

Bemba Pocket Dictionary, Bemba-English and English-Bemba, by Rev. E. Hoch and The Graphical Law

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Abstract

We study the Bemba head entries of the Bemba Pocket Dictionary, Bemba-English and English-Bemba, by Rev. E. Hoch, 1960. We draw the natural logarithm of the number of head entries, normalised, starting with a letter vs the natural logarithm of the rank of the letter, normalised/unnormalised. We find that the Bemba head entries underlie a magnetisation curve of a Spin-Glass in the presence of little external magnetic field.

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A	B	C	D	E	F	G	H	I	J	K	L	M	N	Ng	O	P	Q	R	S	T	U	V	W	X	Y	Z
79	305	441	0	37	139	0	0	253	0	379	358	637	276	14	28	222	0	0	295	197	38	0	15	0	12	0

TABLE I. The Bemba Head entries of Bemba Pocket Dictionary, Bemba-English and English-Bemba, by Rev. E. Hoch

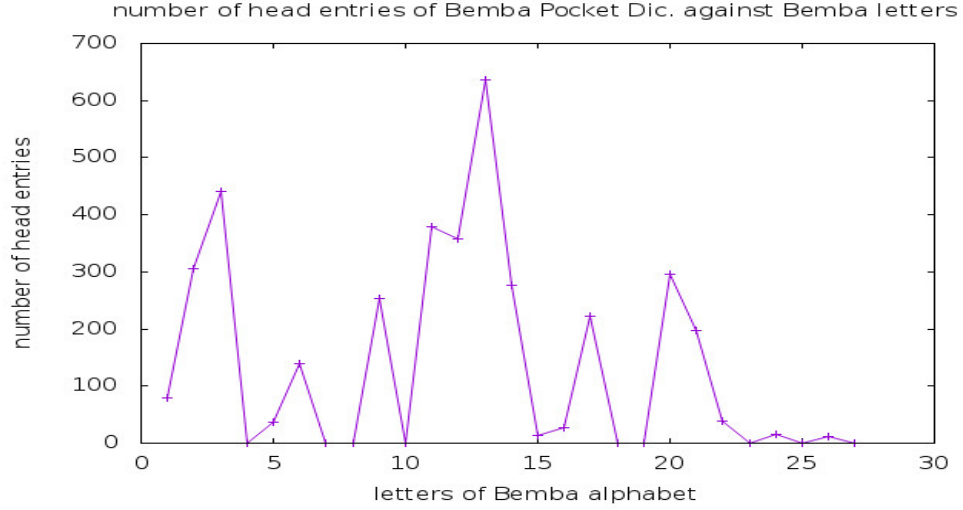


FIG. 1. The vertical axis is number of the Bemba head entries of Bemba Pocket Dictionary, Bemba-English and English-Bemba, by Rev. E. Hoch,[1]. The horizontal axis is the letters of the Bemba alphabet. Letters are represented by the sequence number in the alphabet as it appears in the dictionary,[1].

I. INTRODUCTION

Bemba speaking people of Africa inhabit Zambia and other regions. Bemba is a branch of the Bantu language family. In this paper, we study the bemba language. We count all the Bemba head entries of Bemba Pocket Dictionary, Bemba-English and English-Bemba, by Rev. E. Hoch., [1]. The result is the table, tableI. To visualise we plot the number of head entries against the respective letters in the dictionary sequence, [1] , in the adjoining figure, fig.1.

Next we look for the graphical law. We have started considering magnetic field pattern in [2], in the languages we converse with. We have studied there, a set of natural languages, [2] and have found the existence of a magnetisation curve under each language. We have termed this

phenomenon as the Graphical Law. Then, we moved on to investigate, [3], into dictionaries of five disciplines of knowledge and found the existence of a curve of magnetisation under each discipline. This was followed by finding of the graphical law in references from [4] to [100].

The planning of the paper is as follows. In the next section, we describe the Graphical Law analysis of the Bemba head entries of Bemba Pocket Dictionary, Bemba-English and English-Bemba, by Rev. E. Hoch , [1]. In the section III, we give an introduction to the standard curves of magnetisation of Ising model. The section IV is Acknowledgment. The last section is Bibliography.

II. THE GRAPHICAL LAW ANALYSIS

For the purpose of exploring graphical law, we assort the letters according to the number of head entries, in the descending order, denoted by f and the respective rank, [101], denoted by k . k is a positive integer starting from one. Moreover, the minimum non-zero number of head entries is twelve. Hence, we attach a limiting head entry number one. The limiting rank is maximum rank plus one, here it is nineteen. As a result both $\frac{\ln f}{\ln f_{max}}$ and $\frac{\ln k}{\ln k_{lim}}$ varies from zero to one. Then we tabulate in the adjoining table,II, and plot $\frac{\ln f}{\ln f_{max}}$ against $\frac{\ln k}{\ln k_{lim}}$ in the figure fig.2. We then ignore the letter with the highest number of head entries, tabulate in the adjoining table,II,and redo the plot, normalising the $\ln f$ s with $\ln f_{n-max}$, and starting from $k = 2$ in the figure fig.3. Normalising the $\ln f$ s with $\ln f_{2n-max}$, we tabulate in the adjoining table,II, and starting from $k = 3$ we draw in the figure fig.4. Normalising the $\ln f$ s with $\ln f_{3n-max}$ we record in the adjoining table,II, and plot starting from $k = 4$ in the figure fig.5. In this way we obtain up to the figure fig.7.

