The Excited States of the String Electron

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Abstract

The nonlinear Schrödinger equation is derived for the electron which is represented with the closed circular string. The energies for the stationary states of such system is derived and the corresponding frequencies of the emitting photons are determined.

1 Introduction

One of the fundamental dichotomies of the particle physics is

$$electron(quark)\ is\ a\ point-like\ particle \tag{1}$$

$$electron(quark)\ is\ not\ a\ point-like\ particle. \tag{2}$$

While for many years it has been supposed that electron(quark) is a point-like particle, now, the attention to be concentrated on the electron(quark) as a composite particle. One of the possibilities how to decide whether electron, or, quark is a composite is to investigate their excited states. Such possibility has been discussed since the first proposal given by Low (1965) and later several estimations of masses of the excited leptons and quarks has been obtained (Renard, 1983). In the present article we will concentrate only on the excited states of an electron.The natural reaction leading to such states are obviously the following ones:

$$p + e^- \to p + e^{-*} + \gamma \to p + e^- + \gamma \tag{3}$$

$$\gamma + p \to p + e^+ + e^{-*} + \gamma \to p + e^- + e^+ + \gamma \tag{4}$$

$$e^+ + e^- \to e^+ + e^{-*} \to e^+ + e^- + \gamma. \tag{5}$$

and so on (Renard, 1983).

The coupling between excited spin 1/2 fermions, light fermions and photons is usually in literature considered in the form of the magnetic type with the effective Lagrangian (Kühn et al., 1985):

$$\mathcal{L} = \frac{ef}{2m^*}\bar{e}^*\sigma_{\mu\nu}F^{\mu\nu}e_L + h.c., \tag{6}$$

where m^* denotes the mass of the excited electron e^{-*}, f is some coupling constant e_L is the left handed component of the electron wave function. It is possible also to use the right component e_R.

Similar Lagrangian has been considered for decay of bosons W^-, W^+, Z^0 decaying into the excited particles (Rubbia, 1985).

During the 1983 run, the UA2 group reported an event of the type

$$Z^0 \to e^+ + e^- + \gamma \tag{7}$$

with a resolved $e - \gamma$ pair. This has promoted the speculations that one might have observed e new excited lepton $e^{-*} \to e^- + \gamma$ Similarly a search for

$$W \to e^{-*} + \nu \to e^- + \nu + \gamma \tag{8}$$

exists, however no event has been found so far (Rubbia, 1985). Nevertheless, the question of the excited states of leptons is attractive an it needs attention.

While the description of the excited lepton states is usually involved in the effective Lagrangians, we will solve this problem by supposing the specific internal structure of an electron. In other words, we will suppose, an electron has a form of the closed string. The motivation for a such model is based on the present situation in the particle physics, where strings and the string theory form the fundamental mathematical objects in the particle physics (Artru, 1983).

On the basis of the string model of the electron we will show that the string motion is described by the nonlinear Schrödinger equation, and we will derive an equation for the stationary states. Because of the strong non linearity of the derived equation, we solve the problem of the stationary states only approximately using a method which is described in the Migdal book (Migdal, 1975). To our knowledge, the results obtained in the present article are original.

2 The quantum mechanics of the closed string

Early than we formulate the Schrödinger equation for the open and closed string, let us remember the many-particle interpretation of the Ψ-function.

If we put

$$\Psi = \sqrt{\varrho}e^{\frac{i}{\hbar}S}, \tag{9}$$

where ϱ and S are real functions, into the original Schrödinger equation

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\Delta\Psi + V\Psi \tag{10}$$

we get after separation of the real and imaginary part of the solution, the hydrodynamical equation and the equation of continuity (Wilhelm, 1970; Rosen, 1974). It means that the

Schrödinger equation describes not only the behavior a single particle, but at the same time the system of particles moving in the potential field V. The many-particle description of the physical reality was for instance successful used in the superconductivity (Feynman, 1972) and now the question arises, how to apply it for the motion of the string.

Let us replace the string by the linear chain consisting of the massive points with masses m, which are at the equilibrium state at points $x_k = k.a$, where a is the lattice constant of the chain and the left end of the chain is fixed at point $x_0 = 0$. Then the potential energy of the chain is

$$V = \frac{1}{2}\kappa \sum_{i=1} (\eta_i - \eta_{i-1})^2; \quad \eta_0 = 0, \tag{11}$$

where κ is some elasticity constant and η_i is deflection of a mass point with index i from the equilibrium position.

Let us the equilibrium length of the chain is $l_0 = k.a$ and l is is length of the uniformly stretched chain. Then we have

$$\eta_i - \eta_{i-1} = \delta_i = \delta = \frac{l - l_0}{N} \tag{12}$$

and

$$V = \frac{1}{2}\kappa N \delta^2 = \frac{1}{2}\kappa N \left(\frac{1}{\varrho} - \frac{1}{\varrho_0}\right)^2, \tag{13}$$

where ϱ and ϱ_0 are densities of of points of the stretched and equilibrium chain.

