The Apex Proof to Fermat’s Last Theorem

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Abstract-Hypothesis:

This paper shall show with Diophantine equations, which will be shown to be derived from $A^P + B^P = C^P$, which is analyzed more quickly as $A^P + B^P + C^P = 0$, C being negative. That the 3 base variables A, B and C are congruent, expressed formulaically:

$A = B = C \text{ Mod } P$ *for Sophie Germane Case 1, when P >= 5*
Thus establishing that $A + B + C \neq 0 \text{ Mod } P$

as well as:

$A = B \text{ Mod } P$ *for Sophie Germane Case 2, when P >= 3*
Thus establishing that $A + B + C \neq 0 \text{ Mod } P$

$A + B + C = 0 \text{ Mod } P$ of course being one of the prerequisite equations for FLT, based on Sophie Germain’s first Axiom. And this prototypical formula is quite easily established by Fermat’s Little Theorem.

The solution will involve establishing two factors for each of the key variables A, B and C, which will be denoted by subscripts.

While FLT was proved quite some time ago by Wiles/Taylor, it remains out of reach for the vast majority of mathematicians, due to the need of a strong background in modularity theory for *elliptic* curves, and other arcane branches of Number Theory. Thus most mathematicians are hoping for a proof that is a little easier to comprehend using Diophantine equations. This paper is intended to satisfy that need.

I have tried hard to making the writing light and entertaining. Writing this paper was like writing a book, a tremendous amount of blood, sweat and tears went into it’s construction. Thousands of hours of math work. Do not feel the need to try to rush thru it, three subsequent readings of perhaps an hour each should allow complete absorption of this creative work of mathematics art.

The basic formula $A^P + B^P = C^P$, is non-symmetrical in presentation. This exposition on FLT, for the most part makes use of the symmetrical presentation in the form $A^P + B^P + C^P = 0$, with C being considered to be a negative integer value. This approach method was also used by Euler, who was the first recorded mathematician to prove the case P = 3 for Fermat’s Last theorem.

In my earlier 9th proof attempt, which I wrote up several months ago, I used a metaphor of climbing Mount Everest liberally throughout the proof in various places, and I will reuse much of that proof in this new document. I hope you find the reading of this proof entertaining and sparkling. Or at least you may find it more entertaining and sparkling than your average Diophantine proof you may find on arXiv. For quite certainly, it is highly conceivable that others could have discovered a similar proof years before, but due to an inability to promote their ideas to the world at large, a proof would have gone unnoticed. Note, mathematics manipulation is only a way to pass the time for me, my true skills lie in music creation and engineering, thus you may find my notations somewhat arcane, for which I apologize in advance.

Basic knowledge regarding the exponent value. For any case of $A^N + B^N = C^N$, where N is >=3, it is relatively easy to show that it is only necessary to prove FLT for prime number exponents. Additionally, it is only necessary to prove FLT for A, B and C being coprime for obvious reasons. For even number value exponents, any that are composite and have an odd number factor will be provable by the odd number having a prime number factor, and if N = 4, 8, 16, 32 etcetera, Fermat's proof for N =4 by Infinite Descent serves as the simple basis of a proof. I will not elaborate on the above statements in this paragraph, as the proofs are very simple and can be viewed on a 1000 different web portals.

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Conventions used in this Paper:

Please note that instead of using the congruence operator of 3 parallel lines, I will instead be using a standard equality operator, for all modulus equations, as was the practice used regularly in the somewhat distant past. This will save me considerable mouse clicks during the creation of this document.

The abbreviation FLT will be used to indicate Fermat's Last Theory.

In the last 20 years of working on this theory, I have become accustomed to using a Symmetrical Form of the presentation of FLT, as follows: $A^P + B^P + C^P = 0$, this form has the benefit of reducing the amount of analysis when dealing with a symmetrical problem such as FLT. It should be mentioned the first Mathematician to seriously do some work on this problem other than Pierre Fermat himself was Leonard Euler, and he wrote his proof for the case N = 3 in the Symmetrical form as well. At times I may switch over to the non-symmetrical standard form of $A^P + B^P = C^P$, when the NSF (*non-Symmetrical Form*) may yield better clarity in an explanation.

Finally, the variables A, B and C are broken down into factors A_1, A_2, B_1, B_2, C_1 and C_2. The subscripts help to organize the factoring and memorizing of these 6 variables.

FOUNDATION THEORY, Necessary to Gain Basic Skills to understanding Fermat's Last Theorem

Note, there is a certain amount of repetition in this section, and some of the final forms referred to as "Presentation of D", may be not actually be required to be absorbed for a clear understanding of the two final SGC (*Sophie Germain Case*) proofs, but are of interest in gaining a solid foothold into the fundamentals, none-the-less. The first 12 pages are presented in a Classroom Lesson type presentation style, with use of metaphor to enhance the reading experience.

These next few pages will give the basic equational tools and gear necessary for climbing to the peak of the Mount Everest of math problems. Note the Himalaya's peaks are many and this Sherpa can only explore a limited number of them. I have found two routes to the summit, from which an inspiring view and feeling well being may spring. The climb is not without ardor, and to try to push to quickly to the summit may find one out of breath, and a fuzzy mind. Thus it is essential to accumulate these basic equational tools and commit them to memory. In further documents in this proof, the level of detail that will be expressed DEPENDS on a deep internal mathematics absorption of this foundational base.

At the completion of this portion of the proof we will be at Base Camp, and prepared to ascend to the heights of Everest.

The starting point will be defining the problem. It is normally defined as follows:

$$X^N + Y^N = Z^N \qquad \textbf{E4a}$$

With X, Y and Z being positive integer values, and N being an integer value >= 3. That there exist no possible solutions.

A proof for the case for N = 4 was shown by Fermat in a margin of his copy of Arithmetica, and later published by his son, after his death. Adjacent to the short detailed proof which makes use of the technique of Infinite Descent, is a comment that there are no solutions for any other higher exponent than 2, and that the margin of the paper is to small to hold this proof. Hard to say one way or another if he had a rock solid proof.

Anyway moving on, if N is any power of 2 >= 4 the proof would also hold, based upon simple algebraic use of exponent rules. Using similar reasoning, we can prove that any odd number exponent which is a composite number, will also hold true, if we can establish a proof for either of the factors for that composite number. And of course any even number which is a product of an odd prime number or odd composite number will also be “covered” by a proof for prime numbers which are >=3.