k	lnk	lnk/ lnk_{lim}	f	lnf	lnf/ lnf_{max}	lnf/ lnf_{nmax}	lnf/ lnf_{2nmax}	lnf/ lnf_{3nmax}	lnf/ lnf_{4nmax}	lnf/ lnf_{5nmax}
1	0	0	637	6.457	1	Blank	Blank	Blank	Blank	Blank
2	0.69	0.235	441	6.089	0.943	1	Blank	Blank	Blank	Blank
3	1.10	0.373	379	5.938	0.920	0.975	1	Blank	Blank	Blank
4	1.39	0.471	358	5.881	0.911	0.966	0.990	1	Blank	Blank
5	1.61	0.547	305	5.720	0.886	0.939	0.963	0.973	1	Blank
6	1.79	0.609	295	5.687	0.881	0.934	0.958	0.967	0.994	1
7	1.95	0.661	276	5.620	0.870	0.923	0.946	0.956	0.983	0.988
8	2.08	0.706	253	5.533	0.857	0.909	0.932	0.941	0.967	0.973
9	2.20	0.746	222	5.403	0.837	0.887	0.910	0.919	0.945	0.950
10	2.30	0.782	197	5.283	0.818	0.868	0.890	0.898	0.924	0.929
11	2.40	0.815	139	4.934	0.764	0.810	0.831	0.839	0.863	0.868
12	2.48	0.844	79	4.369	0.677	0.718	0.736	0.743	0.764	0.768
13	2.56	0.871	38	3.638	0.563	0.597	0.613	0.619	0.636	0.640
14	2.64	0.896	37	3.611	0.559	0.593	0.608	0.614	0.631	0.635
15	2.71	0.920	28	3.332	0.516	0.547	0.561	0.567	0.583	0.586
16	2.77	0.942	15	2.708	0.419	0.445	0.456	0.460	0.473	0.476
17	2.83	0.962	14	2.639	0.409	0.433	0.444	0.449	0.461	0.464
18	2.89	0.982	12	2.485	0.385	0.408	0.418	0.423	0.434	0.437
19	2.94	1	1	0	0	0	0	0	0	0

TABLE II. The Bemba head entries of Bemba Pocket Dictionary, Bemba-English and English-Bemba, by Rev. E. Hoch, [1]: ranking, natural logarithms, normalisations

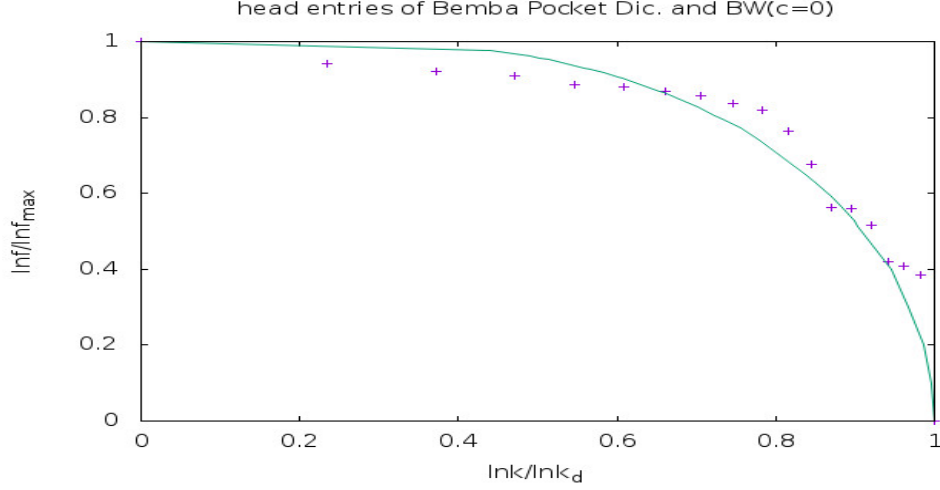


FIG. 2. The vertical axis is $\frac{\ln f}{\ln f_{\max}}$ and the horizontal axis is $\frac{\ln k}{\ln k_{\lim}}$. The + points represent the Bemba head entries of Bemba Pocket Dictionary, Bemba-English and English-Bemba, by Rev. E. Hoch, [1], with the fit curve BW($c=0$), being the curve in the Bragg-Williams approximation of the Ising Model in the absence of external magnetic field, H , with $c = \frac{H}{\gamma\epsilon} = 0$.

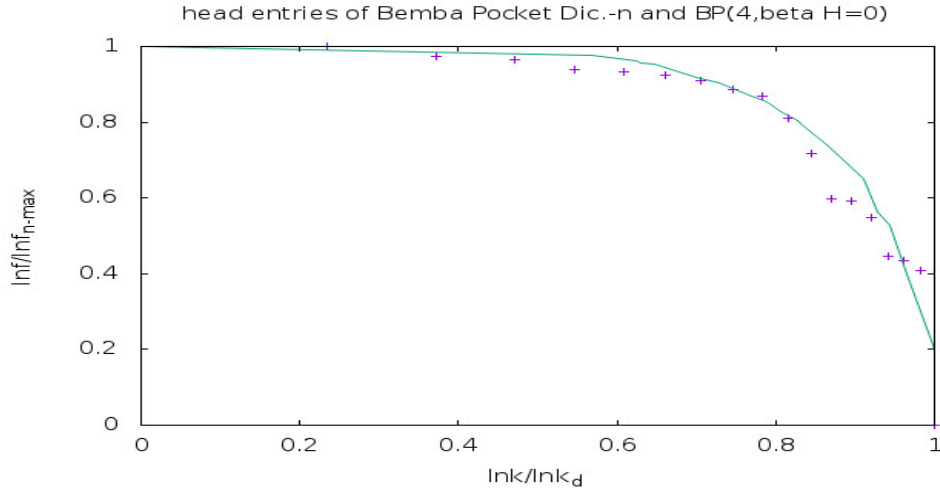


FIG. 3. The vertical axis is $\frac{\ln f}{\ln f_{n-\max}}$ and the horizontal axis is $\frac{\ln k}{\ln k_{\lim}}$. The + points represent the Bemba head entries of Bemba Pocket Dictionary, Bemba-English and English-Bemba, by Rev. E. Hoch, [1], with the fit curve BP(4, $\beta H = 0$), being the Bethe-Peierls curve of the Ising Model in the presence of four nearest neighbours and in the absence of external magnetic field, H , i.e. $\beta H = 0$.

