

# On the Gravitational Analog of Continuous Spacetime Dimensions

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## Abstract

The object of this brief note is to point out that a spacetime endowed with continuous dimensions can be interpreted as dual to classical gravitation in four dimensions. This observation sheds unexpected light onto the gravitational behavior of Cantor Dust (CD), a hypothetical *large-scale topological condensate* of relic dimensions left over from the primordial stages of Universe evolution.

**Key words:** effective gravitational coupling, Dark Matter, Cantor Dust, continuous spacetime dimensions.

A straightforward comparison of the Newton potential

$$V_N = -\frac{G_N m^2}{r} \quad (1)$$

with the Coulomb potential expressed in natural units ( $\hbar = c = 1$ )

$$V_c = -\frac{e^2}{4\pi r} = -\frac{\alpha}{r} \quad (2)$$

is consistent with the concept of Planck mass defined as

$$M_{Pl} = \sqrt{\frac{\hbar c}{G_N}} = \sqrt{\frac{1}{G_N}} \quad (3)$$

By analogy with the fine structure constant  $\alpha$ , (1) – (3) suggest the introduction of the *effective gravitational coupling* [1]

$$\alpha_G = \frac{m^2}{M_{Pl}^2} \quad (4)$$

The usual interpretation of (4) is that gravitational effects are practically irrelevant to the low-energy sector of Quantum Field Theory (QFT) and the Standard Model of Particle Physics. Since, in QFT mass runs with the observation scale ( $\mu$ ), a natural extrapolation of (4) is given by

$$\alpha_G(\mu) = \frac{m^2(\mu)}{M_{Pl}^2} \quad (5)$$

Following [2 - 3], dimensional regularization of QFT indicates that the continuous deviation from  $D=4$  spacetime dimensions takes the form

$$\varepsilon(\mu) = 4 - D(\mu) = O(m^2(\mu)/\Lambda_{UV}^2) \ll 1 \quad (6)$$

in which  $\Lambda_{UV}$  is the high energy cutoff of the theory. By (5), under the identification  $\Lambda_{UV} = O(M_{Pl})$  and taking the far asymptotic limit  $m \ll M_{Pl}$  yields

$$\boxed{\alpha_G(\mu) = O(\varepsilon(\mu))} \quad (7)$$

Consider now the metric term describing the Newtonian potential created by a point-source of mass  $m$  at a distance  $R$

$$g_{00} = 1 + 2V_N = 1 - 2G_N \frac{m}{R} \quad (8)$$

or, on account of (3),

$$g_{00} = 1 - 2 \frac{m}{M_{Pl}^2 R} \quad (9)$$

For a particle-antiparticle pair produced by vacuum fluctuations and separated by distance having the same order of magnitude as their Compton wavelength  $\lambda_C$ , that is,  $R = O(\lambda_C) = O(1/m)$ , (9) turns into

$$g_{00}(\mu) = 1 - 2 \frac{m^2(\mu)}{M_{Pl}^2} \approx 1 - 2\varepsilon(\mu) \quad (10)$$

or

$$\boxed{\varepsilon(\mu) \approx \frac{1}{2}(1 - g_{00}(\mu))} \quad (11)$$

Loosely speaking, (11) may be associated with the *gravitational coupling of a particle-antiparticle pair separated by a distance scale comparable with their Compton wavelength*. This observation sets the stage for the gravitational interpretation of Cantor Dust (CD), under the assumption that CD acts as a large-scale cluster of ultralight Dark Matter particles such as self-interacting bosons, axions, fuzzy condensates or 3D anyons [4].

On the same note, a noteworthy hypothesis is that CD equations may generate the *stochastic gravitational background* recently reported by several international collaborations [5].

## **References**

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