Based upon the above, and my personal preferences, we may rewrite the starting point equation as:

$$A^P + B^P = C^P \qquad \textbf{E4b}$$

In this presentation, the exponent P represents a prime number >=3, and A, B and C as coprime integers.
The fundamental reasoning that A, B and C are considered as coprime, is that if A and B had a common factor, then C would also, and then we could remove this factor from all 3 variables, and rewrite.

Again based upon personal preference we may rewrite the equation in the symmetrical form as:

$$A^P + B^P + C^P = 0 \qquad \textbf{E4c}$$

In this presentation, we presume one of the 3 variables A, B and C must be negative. For convenience sake we will assume that C has a negative value. It should be noted that Euler was the first mathematician to find a proof for the case P = 3, and his proof used the symmetrical form. In other words, good historical precedent to proceed along this approach vector to the solution.

At this point maybe good to throw in some philosophy (*OH NOOOOOOO!*) Oh yes, consider the following.

This proof could also be for two negative numbers and one positive number, and be equally valid. And if we conveniently ignore the trivial solution aspect, the potential values and polarities of ***negative, zero and positive*** sort of make up a spectrum analogy of the human race coloration and sexual orientation. (*Note, this paper may be burned in “Fahrenheit 451ish fashion” in some* *fundamentalist republic provinces, and produce lots of heat, and additional CO_2 for our sky*.) So much for my comedic relief, back to reality.

Sophie Germain around the year 1800 was working on a number of mathematical and physics problems, her work on Fermat's Last Theorem has had a profound effect on the understanding of the underlying aspects of the problem. And her definition of Case 1 and Case 2 analysis of the famous equation is a starting point in understanding the two fundamental analysis approaches which must be employed.

Case 1, is when **none** of the integer variables A, B or C contains a factor of P.

Case 2, is when **one** of the integer variables A, B or C contains a factor of P.

Other than this simple branching aspect of the proof definition, no other aspects of Sophie Germain's extensive work on Fermat's Last Theory are utilized, in this exposition.

FACTORING $A^P + B^P + C^P = 0$

Consider $G^P + H^P$ and $G^P – H^P$ each consists of two factors as follows:

$G^P + H^P = (G + H)(G^{P-1} – G^{P-2}H + G^{P-3}H^2 – \ldots\ldots\ldots + G^2H^{P-3} – GH^{P-2} + H^{P-1})$ **E5a**

Note, alternating sign polarities in factor 2

$G^P - H^P = (G – H)(G^{P-1} + G^{P-2}H + G^{P-3}H^2 + \ldots\ldots\ldots + G^2H^{P-3} + GH^{P-2} + H^{P-1})$ **E5b**

Note, same polarities in factor 2

Note, writing out the above right side factor 2 is time consuming to write, so as a shortcut, we may consider using the following functions instead:

$f_a(G, H, P) = (G^{P-1} – G^{P-2}H + G^{P-3}H^2 - \ldots\ldots\ldots + G^2H^{P-3} – GH^{P-2} + H^{P-1})$ **E5c**

(f_a being the additive function factor of $G^P + H^P$)

$f_s(G, H, P) = (G^{P-1} + G^{P-2}H + G^{P-3}H^2 + \ldots\ldots\ldots + G^2H^{P-3} + GH^{P-2} + H^{P-1})$ **E5d**

(f_s being the subtractive function factor of $G^P – H^P$)

While working in the symmetrical presentation of Fermat's Last Theory I do not show the subscript "a" or "s", since all factoring work is from an additive point of view.

We may now expand the presentation form for Sophie Germain Case 1, using the above factoring Concepts.

> **Please bear in mind that G + H, may only divide once into $G^N + H^N$, and that for SGC1 there can be no common factors that exist between G + H and f_a(G, H, P). This is shown in Lemma T3 on page 12. Regarding SGC2, this T3 Lemma also shows that if G + H contains one or more P factors then f_a(G, H, P) must contain exactly one factor of P.**

$A_1^P A_2^P + B_1^P B_2^P + C_1^P C_2^P = 0$ (*Specific to SGC1*) **E6a**

where $A_1^P = -(B + C)$ and $A_2^P = f(B, C, P)$

and $B_1^P = -(A + C)$ and $B_2^P = f(A, C, P)$

and $C_1^P = -(A + B)$ and $C_2^P = f(A, B, P)$

Similarly, we may expand the presentation for Sophie Germain Case 2:

$A_1^P A_2^P + B_1^P B_2^P + \mathbf{P_1^P} C_1^P C_2^P = 0$ (*Specific to SGC2*) **E6b**

where $A_1^P = -(B + C)$ and $A_2^P = f(B, C, P)$

and $B_1^P = -(A + C)$ and $B_2^P = f(A, C, P)$

and $\mathbf{P^{P-1}} C_1^P = -(A + B)$ and $\mathbf{P} C_2^P = f(A, B, P)$

At this point, I suppose a simple presentation that can be written out on a blackboard for the class is needed. Let's look at the simpler case of SGC1 first, for P=5.

$A^5 + B^5 + C^5 = 0 = (A+B)(A^4 - A^3B + A^2B^2 - A^3B + B^4) + C^5$ **E6c**

and we could rewrite this as $(A+B)(A^4 - A^3B + A^2B^2 - A^3B + B^4) = -C^5$ **E6d**

The above form looks pretty basic, of course if we used the typical non-symmetrical presentation form instead of $-C^5$ we would simply have C^5. At this point you may wonder, why deal with a symmetrical form at all, which has positive and negative integer variables. Well, when the algebraic juggling gets super complex, using a somewhat simpler form helps to keep the polarity errors from creeping in to the analysis. Of course at this point in the exposition, everything is pretty simple. When we get to the trinomial expansion of $(A + B + C)^P$, the symmetrical form starts to look more appealing.