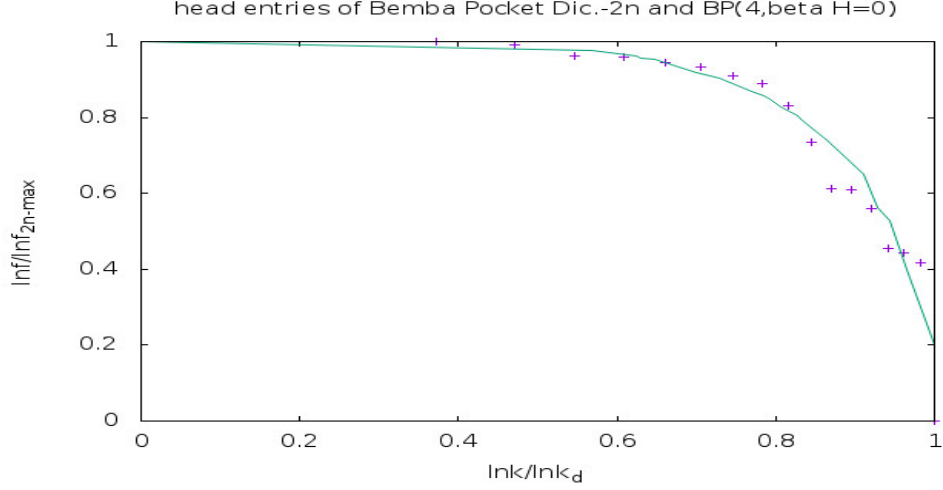


FIG. 4. The vertical axis is $\frac{\ln f}{\ln f_{2n-\max}}$ and the horizontal axis is $\frac{\ln k}{\ln k_{lim}}$. The + points represent the Bemba head entries of Bemba Pocket Dictionary, Bemba-English and English-Bemba, by Rev. E. Hoch, [1], with the fit curve $BP(4, \beta H = 0)$, being the Bethe-Peierls curve of the Ising Model in the presence of four nearest neighbours and in the absence of external magnetic field, H , i.e. $\beta H = 0$.

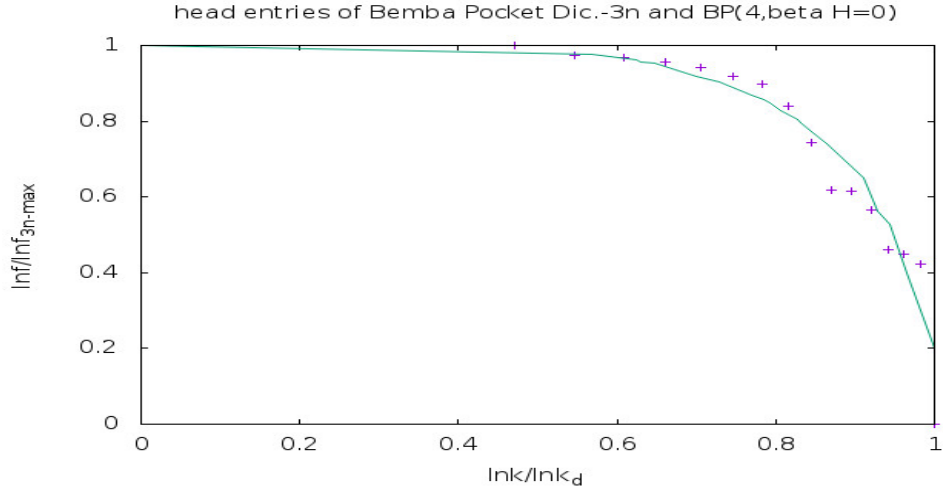


FIG. 5. The vertical axis is $\frac{\ln f}{\ln f_{3n-\max}}$ and the horizontal axis is $\frac{\ln k}{\ln k_{lim}}$. The + points represent the Bemba head entries of Bemba Pocket Dictionary, Bemba-English and English-Bemba, by Rev. E. Hoch, [1], with the fit curve $BP(4, \beta H = 0)$, being the Bethe-Peierls curve of the Ising Model in the presence of four nearest neighbours and in the absence of external magnetic field, H , i.e. $\beta H = 0$.