It is easy to see that it follows from eq. (13) that for the string with the limit $N \to \infty$, the potential energy is of the form:

$$V = \left(\frac{\alpha}{\varrho} - \beta\right)^2 \tag{14}$$

and the corresponding Schrödinger equation of the one-dimensional string is as follows:

$$i\hbar\frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\Delta\Psi + V\Psi + \left(\frac{\alpha}{\varrho} - \beta\right)^2 \Psi. \tag{15}$$

3 The stationary states of the string

Inserting

$$\Psi = \varphi(x) e^{-\frac{i}{\hbar}Et}, \tag{16}$$

into eq. (15), we get

$$\varphi'' = f(\varphi), \tag{17}$$

where

$$f(\varphi) = -\omega_0^2\varphi + \frac{2m}{\hbar^2}\left(\frac{\alpha}{\varrho} - \beta\right)^2 \varphi \tag{18}$$

and

$$\omega_0^2 = \frac{2mE}{\hbar^2}. \tag{19}$$

We get also instead of eq.(17)

$$d\varphi'^2 = 2f(\varphi)d\varphi, \tag{20}$$

or,

$$\varphi'^2 = 2(\varphi)d\varphi + C, \tag{21}$$

or,

$$x = \int \frac{d\varphi}{\left(2\int f(\varphi)d\varphi + C\right)^{1/2}} + D, \tag{22}$$

where C and D are some constants of integration. It is easy to see that the integral in eq. (22) cannot be evaluated by the elementary methods. This is why we approach the solving the problem by some approximate method.

It holds good $V(\varrho_0)$ for the equilibrium state of the string, where $\varrho_0 = \alpha/\beta$. Furthermore we have

$$V'(\varrho_0) = 0 \tag{23}$$

and therefore

$$V \approx \frac{V'(\varrho_0)}{2!}\varrho_0^2 = \frac{\beta^4}{\alpha^2}\varrho_0^2 \tag{24}$$

Now, we have instead of eq. (17) the following equation

$$\varphi'' = -\omega_0^2\varphi + \frac{2m}{\hbar^2}\frac{\beta^4}{\alpha^2}\varphi^5, \tag{25}$$

or,

$$\varphi'' + \omega_0^2\varphi = A\varphi^5; \; A = \frac{2m}{\hbar^2}\frac{\beta^4}{\alpha^2} \tag{26}$$

The first iteration equation corresponding to eq.(26) is obviously

$$\varphi_1'' + \omega_0^2\varphi_1 = A\varphi_0^5, \tag{27}$$

where

$$\varphi_0 = a\sin(\omega_0 x) \tag{28}$$

is the solution of the homogenous equation (26). After some modification, we have instead of eq. (27) the following equation:

$$\varphi_1'' + \omega^2\varphi_1 = \frac{Aa^5}{16}\left(10\sin(\omega_0 x) - 5\sin(3\omega_0 x) + \sin(5\omega_0 x)\right). \tag{29}$$

However, function $\sin(\omega_0 x)$ forms the resonance solution of eq. (29), which means we are forced to modify the iteration method. We use here the prescription of the method described in the Migdal book (Migdal, 1975) and write instead of eq. (26)

$$\varphi'' + \omega^2\varphi = A\varphi^5 + (\omega^2 - \omega_0^2)\varphi, \tag{30}$$

where $\omega \neq \omega_0$ and the equation of the iteration is now of the form:

$$\varphi_1'' + \omega^2\varphi_1 = \frac{Aa^5}{16}\left(10\sin(\omega x) - 5\sin(3\omega x) + \sin(5\omega x)\right) + \left[a\sin(\omega x)\right](\omega^2 - \omega_0^2), \tag{31}$$

where ω must be determined in order to get no resonance solution. Or,

$$\omega^2 = \omega_0^2 - \frac{5a^4}{8}A. \tag{32}$$

Now, it is obvious that the solution of eq. (31) is of the form:

$$\varphi_1 = c_1\sin(3\omega x) + c_2\sin(5\omega x), \tag{33}$$

where constant c_1, c_2 can be easily determined as

$$c_1 = \frac{5}{128}\frac{1}{\omega^2}Aa^5 \tag{34}$$

$$c_2 = -\frac{1}{384}\frac{1}{\omega^2}Aa^5. \tag{35}$$

4 The stationary states of the closed string

The stationary states of the closed string are determined by the so called Born-Kármán periodicity conditions

$$\varphi(x) = \varphi(x+L); \quad \varphi(x,\omega) = \varphi_0(x,\omega) + \varphi_1(x,\omega), \tag{36}$$

where L is the length of the perimeter of the string.

After inserting of eq. (33) into eq. (36) we get that condition (36) is fulfilled only for the specific ω_n. Namely for

$$\omega_n = \frac{2\pi}{L}n; \quad n = 1, 2, 3, ... \tag{37}$$

Using eq. (32) and eq. (19),we get for the stationary states of the closed string the following formula:

$$E_n = \frac{\hbar^2}{2m}\left(\omega_n^2 + \frac{5a^4}{8}A\right). \tag{38}$$

5 The excited states of an electron

We identify the electron with closed string. It means that the excited states of the electron are the excited states of the closed string. It is usually suppose, in quantum physics, that the change of the stationary state is accompanied by the emission, or, absorption of photon with frequency

$$\omega = \frac{E_n - E_m}{\hbar}; \quad E_n > E_m. \tag{39}$$

and it means that we can await a signal represented by photons, which will inform us about the existence of the excited states of electron. The processes (3–5) which can be possible in the high-temperature plasma of protons, electrons and positrons will probably give us the evidence of the new physical reality.

6 Discussion

In the preceding text we exhibited the model of non-point-like electron in the form of the closed string. The model enables the existence of the excited states which is the measurement problem for particle physics. In general, composites models can be ones in explanation for the observed three generations of standard model (SM) particles. Observation concerns the all excited leptons and excited muons.

At the LHC such particles could be produced in pp collisions under the assumption that leptons are composite objects. Produced excited leptons are expected to transition to their corresponding SM lepton partner via gauge or via contact interaction. CMS has performed a search for such excited states. While no signal was observed, the exclusion results provide the best limits to date (Hoepfner et al., 2021)

The problem can be generalized for the situation with the Dirac equation and using the relativistic methods of quantum electrodynamics (Akhiezer et al., 1965; Berestetskii et al., 1982).

The article is some modification of the original contribution at the international conference on the hadron physics in Prague (Pardy, 1988).

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