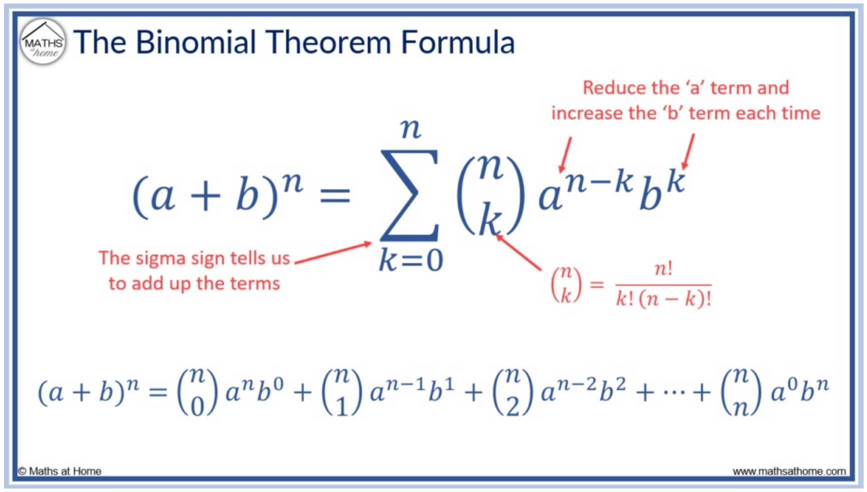
Binomial Expansion of $(a+b)^P$

When $(a+b)^P$ goes thru binomial expansion, the expanded form may be presented/condensed as:

$a^P + P\,(f(a,b)) + b^P$ *(with P (f(a,b)) representing the sum of all center terms)* **E7a**

Basically, all of the center term coefficients will have a prime factor of P.

This may be understood by absorbing the basic standard formula for Binomial Expansion which is noted to the right:



Maybe a little too abstract? Let's try a few prime exponent examples to add light to the concept.

$(a+b)^3 = a^3 + 3a^2b + 3\,ab^2 + b^3$ **E7b**
$(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$ **E7c**

If you study the coefficient formula for a bit (*shown in Red Text above*), it will make sense, that all of the center term coefficients must have a prime factor of P, since a prime factor of n occurs in the numerator and can not occur in the denominator for all center term coefficients.

Below is Pascal's triangle from Wiki which shows all of the term coefficients up to exponent 7:
(*It's a classic math diagram!*) The center term coefficient prime factors are obvious for 3, 5 and 7.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1

Trinomial Expansion of $(A+B+C)^P$

Now for Trinomial Expansion, pretty much the same applies, but we will now have to start thinking somewhat geometrically, but with supportive algebraic logic.

$(A + B + C)^3 =$ *(first diagrams, exponent = 3)* **E8a**

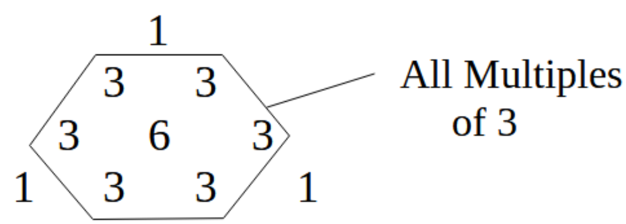
$(A + B + C)^5 =$ *(following diagram, exponent = 5)* **E8b**

$$C^3$$

$$3AC^2 + 3BC^2$$

$$3A^2C + 6ABC + 3B^2C$$

$$A^3 + 3A^2B + 3AB^2 + B^3$$



$$C^5$$

$$5AC^4 + 5BC^4$$

$$10A^2C^3 + 20ABC^3 + 10B^2C^3$$

$$10A^3C^2 + 30A^2BC^2 + 30AB^2C^2 + 10B^3C^2$$

$$5A^4C + 20A^3BC + 30A^2B^2C + 20AB^3C + 5B^4C$$

$$A^5 + 5A^4B + 10A^3B^2 + 10A^2B^3 + 5AB^4 + B^5$$

NOTE, all of the coefficients (*shown in brown text*) for the P=5 trinomial expansion are divisible by 5.

For the general case of any prime number equal to 3 or greater this must also be true, since the center terms of the Binomial expansion are all multiples of the prime exponent factor, when expanded.

From the above rather un-artistic graphics we can gain a foothold into Trinomial expansion coefficients, that they all appear to be multiples of the prime exponent.

Formulaically expressed as:

$(A + B + C)^P = A^P + B^P + C^P + P\,(f(A,B,C,P))$ **E9a**

Where $P\,(f(A,B,C,P))$ is a unique positive integer value function representing the sum of all center terms.

Thus we observe the 3 corner terms have coefficients of 1, and all of the center coefficients are multiples of prime exponent value P.

The graphical view is nice, maybe algebraically you may understand that since all non-corner ***perimeter*** binomial expansions have factors of prime P, when we can multiply any horizontal binomial center row coefficients by the outer perimeter angled vertical row coefficients then all interior term coefficients must also contain a factor of prime P.

Perhaps at this point a more tangible proof of the center none-perimeter coefficients is needed. Supposing we rewrite the starting point equation in this analysis as follows:

$(A + B + C)^P = ((A+B) + C)^P$ and next simply apply Binomial Expansion to (A+B) and C. **E9b**

In this case, if we consider P = 5, and the second row from the bottom, we will see that the coefficient elements will all be multiples of 5. Then once we expand (A+B), all of these coefficients will be multiplied by the factor 5. QED.

Since the summation of A^P, B^P and C^P is supposedly zero, we may now remove the 3 corner elements from the isosceles matrix.

With the 3 Corner Values of A^P, B^P and C^P removed, we find that all remaining elements are divisible by P, additional a careful analysis of a typical binomial expansion shows that the sum of the center terms are also divisible by a + b, therefore we can now show that the expansion of $(A + B + C)^P$ has the following 4 factors:

P (A+B) (B+C) and (C+A)

And bearing in mind the previous work from page 6: $A+B = -C_1^P$, $B+C = -A_1^P$, $C+A = -B_1^P$ **E9c**

Then based upon the knowledge that (A + B + C) must have an initial value which can be raised to the P exponent to $(A + B + C)^P$, we may determine that (A + B + C) must have an alternate form of:

$$A + B + C = P\,A_1\,B_1\,C_1\,K_0$$ **E9d**

with K_0 being an arbitrary integer value which is related to the remaining factor of the division of $(A+B+C)^P$ by $P(A+B)(B+C)(C+A)$

For the case P = 3, K_0 is easily determined for SGC2 and SGC1. However for higher order prime exponents the computation of K_0 as a formula derived from A, B and C becomes more and more difficult as the exponent P increases. Yet we do not need to know the exact value of K_0, only that it is an integer if there would exist a counter-example solution to FLT.

Additionally, the various presentations of A + B + C may be given a single variable designation of **D** to simplify reference to this important variable in the FLT analysis.