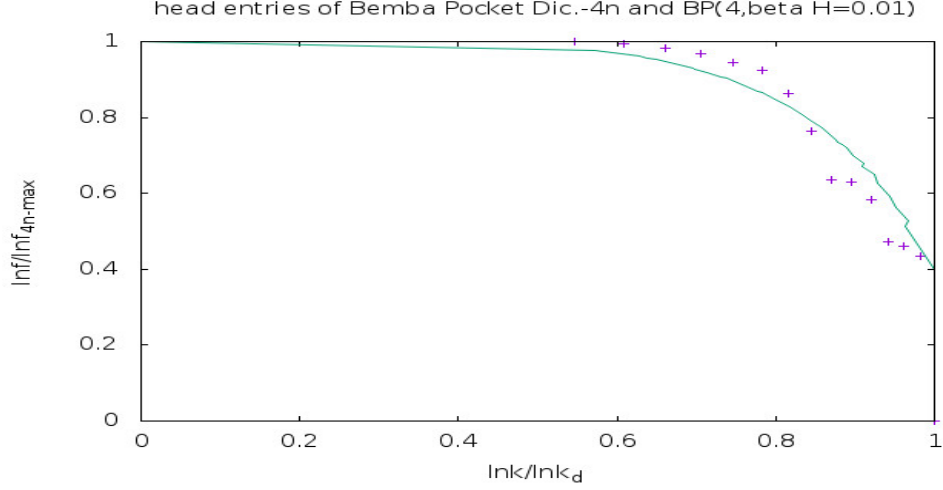


FIG. 6. The vertical axis is $\frac{\ln f}{\ln f_{4n-\max}}$ and the horizontal axis is $\frac{\ln k}{\ln k_{im}}$. The + points represent the Bemba head entries of Bemba Pocket Dictionary, Bemba-English and English-Bemba, by Rev. E. Hoch, [1], with the fit curve $BP(4, \beta H = 0.01)$, being the Bethe-Peierls curve of the Ising Model in the presence of four nearest neighbours and in the presence of external magnetic field, H , with $\beta H = 0.01$.

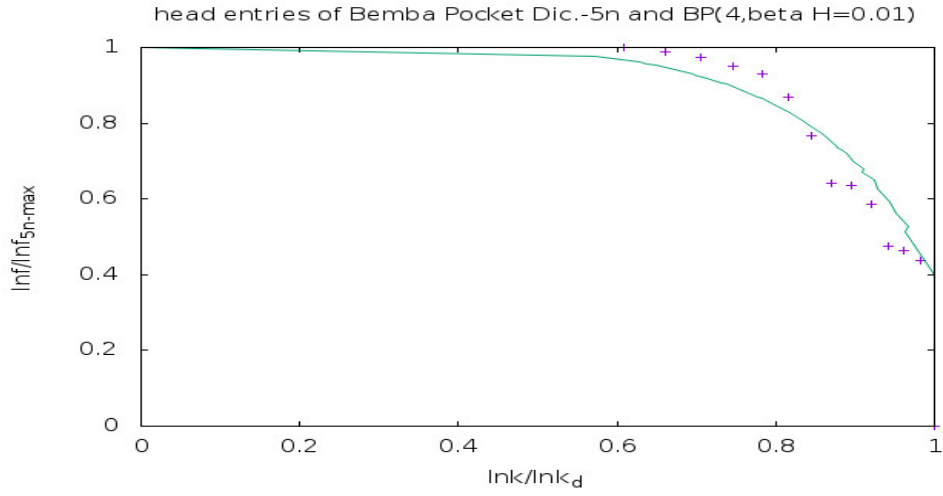


FIG. 7. The vertical axis is $\frac{\ln f}{\ln f_{5n-\max}}$ and the horizontal axis is $\frac{\ln k}{\ln k_{im}}$. The + points represent the Bemba head entries of Bemba Pocket Dictionary, Bemba-English and English-Bemba, by Rev. E. Hoch, [1], with the fit curve, $BP(4, \beta H = 0.01)$, being the Bethe-Peierls curve of the Ising Model in the presence of four nearest neighbours and in the presence of external magnetic field, H , with $\beta H = 0.01$.

A. tentative conclusion

From the figures (fig.2-fig.7), we observe that there seems to be a curve of magnetisation, behind the Bemba head entries of Bemba Pocket Dictionary, Bemba-English and English-Bemba, by Rev. E. Hoch, [1]. This is the magnetisation curve in the Bethe-Peierls approximation of the Ising model, BP(4, $\beta H = 0$), in the presence of four nearest neighbours and in the absence of external magnetic field, H , i.e. $\beta H = 0$, with the associated correspondence,

$$\frac{\ln f}{\ln f_{2n-max}} \longleftrightarrow \frac{M}{M_{max}},$$

$$\ln k \longleftrightarrow T.$$

k corresponds to temperature in an exponential scale, [108].

Matching of the plots in the figures fig.(2-7), with comparator curves i.e. the magnetisation curves of the Ising Model in various approximations, are with dispersions and dispersions do not reduce, significantly, over higher orders of normalisations.

To explore for possible existence of spin-glass transition, in the presence of little external magnetic field, $\frac{\ln f}{\ln f_{r-max}}$ with $r = 0, 1, 2$ are drawn against $\ln k$ in the figures fig.8-fig.10.

