tools to “quantize everything”, while incorporating sources, not as singularities, but rather branching points of the field via Ramification Theory (& Galois-Grothendieck Theory, of course).

Natural Physics Laws are Math Period Th.

- Moreover, Theory of AG-Periods and Pi-Group Theory is being developed to establish the reasons for the "unreasonable effectiveness of Mathematics" (Eugene Wigner): A) There are NO "fine tuned" fundamental constants (see the New SI-unit system and ROCAM talk of Klaus von Klitzing (Nobel Prize for Q-Hall Effect 1985) pg. 11 (pdf p.13) (Also available on YT is the "Quantum Revolution in Metrology", talk at GYSS 2024).
- The progress in this direction is simultaneous with the understanding of what is the *fine structure constant* and how to characterize it mathematically (see LI series of articles).

Supplements

… ideas for some R&D Projects!

Connections with other MP-Structures

- Metric (Geometry), Lagrangian (Dynamics) & Kahler “package”: complex, symplectic & metric … “egg or chicken problem” …

- Complex Structure is “angle” (Geometry) & Einstein-Feynman Local Quantum Time (Quantum Phase exp(iwt) …)

- So: C-structure (U(1)-action), S-structure (pairing “is” & “becomes”) => Contact Structure? Hermitean Metric:

<X,Y>=w(X, JY)

- Think: electric current I,U; sin & cos … Y & JY & vibration!

Lagrangian Mechanics

… is an *Optimization Problem with Constraints* (Stationary Action Principle - action as an "objective function(al)" etc).

See the role of Lagrange multipliers etc.

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