Restating:

$$D = A + B + C = P\, A_1\, B_1\, C_1\, K_0 \qquad \textbf{E10a}$$

Still there are many more Presentations of D, which we will be required to be fluent in, as we forge our way to Base Camp.

<u>Presentations of D:</u>

Perhaps the **most important presentation of D** is as follows, thru substitution:

$$A + B + C = \frac{(A + B) + (B + C) + (A + C)}{2} = \frac{C_1^P + A_1^P + B_1^P}{-2} \qquad \textbf{E10b}$$

(*Note, above form specific to SGC1*)

Although the -2 in the denominator of the far right presentation, appears out of place, it's required to be a negative. Not too hard to show that, if you go back to the beginning of the proof.

This particular form is instrumental to the final proof for SGC2 since it is factorable, and after factoring new transforms are possible which lead directly to the actual proofs, which will be explored in later sections of this document.

These forms can also be expressed in relation to SGC2 as:

$$A + B + C = \frac{(A + B) + (B + C) + (A + C)}{2} = \frac{\mathbf{P^{P-1}}C_1^P + A_1^P + B_1^P}{-2} \qquad \textbf{E11a}$$

It may be noted that this form is less factorable, than the form for SGC1, however $A_1^P + B_1^P$ can be factored!

And there yet remain a few more forms of D, which will be useful gear as we approach Base Camp:

A_1^P = – (B + C) **A + (B + C) = A – A_1^P** **Similar substitutions for B and C arrive at:** **E11b**

$A + B + C = A - A_1^P = B - B_1^P = C - C_1^P$ *This form for* ***SGC1*** **E11c**

and

$A + B + C = A - A_1^P = B - B_1^P = C - \mathbf{P^{P-1}}C_1^P$ *This form for* ***SGC2*** **E11d**

Now these last forms have a use of proving some detail about A_2, B_2 and C_2 for SGC1 as follows:

$A - A_1^P = A_1 (A_2 - A_1^{P-1})$ *Of course same considerations for B and C* **E11e**

Based upon a complete understanding of Fermat's Little Theorem, we can show that:

$A^P = A$ Mod P and less well expounded: $A^{P-1} = 1$ Mod P **E11f**

From the above we can prove for SGC1 that A_2, B_2 and C_2 = 1 Mod P, and for SGC2 if we assume C has the factor P then A_2 and B_2 = 1 Mod P and C_2 is an undefined Modulus of P, which is not 0 Mod P.

Below supporting lemma was written abut 18 months ago, and demonstrates that no common factors can exist between A_1 and A_2 other than P, and similarly for variables B and C. It also shows that if P is a factor of A_1, then it must also be a factor of A_2.
Below simple Axioms are demonstrated by the T3 Lemma.

Axiom 1: *with the precondition that J+K ≠ 0 Mod P, with P being an odd prime number,* *SGC1*
$J^P + K^P$ is divisible by J+K, and can not be divisible by any additional factors within J, K, J+K or P.

Axiom 2: *with the precondition that J+K ≠ 0 Mod P, with P being an odd prime number,* *SGC1*
J+K is coprime to $(J^{P-1} - J^{P-2}K + J^{P-3}K^2 - \dots\dots\dots + J^2K^{P-3} - JK^{P-2} + K^{P-1})$

Axiom 3: *with the precondition that J+K = 0 Mod P, with P being an odd prime number,* *SGC2*
$J^P + K^P$ is divisible by J+K, but can not be divisible by any additional factors within J, K or J+K, besides P.

Axiom 4: *with the precondition that J+K = 0 Mod P, with P being an odd prime number,* *SGC2*
When $J^P + K^P$ is divided by J+K the result f_α(J, K, P), can only contain a single factor of P, any other possible factors of P, must be contained within J+K.

Axiom 5: *with the precondition that J+K = 0 Mod P, with P being an odd prime number,* *SGC2*
With the exception of P, J+K is coprime to $(J^{P-1} - J^{P-2}K + J^{P-3}K^2 - \dots\dots\dots + J^2K^{P-3} - JK^{P-2} + K^{P-1})$

Further information regarding this Lemma is explained in the "Elucidation on the T3 Lemma" paper.

T3 lemma

Binomial Expansion & Subduction of $J^P + K^P$

It is generally well known in number theory, proper factoring of $J^P + K^P$, and limits of prime cofactors when J and K are coprime. However this common knowledge is repeated below in a somewhat abbreviated form. I use the term Subduction here, as an indication of the application of subtractive and deductive reasoning processes.

And obviously, the same method of proof would apply to $J^P - K^P$

Similar to the form on pages 1 to 4, $J^{P-1} - J^{P-2}K + J^{P-3}K^2 \ldots K^{P-1}$ is simply represented by $f(J,K)$.

For the case P=5 as an example, it is given

$J^P + K^P$ Factors Into:

$$(J+K)(J^4 - J^3K + J^2K^2 - JK^3 + K^4)$$

However $(J+K)$ can not have any prime co-factor within $(J^4 - J^3K + J^2K^2 - JK^3 + K^4)$ except P as follows,

If attempting to divide J+K into $(J^4 - J^3K + J^2K^2 - JK^3 + K^4)$, *(this detailed on pg 6 to right)*

J+K Long Division	Coefficients only shown					
	1	-1	1	-1	1	
Subtr $J^3(J+K)$* 1	1	1				

	0	-2				
Subtr $J^2K(J+K)$* -2		-2	-2			

		0	3			
Subt $JK^2(J+K)$* 3			3	3		

			0	-4		
Subt K3(J+K)* -4				-4	-4	

				0	5	

Here the remainder (*AKA residue*) is $5K^4$. Similarly, by successive J+K factor subtraction (*long division*), the remaining may be shown alternately as $5J^4$ or $5J^2K^2$.

The remainder is not fully divisible into J+K.

However it is easy to show any prime cofactors would need to exist between J+K and (*with symmetrical form*) $5J^2K^2$,

Thus $\frac{5J^2K^2}{J+K}$ would have to have these cofactors.

The only cofactor can be P (*or 5 in this case*).
J^2 and K^2 can not contain any cofactors to J+K, by reciprocity.
Such that $\frac{J+K}{JK}$ can not have any cofactors since
it can be rewritten/understood that K is stated to be relatively prime (*coprime*) to J.

Then due to the simplicity of the subduction process:

$\frac{PJK}{J+K}$ may only have a single cofactor of P.