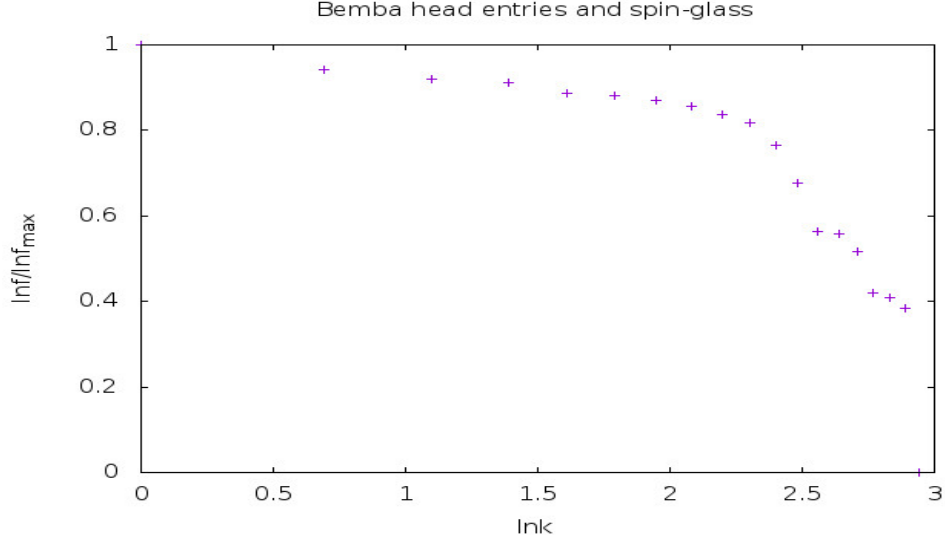


FIG. 8. The vertical axis is $\frac{\ln f}{\ln f_{\max}}$ and the horizontal axis is $\ln k$. The + points represent the the Bemba head entries of Bemba Pocket Dictionary, Bemba-English and English-Bemba, by Rev. E. Hoch, [1].

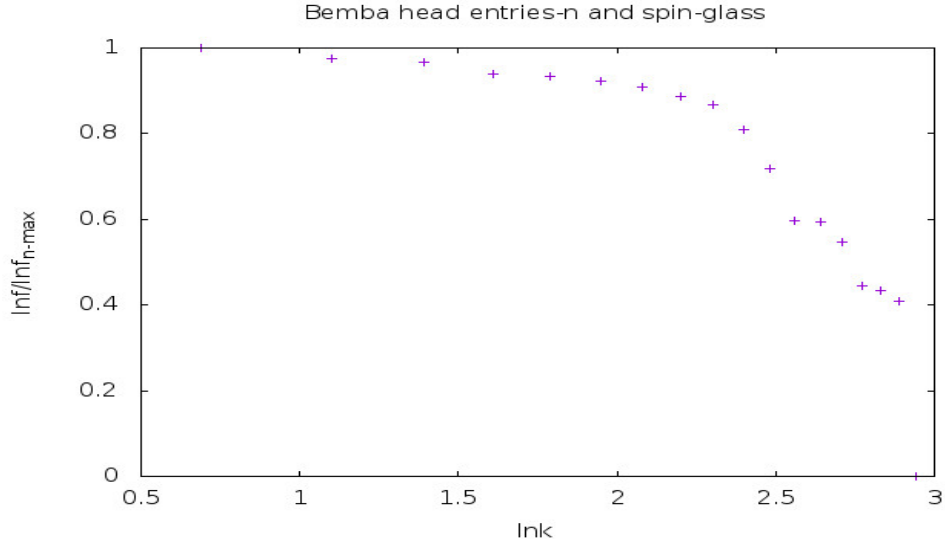


FIG. 9. The vertical axis is $\frac{\ln f}{\ln f_{n-\max}}$ and the horizontal axis is $\ln k$. the Bemba head entries of Bemba Pocket Dictionary, Bemba-English and English-Bemba, by Rev. E. Hoch, [1].

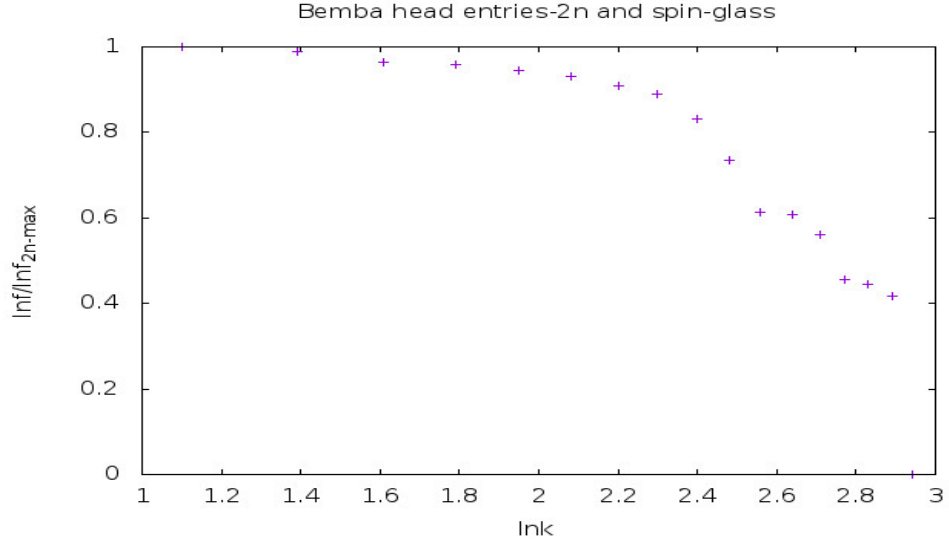


FIG. 10. The vertical axis is $\frac{\ln f}{\ln f_{2n-\max}}$ and the horizontal axis is $\ln k$. The + points represent the Bemba head entries of Bemba Pocket Dictionary, Bemba-English and English-Bemba, by Rev. E. Hoch, [1].

B. conclusion

In the figures Fig.8-Fig.10, the points has a smoothed transition, [124]. Above the transition point(s), the lines are almost horizontal and below the transition point(s), points-line rises like the branch of a rectangular hyperbola. Hence, the Bemba head entries, [1], are better described by a Spin-Glass magnetisation curve, [109], in the presence of little external magnetic field. Moreover, the associated correspondence is,

$$\frac{\ln f}{\ln f_{max}} \longleftrightarrow \frac{M}{M_{max}},$$
$$\ln k \longleftrightarrow T.$$

k corresponds to temperature in an exponential scale, [108].