Thus J^P+K^P can only be factored as:

Case 1: $(J+K) \cdot f(J,K)$ with no common factor P

Or Case 2: $(J+K) \cdot f(J,K)$ with a common factor P

With $f(J,K)$ only able to contain a single factor of P

Detailed example of long division by J+K shown below, for clarity of understanding:

$$J^4 - J^3K + J^2K^2 - JK^3 + K^4 \;/\; (J + K)$$

$$\begin{array}{r}
J^4 - J^3K + J^2K^2 - JK^3 + K^4 \\
-\ J^3(J+K) \\
\hline
-2J^3K + J^2K^2 \\
+\ 2J^2K(J+K) \\
\hline
3J^2K^2 - JK^3 \\
-\ 3JK^2(J+K) \\
\hline
-4JK^3 + K^4 \\
+\ 4K^3(J+K) \\
\hline
5K^4
\end{array}$$

*(note, -1 * -1 = +1)*

*(note, -1 * -1 = +1)*

Thus showing that P, in this case 5, is the only remainder when divided by J + K, similarly if dividing right to left the remainder will be $5J^4$, and if dividing symmetrically from both ends simultaneously, the result will be $5J^2K^2$. In all 3 cases, the only possible cofactor to J +K is 5 in essence P.

This T3 Lemma is fundamentally written to show that there are no possible common factors between A_1, A_2, B_1, B_2, C_1 and C_2 except the possibility of a factor of P.

I coined the term “Subduction” as being Subtraction/Deduction combined.

It should be somewhat obvious from the above analysis that if $J^P + K^P$ can not have a *single* factor of P, since both factors of it must contain a factor of P. Of course J + K could contain multiple factors of P, but f_A(J,K,P) may only contain a single factor of P.

The long division presented above, dividing J + K into f_A(J,K,P), can be done from left to right, right to left or may simultaneously be approached from both left and right sides. Although it is clearly intuitively obvious that J+K can not divided into f_A(J,K,P) with the exception of factor P, this Lemma drives the point home using Long Division.

My first writeup on this in my NoteBook was for the case P = 7, with the Long division approached from both left and right sides simultaneously. Quite naturally, the residue was $7J^3K^{3.}$

BASE CAMP REVIEW:

<u>Presentations of D for SGC1</u> **E13a**

$$D = A + B + C \;=\; P A_1 B_1 C_1 K_0 = \frac{(A + B) + (B + C) + (A + C)}{2} = \frac{C_1^P + A_1^P + B_1^P}{-\,2}$$

$$D = A + B + C \;=\; A - A_1^P = A_1(A_2 - A_1^{P-1}) \;=\; B - B_1^P = B_1(B_2 - B_1^{P-1}) \;=\; C - C_1^P = C_1(C_2 - C_1^{P-1})$$

$A_2 = B_2 = C_2 = 1$ Mod P (see **E11f**)

<u>Presentations of D for SGC2,</u> (*C = PC₁C₂*) **E13b**

$$D = A + B + C \;=\; P A_1 B_1 C_1 K_0 = \frac{(A + B) + (B + C) + (A + C)}{2} = \frac{\mathbf{P}^{\mathbf{P-1}} C_1^P + A_1^P + B_1^P}{-\,2}$$

$$A + B + C \;=\; A - A_1^P = A_1(A_2 - A_1^{P-1}) \;=\; B - B_1^P = B_1(B_2 - B_1^{P-1}) \;=\; C - \mathbf{P}^{\mathbf{P-1}} C_1^P = \mathbf{P}C_1(C_2 - \mathbf{P}^{\mathbf{P-2}} C_1^{P-1})$$

$A_2 = B_2 = 1$ Mod P (see **E11f**)

The Apex Proof

Now that you have persevered through an arduous climb of historic proportions, struggled thru a labyrinth of abstruse equations, and finally reached the plateau where we may climb the final ascent, it is clear your strong determination to succeed in climbing Mount Everest is ever-shining.

Pierre Fermat himself, if were here today, would be proud of you. The final 100 meters of ascent will take us to the apex.

We will start the analysis at P = 5.

$$D = A+B+C = A - A_1^5 = B - B_1^5 = C - C_1^5 = 5A_1B_1C_1K_0 \qquad \textbf{E14a}$$

$$A - A_1^5 = B - B_1^5 \qquad \textbf{E14b}$$

$$A_1^5 - B_1^5 = A - B \qquad \textbf{E14c}$$

$$(A_1 - B_1)(A_1^4 + A_1^3B_1 + A_1^2B_1^2 + A_1B_1^3 + B_1^4) = A - B = A_1A_2 - B_1B_2 \qquad \textbf{E14d}$$

$$A_1^4 + A_1^3B_1 + A_1^2B_1^2 + A_1B_1^3 + B_1^4 = \frac{A_1A_2 - B_1B_2}{A_1 - B_1} \qquad \textbf{E14e}$$

Evaluation of the RHS above equation. $$\frac{A_1A_2 - B_1B_2}{A_1 - B_1} \qquad \textbf{E14f}$$

We may surmise with a quick inspection that in order for $A_1 - B_1$ to be divisible into $A_1A_2 - B_1B_2$, if $A_2 = B_2$ then the denominator would divided into the numerator, however in this case A and B would not be coprime. In a more general sense,

$(A_1 - B_1)(A_2 + B_2 + X)$ can be shown to be equal to $\mathbf{A_1A_2 - B_1B_2} + (A_1(B_2 + X) - B_1(A_2 + X))$, thus if: $A_1(B_2 + X) - B_1(A_2 + X) = 0$ **E14g**

we can use this form to approach the apex proof.

Thus, in order for $\frac{A_1A_2 - B_1B_2}{A_1 - B_1}$ to be divisible by $A_1 - B_1$, it is necessary that $(A_1(B_2 + X) - B_1(A_2 + X)) = 0$ **E14h**

As stated earlier in the Base Camp Foundation, A_2, B_2 and C_2 for SGC1 must all be of the form 1 Mod P. And this is certainly obvious, with a fundamental understanding of the form of Fermat's Last Theorem.

$$A_1(B_2 + X) – B_1(A_2 + X) = 0 \qquad \textbf{E15a}$$

$$A_1(B_2 + X) = B_1(A_2 + X)$$

Since A_1 must be coprime to B_1: $A_1 = A_2 + X$ and $B_1 = B_2 + X$ **E15b**

Now we can solve for X: $X = A_1 – A_2 = B_1 – B_2$ **E15c**

Let's remove X and rearrange: $A_1 – B_1 = A_2 – B_2$ **E15d**

From here we see the big affect of the A_1 coprimeness to B_1 step above, which now shows that since $A_2 – B_2$ is 0 Mod P, then it must also be true that A_1 is congruent to B_1, which if we now rotate the 3 variables A, B and C and present in SGC1 form, we get:

$$A_1 = B_1 = C_1 \text{ Mod P} \qquad \textbf{E15e}$$

All 3 variables are congruent (*same modulus of P*) thus analyzing from the basic presentation of D:

$A_1A_2 + B_1B_2 + C_1C_2$ =0 Mod 5, we can see a dilemma, since A2, B2 and C2 are equal to 1 Mod 5, there can be no solution to the D equation, if P is >= 5. Only for the case of P = 3 is there any imperfection in the analysis, in SGC1. And this specific exception will be proved later on in this paper, on a future rewrite.

For SGC1: Reductio Ad Absurdum P=5, and by logical extension all other primes greater than 5.

Now for SGC2 we will find if the factor of P resides in C, that $A_1 = B_1$ Mod P, and we will see that for the D basis equation:

$A_1A_2 + B_1B_2 + C_1PC_2$ =0 Mod P, that A and B are congruent, then this leads to $A_1A_2 + B_1B_2 \neq 0$ Mod P, and we can surmise then that for SGC2 that the proof will stand for all prime exponents >= 3.

For SGC2: Reductio Ad Absurdum P=3, and by logical extension all other primes greater than 3.

ADDENDUM

-A- A Brief Proof for N=3 for Sophie Germain Case 1

Mar 25, 2025 D. Ross Randolph Updated April 1, 2025

Can $A^3 + B^3 + C^3 = 0$ (*C being negative*) have a finite solution?

<u>Sophie Germain Case 1</u>, none of the coprime variables have a factor of 3
A+B+C = 0 Mod 3 (by virtue of Fermat's Little Theorem)
$(A+B+C)^3 = A^3 + B^3 + C^3 + 3(A+B)(B+C)(C+A)$

$(A+B+C)^3$

$$\begin{array}{ccccccc} & & & C^3 & & & \\ & & & \mathbf{1} & & & \\ & & \mathbf{3} & & \mathbf{3} & & \\ & \mathbf{3} & & \mathbf{6} & & \mathbf{3} & \\ \mathbf{1} & & \mathbf{3} & & \mathbf{3} & & \mathbf{1} \\ A^3 & & & & & & B^3 \end{array}$$

$(A+B)(B+C)(C+A)$ *(8 terms)*

$$\begin{array}{ccccccc} & & & C^3 & & & \\ & & & \mathbf{0} & & & \\ & & \mathbf{1} & & \mathbf{1} & & \\ & \mathbf{1} & & \mathbf{2} & & \mathbf{1} & \\ \mathbf{0} & & \mathbf{1} & & \mathbf{1} & & \mathbf{0} \\ A^3 & & & & & & B^3 \end{array}$$

From the above trinomial expansion diagrams which *only show* the coefficients, we can easily conceptualize, that if we multiply (A+B)(B+C)(C+A) by 3, and then add the corner coefficients for A^3, B^3 and C^3 that the resulting diagram will be equal to $(A+B+C)^3$.

Next,

If $A^3 + B^3 + C^3 = 0$ then,
$(A+B+C)^3 = 3(A+B)(B+C)(C+A)$

$(A+B+C)^3 = 0$ Mod 27
3(A+B)(B+C)(C+A) = 0 Mod 3 ≠ 0 Mod 9

0 Mod 27 ≠ 0 Mod 3 **Reductio Ad Absurdum**

-B- A Somewhat Geometric Proof for Fermat's Last Theorem for N = 4

June 8th^{t}, 2025 D.Ross Randolph

"*It is impossible to separate a cube into two cubes, a fourth power into two fourth powers, or generally, any power above the second into two powers of the same degree*", Fermat wrote this in the margin of his copy of an ancient Greek math book written by Diophantus, titled Arithmetica.

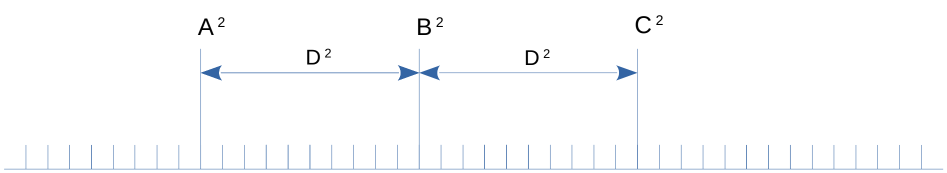
This proposition was first stated as a theorem by Pierre de Fermat around 1637. And it is known that Fermat used the method of Infinite Descent to prove this statement for N=4 using a logical geometric approach.

The method I will use will be somewhat geometric, and will not use infinite descent.

Can $X^4 + Y^4 = Z^4$ have a finite solution, for all pair-wise coprime integers?

We will morph the above equation into the following form:

$A^2 + D^2 = B^2$, $B^2 + D^2 = C^2$ *(as diagrammed pictorially below)*



From this form we will extract the proof.

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Consider Z^4 can not be even since, $X^4 + Y^4$ can only be divisible by 2 if both odd, therefore we will select Y without loss of generality to be even parity.

$X^4 = Z^4 - Y^4 = (Z^2+Y^2)(Z^2-Y^2)$

Since Z is coprime to Y, Z^2+Y^2 must be coprime to $Z^2 - Y^2$.

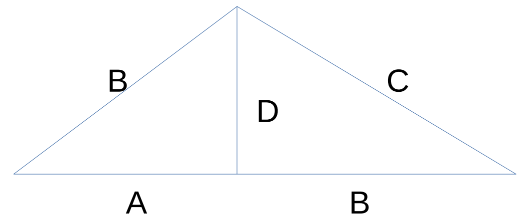
Now we can factor X^4,

$X_1^4 = Z^2+Y^2$ and $X_2^4 = Z^2-Y^2$

Now we will assign to A, B, C and D.

$X_1^4 = A^2$ $Z^2 = B^2$ $Y^2 = D^2$ $X_1^4 = C^2$ *(Note: A,B,C,D are all pair-wise coprime)*

Next we can draw a geometric grouping of right triangles, bear in mind since A, B and C are odd parity, therefore D must be even parity.