III. APENDIX: MAGNETISATION

A. Bragg-Williams approximation

Let us consider a coin. Let us toss it many times. Probability of getting head or, tale is half i.e. we will get head and tale equal number of times. If we attach value one to head, minus one to tale, the average value we obtain, after many tossing is zero. Instead let us consider a one-sided loaded coin, say on the head side. The probability of getting head is more than one half, getting tale is less than one-half. Average value, in this case, after many tossing we obtain is non-zero, the precise number depends on the loading. The loaded coin is like ferromagnet, the unloaded coin is like para magnet, at zero external magnetic field. Average value we obtain is like magnetisation, loading is like coupling among the spins of the ferromagnetic units. Outcome of single coin toss is random, but average value we get after long sequence of tossing is fixed. This is long-range order. But if we take a small sequence of tossing, say, three consecutive tossing, the average value we obtain is not fixed, can be anything. There is no short-range order.

Let us consider a row of spins, one can imagine them as spears which can be vertically up or, down. Assume there is a long-range order with probability to get a spin up is two third. That would mean when we consider a long sequence of spins, two third of those are with spin up. Moreover, assign with each up spin a value one and a down spin a value minus one. Then total spin we obtain is one third. This value is referred to as the value of long-range order parameter. Now consider a short-range order existing which is identical with the long-range order. That would mean if we pick up any three consecutive spins, two will be up, one down. Bragg-Williams approximation means short-range order is identical with long-range order, applied to a lattice of spins, in general. Row of spins is a lattice of one dimension.

Now let us imagine an arbitrary lattice, with each up spin assigned a value one and a down spin a value minus one, with an unspecified long-range order parameter defined as above by $L = \frac{1}{N} \sum_i \sigma_i$, where σ_i is i-th spin, N being total number of spins. L can vary from minus one to one. $N = N_+ + N_-$, where N_+ is the number of up spins, N_- is the number of down spins. $L = \frac{1}{N}(N_+ - N_-)$. As a result, $N_+ = \frac{N}{2}(1 + L)$ and $N_- = \frac{N}{2}(1 - L)$. Magnetisation or, net magnetic moment, M is $\mu \sum_i \sigma_i$ or, $\mu(N_+ - N_-)$ or, $\mu N L$, $M_{max} = \mu N$. $\frac{M}{M_{max}} = L$.

$\frac{M}{M_{max}}$ is referred to as reduced magnetisation. Moreover, the Ising Hamiltonian,[102], for the lattice of spins, setting μ to one, is $-\epsilon \sum_{n,n} \sigma_i \sigma_j - H \sum_i \sigma_i$, where n.n refers to nearest neighbour pairs.

The difference ΔE of energy if we flip an up spin to down spin is, [103], $2\epsilon\gamma\bar{\sigma} + 2H$, where γ is the number of nearest neighbours of a spin. According to Boltzmann principle, $\frac{N_-}{N_+}$ equals $\exp(-\frac{\Delta E}{k_B T})$, [104]. In the Bragg-Williams approximation,[105], $\bar{\sigma} = L$, considered in the thermal average sense. Consequently,

$$\ln \frac{1+L}{1-L} = 2 \frac{\gamma\epsilon L + H}{k_B T} = 2 \frac{L + \frac{H}{\gamma\epsilon}}{\frac{T}{\gamma\epsilon/k_B}} = 2 \frac{L + c}{\frac{T}{T_c}} \quad (1)$$

where, $c = \frac{H}{\gamma\epsilon}$, $T_c = \gamma\epsilon/k_B$, [106]. $\frac{T}{T_c}$ is referred to as reduced temperature.

Plot of L vs $\frac{T}{T_c}$ or, reduced magnetisation vs. reduced temperature is used as reference curve. In the presence of magnetic field, $c \neq 0$, the curve bulges outward. Bragg-Williams is a Mean Field approximation. This approximation holds when number of neighbours interacting with a site is very large, reducing the importance of local fluctuation or, local order, making the long-range order or, average degree of freedom as the only degree of freedom of the lattice. To have a feeling how this approximation leads to matching between experimental and Ising model prediction one can refer to FIG.12.12 of [103]. W. L. Bragg was a professor of Hans Bethe. Rudolf Peierls was a friend of Hans Bethe. At the suggestion of W. L. Bragg, Rudolf Peierls following Hans Bethe improved the approximation scheme, applying quasi-chemical method.

B. Bethe-peierls approximation in presence of four nearest neighbours, in absence of external magnetic field

In the approximation scheme which is improvement over the Bragg-Williams, [102],[103],[104],[105],[106], due to Bethe-Peierls, [107], reduced magnetisation varies with reduced temperature, for γ neighbours, in absence of external magnetic field, as

$$\frac{\ln \frac{\gamma}{\gamma-2}}{\ln \frac{factor-1}{factor^{\frac{\gamma-1}{\gamma}} - factor^{\frac{1}{\gamma}}}} = \frac{T}{T_c}; factor = \frac{\frac{M}{M_{max}} + 1}{1 - \frac{M}{M_{max}}} \quad (2)$$

$\ln \frac{\gamma}{\gamma-2}$ for four nearest neighbours i.e. for $\gamma = 4$ is 0.693. For a snapshot of different kind of magnetisation curves for magnetic materials the reader is urged to give a google