Based upon the standard Pythagorean triplet formula below:

a, b, c = $m^2 - n^2$, $2mn$, $m^2 + n^2$

Note, that in order for a, b and c to be pair-wise coprime, it is a necessary precondition that m and n are coprime.

We will analyze the geometric diagram. Note, variable D appears in both right angle triangles.

$D = 2\,(M_1M_2)(N_1N_2) = 2\,(M_1N_1)(M_2N_2)$ *(This form allows examination of every possible combination case)*

$B = M_1^2M_2^2 - N_1^2N_2^2$ *(Triangle D,B,C)*

$B = M_1^2N_1^2 + M_2^2N_2^2$ *(Triangle D,A,B)*

$B = M_1^2M_2^2 - N_1^2N_2^2 = M_1^2N_1^2 + M_2^2N_2^2$ *(This form may be transformed into the 2 below forms)*

$M_1^2(M_2^2 - N_1^2) = N_2^2(M_2^2 + N_1^2)$ as well as $M_2^2(M_1^2 - N_2^2) = N_1^2(M_1^2 + N_2^2)$

Now since M_1^2 is coprime to N_2^2:

$M_1^2 = M_2^2 + N_1^2$
$N_2^2 = M_2^2 - N_1^2$
-- (summing)
$M_1^2 + N_2^2 = 2M_2^2$

$2M_2^2 = M_1^2 + N_2^2$

Now since M_2^2 is coprime to N_1^2:

$M_2^2 = M_1^2 + N_2^2$
$N_1^2 = M_1^2 - N_2^2$

By Contradiction $2M_2^2 \neq M_2^2$ Reductio Ad Absurdum

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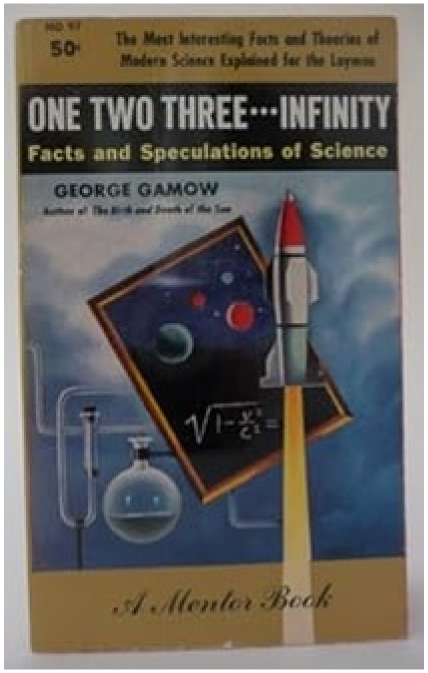
As a closing effort to proving Fermat's Last Theorem for all exponents 3 thru infinity, I felt it was necessary to add a N=4 proof to the mix. I have tried to present it in a unique way, to thus give it my own little signature of originality.
D.Ross.Randolph345@gmail.com

-C- References and Suggested Reading



George Gamow, “One Two Three, Infinity”, 1959
A plain look at the outer-universe, the inner-universe, the expansion of space time, and infinity. Out-of-print, for quite a few years now, good luck finding a copy.

References:

[1] Weisstein, Eric W. "Fermat's Last Theorem". MathWorld.
(mathworld.wolfram.com/FermatsLastTheorem.html)

[2] Wikipedia, “Fermat’s Last Theorem”, (en.wikipedia.org/wiki/Fermat's_Last_Theorem)

[3] Edwards, Harold M. “Fermat's Last Theorem”, Scientific American Vol. 239, No. 4 (October 1978), pp. 104-123 (www.jstor.org/stable/24955826)

[5] Hanna Kagele, “Sophie Germain, The Princess of Mathematics and Fermat’s Last Theorem”
https://www.gcsu.edu/sites/files/page-assets/node-808/attachments/kagele.pdf

[6] Colleen Alkalay-Houlihan, “Sophie Germain and Special Cases of Fermat's Last Theorem”
https://www.math.mcgill.ca/darmon/courses/12-13/nt/projects/Colleen-Alkalay-Houlihan.pdf

[7] D. Ross Randolph
Symmetrical versus non-symmetrical variable approach to FLT
Congruence equality operator and redundancy to the Mod operator
Elucidation on the T3 Lemma
How does it work, a Block Diagram Over-View

Mathematicians thru history whose work is foundational to this exposition.
Wikipedia Links:

Diophantus	Leonard Euler	Pierre Fermat
Euclid	Sophie Germain	Al-Khwarizmi
Pythagoras of Samos	Blaise Pascal	

-D- For the near future, I may be contacted by email at: D.Ross.Randolph345@Gmail.com Feel free to establish contact. I can assist you with further explanation/clarification of any murky areas within the proof.

If you have read and understood this proof, you may wish to contact the Fermat Museum in Beaumont-de-Lomagne, France at: Contact@Fermat-Science.com and/or contact@museefermat.com They may perhaps be interested to learn of this comprehensible proof.

-E- Individuals who have assisted me in my quest, who are worthy of my mention

Quoran, Will Jadson of Brazil: A mathematician enthusiast, who has derived an interesting limited case proof to FLT, which is presented in a web page dangling off the sitemap.

Quoran, David Smith of Gloucestershire, UK : Excellent trained mathematician with an inherent curiosity, who was quite central in my proof analysis in the summer of 2024. With simplicity, he demonstrated a fundamental modularity concept, which I needed to absorb, at a deep grey matter cellular level.

Reddit, Edderiofer and Xhiw usernames, unidentified and well intentioned individuals.

arXiv, Giulio Morpurgo, a retired Physicist and Statistician from the EU, who was the first commentator re my early work

THE DA VINCI CODE EASTER EGG. THE ILLUMINATI HAVE GATHERED IN ATLANTIS TO DISCUSS DEPLOYMENT OF THE DEEPMIND COMPUTER NETWORK WHICH IS POISED TO TAKE OVER THE WORLD IN THE YEAR OF OUR LORD 2026, AT WHICH TIME ALL MATHEMATICS AS WE NOW KNOW IT WILL OBLITERATED FROM EVERY TEXT BOOK AND COMPUTER IN THE CIVILIZED WORLD. THE FLOOR GOES TO THE ESTEEMED MR. LEONARD BONACCI, THE SECRETARY AND FOREMOST MEMBER "IT IS MY BELIEF THAT IT WILL BE NECESSARY TO INSERT A VIRUS INTO DEEPMIND SUCH THAT ANYONE WHO CLAIMS TO HAVE A PROOF OF MR. PIERRE FERMAT'S FAMOUS THEOREM WILL IMMEDIATELY BE NOTIFIED THRU THE BROWSER THAT A COUNTDOWN HAS NOW STARTED STARTING AT T-100 HOURS, AND WHEN THE ZERO MINUTE IS REACHED A FINAL EVENT WILL OCCUR, THE LIKES OF WHICH HAVE NEVER BEFORE BEEN EXPERIENCED BY THE HUMAN RACE. SPACE ALIENS WILL SIMULTANEOUSLY INVADE ALL OF THE SUBCONSCIOUS MINDS ON THE PLANET EARTH, AND STEER THE FUTURE OF MANKIND IN A MOST DEVASTATING DIRECTION." "THEREFORE YOU MUST NOT, I REPEAT NOT, EVER DIVULGE THIS FLT PROOF TO OTHER INHABITANTS OF YOUR HOME PLANET. THE PERIL TO OTHERS ON THE PLANET WILL BE HORRENDOUS. JUST DO NOT DO IT!" AT THIS POINT, A QUIET RESTLESSNESS PERVADES THE ATMOSPHERE OF THE GROUP, AND A DARN NICE LOOKING MERMAID ENTERS THE LIBRARY WHERE THE ILLUMINATI TYPICALLY MEET. SHE SAYS "DO ANY OF YOU LADIES OR GENTLEMEN REQUIRE THE SERVICES OF OUR MASSAGE TEAM, OR REFRESHMENTS?" EULER, THAT CHRONIC JOKER RESPONDS "COULD YOU ENGINEER FOR ME SOMETHING SOFT AND SWEET THAT TOUCHES MY TASTE BUDS, AND WILL RELEASE MY STRESS LEVEL THIS NUTTY GROUP IS CONTINUOUSLY PUSHING ON OUR PLANET?" SHE BLUSHES, AND CURTLY RESPONDS, "KEEP IT IN YOUR PANTS MR. EULER! I'LL GET YOU ANOTHER CUP OF LIQUID VIXEN, JUST THE WAY YOU LIKE IT." EULER SMILES, AND MS SOPHIE GERMAIN TAKES THE FLOOR, "EULER YOU SKINNY EFFIN SKUNK, WILL YOU SETTLE DOWN. HONESTLY, A HORNIER OLD GOAT DOES NOT EXIST IN THIS CORNER OF THE GALAXY!" "GENTLEMEN, GENTLEMEN" IS PROCLAIMED BY LEONARD BONACCI, "I SEE THAT MR. ZARATHUSHTRA HAS SOMETHING TO SAY. THUS SPOKE ZARATHUSHTRA, "IT HAS COME TO MY ATTENTION THAT OUR SPACE ALIEN FRIENDS FROM THE BLACK STAR X, HAVE RECENTLY EXPRESSED AN INTEREST IN THE MATHEMATICS OF PLANET EARTH, IN PARTICULAR THE PROOF BY THE INTERNET SENSATION MATHEMATICIAN BRITNEY SPEARS. HER ENTROPY BASED PROOF TO FERMAT'S LAST THEOREM IS TOTALLY RAD AND TUBULAR, IT'S THE BEES KNEES." AT THIS POINT PIERRE F. YELLS OUT "WTF, IT'S A BLOODY PIECE OF EXCREMENT, BUT WHAT I WOULD DO WITH THAT TIGHT LITTLE ASS IF GIVEN HALF A CHANCE". FERMAT IS RIGHTLY INTERRUPTED AND B. PASCAL CHIMES IN "YOU MAY BE CORRECT SIR, BUT PLEASE, , THERE ARE FEMALE GENIUS MATHEMATICIANS HERE, AND SOME OF THEM KNOW A BIT OF MIND CONTROL, TAUGHT TO THEM BY THE ESTEEMED MENTALIST JL, WHO'S NAME SHALL NOT BE MENTIONED, AS IT MAY BRING AN UNDERWATER TSUNAMI TO ATLANTIS, WHICH WE MUST AVOID." FINALLY D. HILBERT SPEAKS UP " COULD WE PLEASE FINISH THE PROCEEDINGS, I FEEL THE NEED TO TAKE A PHYSICAL FORM, AND DO A LITTLE UNDERWATER SWIMMING, AND MAYBE GET TOGETHER WITH FLIPPER'S HOT SISTER MELISSA. ANYONE FOR A DIP?" THE GROUP ALL AGREE TO CONVENE THE FOLLOWING YEAR AT STARBUCK'S IN JERSEY CITY, SAME BAT TIME, SAME BAT CHANNEL.

CHANGE LOG:
March 3, 2025- A new proof origin is started using Trinomial expansion of S – (A+B).

March 4, 2025 – Added proof for the general case any exponent P.
March 21, 2025 – Resolved previous errors, new theme based upon congruence contradiction modulus P.
March 23, 2025 – Equation numbering in **light blue bold text** added to pages 4 thru 15.
March 29, 2025 – Fixed a spelling error in Sophie Germain's name.
April 1, 2025 – Axioms added on page 11 for the T3 Lemma.
April 3, 2025 – Major cleanup of the Apex proof section, and a new high clarity Abstract added to page 1.
April 11, 2025 – Cleaned up the old Addendum A, Fermat's Little Theorem presentation, which had 2 small errors.
April 14, 2025 – Big clean up pgs 11,13,14 &15 regarding clarity of A_2, B_2 and C_2 and added Presentations of D Review. Removed most of the fog in the structural explanation/organization towards the end of the proof, starting from the T3 Lemma.
April 17, 2025 - Added a references section to Addendum B, six references for Fermat's Last Theorem and Sophie Germain.
April 23, 2025 – Added contact info for the Fermat Museum in France.
April 24, 2025 – Added further clarification re the T3 Lemma.
April 27, 2025 – Added Axiom 5 for the T3 Lemma, to try to make T3 easier to absorb for neophytes.
May 25, 2025 – Replaced Addendum A.
June 7th, 2025 – Added N=4 proof to the Addendum
June 10th, 2025 – Added references to my own work, regarding underlying aspects of this paper in Addendum C.
June 12th, 2025 – Improved line work for N=4 proof, added Reductio Ad Absurdum to end of that section. Fixed March 21st change log entry.