BW	BW($c=0.01$)	BP($4, \beta H = 0$)	reduced magnetisation
0	0	0	1
0.435	0.439	0.563	0.978
0.439	0.443	0.568	0.977
0.491	0.495	0.624	0.961
0.501	0.507	0.630	0.957
0.514	0.519	0.648	0.952
0.559	0.566	0.654	0.931
0.566	0.573	0.7	0.927
0.584	0.590	0.7	0.917
0.601	0.607	0.722	0.907
0.607	0.613	0.729	0.903
0.653	0.661	0.770	0.869
0.659	0.668	0.773	0.865
0.669	0.676	0.784	0.856
0.679	0.688	0.792	0.847
0.701	0.710	0.807	0.828
0.723	0.731	0.828	0.805
0.732	0.743	0.832	0.796
0.756	0.766	0.845	0.772
0.779	0.788	0.864	0.740
0.838	0.853	0.911	0.651
0.850	0.861	0.911	0.628
0.870	0.885	0.923	0.592
0.883	0.895	0.928	0.564
0.899	0.918		0.527
0.904	0.926	0.941	0.513
0.946	0.968	0.965	0.400
0.967	0.998	0.965	0.300
0.987		1	0.200
0.997		1	0.100
1	1	1	0

TABLE III. Reduced magnetisation vs reduced temperature data s for Bragg-Williams approximation, in absence of and in presence of magnetic field, $c = \frac{H}{\gamma\epsilon} = 0.01$, and Bethe-Peierls approximation in absence of magnetic field, for four nearest neighbours.

search "reduced magnetisation vs reduced temperature curve". In the following, we describe data s generated from the equation(1) and the equation(2) in the table, III, and curves of magnetisation plotted on the basis of those data s. BW stands for reduced temperature in Bragg-Williams approximation, calculated from the equation(1). BP(4) represents reduced temperature in the Bethe-Peierls approximation, for four nearest neighbours, computed from the equation(2). The data set is used to plot fig.11. Empty spaces in the table, III, mean corresponding point pairs were not used for plotting a line.

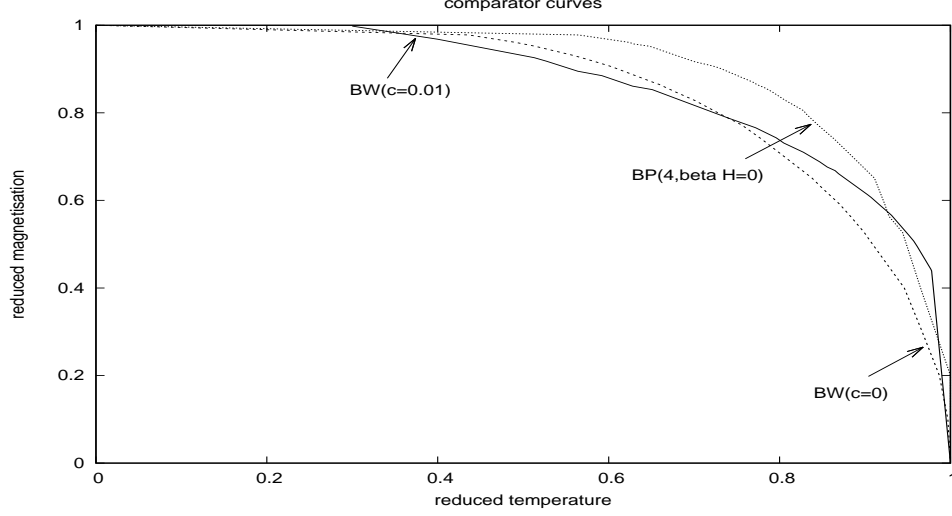


FIG. 11. Reduced magnetisation vs reduced temperature curves for Bragg-Williams approximation, in absence(dark) of and presence(inner in the top) of magnetic field, $c = \frac{H}{\gamma\epsilon} = 0.01$, and Bethe-Peierls approximation in absence of magnetic field, for four nearest neighbours (outer in the top).

C. Bethe-peierls approximation in presence of four nearest neighbours, in the presence of external magnetic field

In the Bethe-Peierls approximation scheme , [107], reduced magnetisation varies with reduced temperature, for γ neighbours, in presence of external magnetic field, as

$$\frac{\ln \frac{\gamma}{\gamma-2}}{\ln \frac{e^{-\frac{2\beta H}{\gamma}} factor^{\frac{\gamma-1}{\gamma}} - e^{-\frac{2\beta H}{\gamma}} factor^{\frac{1}{\gamma}}}{factor-1}} = \frac{T}{T_c}; factor = \frac{\frac{M}{M_{max}} + 1}{1 - \frac{M}{M_{max}}}. \quad (3)$$

Derivation of this formula ala [107] is given in the appendix of [7].

$\ln \frac{\gamma}{\gamma-2}$ for four nearest neighbours i.e. for $\gamma = 4$ is 0.693. For four neighbours,

$$\frac{0.693}{\ln \frac{e^{-\frac{2\beta H}{\gamma}} factor^{\frac{\gamma-1}{\gamma}} - e^{-\frac{2\beta H}{\gamma}} factor^{\frac{1}{\gamma}}}{factor-1}} = \frac{T}{T_c}; factor = \frac{\frac{M}{M_{max}} + 1}{1 - \frac{M}{M_{max}}}. \quad (4)$$

In the following, we describe datas in the table, IV, generated from the equation(4) and curves of magnetisation plotted on the basis of those datas. BP(m=0.03) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.06$. calculated from the equation(4). BP(m=0.025) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that

$\beta H = 0.05$. calculated from the equation(4). BP(m=0.02) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.04$. calculated from the equation(4). BP(m=0.01) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.02$. calculated from the equation(4). BP(m=0.005) stands for reduced temperature in Bethe-Peierls approximation, for four nearest neighbours, in presence of a variable external magnetic field, H, such that $\beta H = 0.01$. calculated from the equation(4). The data set is used to plot fig.9. Empty spaces in the table, IV, mean corresponding point pairs were not used for plotting a line.

BP(m=0.03)	BP(m=0.025)	BP(m=0.02)	BP(m=0.01)	BP(m=0.005)	reduced magnetisation
0	0	0	0	0	1
0.583	0.580	0.577	0.572	0.569	0.978
0.587	0.584	0.581	0.575	0.572	0.977
0.647	0.643	0.639	0.632	0.628	0.961
0.657	0.653	0.649	0.641	0.637	0.957
0.671	0.667		0.654	0.650	0.952
	0.716			0.696	0.931
0.723	0.718	0.713	0.702	0.697	0.927
0.743	0.737	0.731	0.720	0.714	0.917
0.762	0.756	0.749	0.737	0.731	0.907
0.770	0.764	0.757	0.745	0.738	0.903
0.816	0.808	0.800	0.785	0.778	0.869
0.821	0.813	0.805	0.789	0.782	0.865
0.832	0.823	0.815	0.799	0.791	0.856
0.841	0.833	0.824	0.807	0.799	0.847
0.863	0.853	0.844	0.826	0.817	0.828
0.887	0.876	0.866	0.846	0.836	0.805
0.895	0.884	0.873	0.852	0.842	0.796
0.916	0.904	0.892	0.869	0.858	0.772
0.940	0.926	0.914	0.888	0.876	0.740
	0.929			0.877	0.735
	0.936			0.883	0.730
	0.944			0.889	0.720
	0.945				0.710
	0.955			0.897	0.700
	0.963			0.903	0.690
	0.973			0.910	0.680
				0.909	0.670
	0.993			0.925	0.650
		0.976	0.942		0.651
	1.00				0.640
		0.983	0.946	0.928	0.628
		1.00	0.963	0.943	0.592
			0.972	0.951	0.564
			0.990	0.967	0.527
				0.964	0.513
			1.00		0.500
				1.00	0.400
					0.300
					0.200
					0.100
					0

TABLE IV. Bethe-Peierls approx. in presence of little external magnetic fields

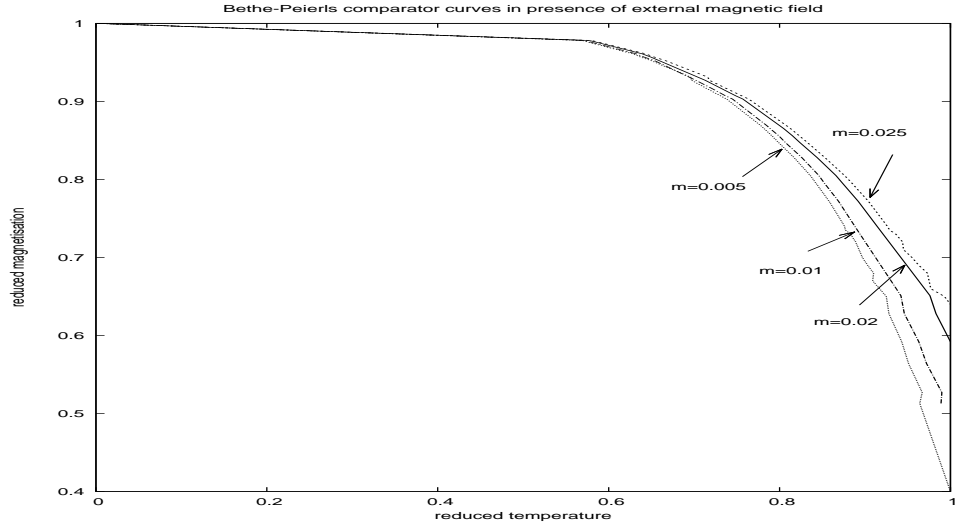


FIG. 12. Reduced magnetisation vs reduced temperature curves for Bethe-Peierls approximation in presence of little external magnetic fields, for four nearest neighbours, with $\beta H = 2m$.

D. Spin-Glass

In the case coupling between(among) the spins, not necessarily n.n, for the Ising model is(are) random, we get Spin-Glass. When a lattice of spins randomly coupled and in an external magnetic field, goes over to the Spin-Glass phase, magnetisation increases steeply like $\frac{1}{T-T_c}$ i.e. like the branch of rectangular hyperbola, up to the the phase transition temperature, followed by very little increase,[109–111], in magnetisation, as the ambient temperature continues to drop.

Theoretical study of Spin Glass started with the paper by Edwards, Anderson,[112]. They were trying to explain two experimental results concerning continuous disordered freezing(phase transition) and sharp cusp in static magnetic susceptibility. This was followed by a paper by Sherrington, Kickpatrick, [113], who dealt with Ising model with interactions being present among all neighbours. The interaction is random, follows Gaussian distribution and does not distinguish one pair of neighbours from another pair of neighbours, irrespective of the distance between two neighbours. In presence of external magnetic field, they predicted in their next paper, [114], below spin-glass transition temperature a spin-glass phase with non-zero magnetisation. Almeida et al, [115], Gray and Moore, [116], finally Parisi, [117], [118] improved and gave final touch, [119], to their line of work. Parisi and collaborators, [120]–[124], wrote a series of papers in postscript, all revolving around a consistent assumption of constant magnetisation in the spin-glass phase in presence of little constant external magnetic field.

In another sequence of theoretical work, by Fisher et al,[125–127], concluded that for Ising model with nearest neighbour or, short range interaction of random type spin-glass phase does not exist in presence of external magnetic field.

For recent series of experiments on spin-glass, the references, [128, 129], are the places to look into.

For an in depth account, accessible to a commoner, the series of articles by late P. W. Anderson in Physics Today, [130]–[136], is probably the best place to look into. For a book to enter into the subject of spin-glass, one may start at [137].

IV. ACKNOWLEDGMENT

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