A RIGHT TRIANGLE AS AN ADDER OF THE AREA OF REGULAR TWO-DIMENSIONAL AND THREE-DIMENSIONAL GEOMETRIC SHAPES

Andrey V. Voron (anvoron1@yandex.ru)

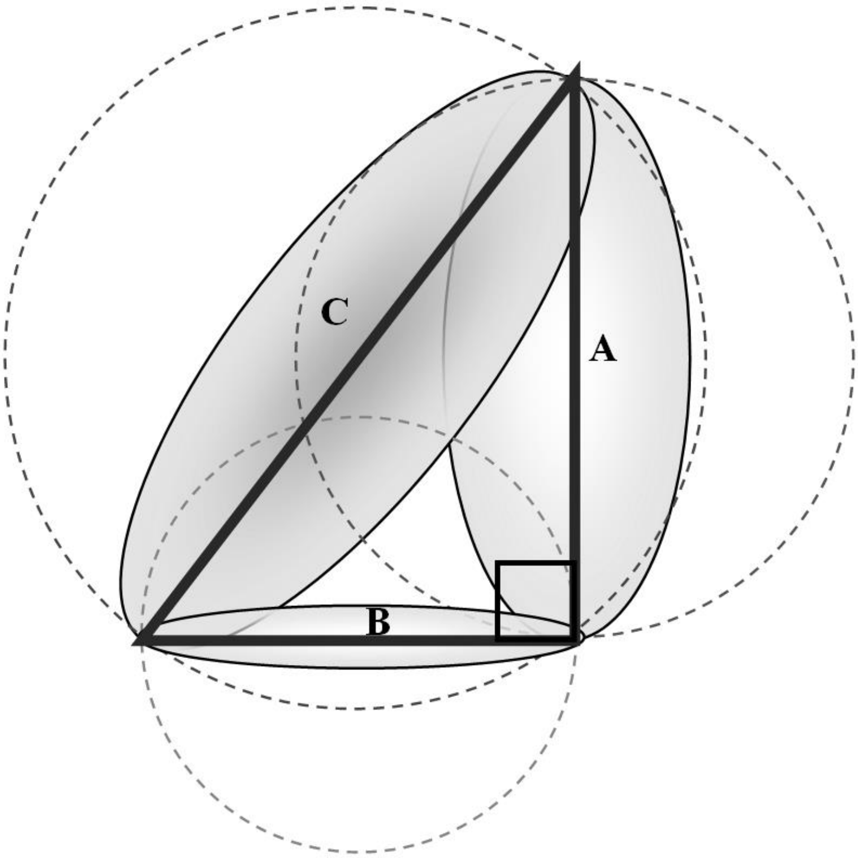
May 15, 2025

Abstract

The article shows the possibility of using a right triangle and the logic of the Pythagorean theorem to find the areas of regular two-dimensional and three-dimensional geometric shapes (in particular, for Platonic solids).

1 Introduction

It is assumed that the Pythagorean theorem is a special case of a more general mathematical pattern of summing the area of regular two–dimensional and three-dimensional geometric shapes in a certain geometric way. That is: the logic of the Pythagorean theorem can be applied to find the areas of regular two–dimensional and three–dimensional geometric shapes (in particular, for Platonic solids) if the values of the length of the sides or edges of these shapes, the height of the shapes, as well as the value of one of the radii - external, average or internal - are applied as the dimension of the legs. The figure, for example, shows a model of using the Pythagorean theorem to find the areas of a circle and a sphere.



The drawing is a model of using the Pythagorean theorem to find the areas of a circle and a sphere, where a solid line represents a right triangle and circles A, B, C, and a discontinuous line of various colors represents spheres.

Checking the above assumption by calculating the values of the areas of regular geometric shapes for the values of the natural series (from 1 to 5) of the lengths of the sides or the diameter of the circle (for two-dimensional shapes), the lengths of the edges or the diameter of the sphere (for three-dimensional shapes) showed its validity (table).

The table shows the values of the areas of regular geometric shapes with values of the natural series from 1 to 5 side lengths or circle diameter (for two–dimensional shapes), edge lengths or sphere diameter (for three-dimensional shapes)

Shapes	Area parameters of regular geometric shapes, (S), conventional units									
	Two-dimensional				Three- dimensional					
Side length, rib or inner diameter, (L), conventional units	Circle	Equilateral Triangle	Square	Pentagon	Cube	Sphere	Tetrahedron	Octahedron	Icosahedron	Dodecahedron
1	0,785… 0,25π	0,433…	1	1,720…	6	3,141… =1π	1,732… =√3	3,464… =√12	8,66… =√75	20,646..
2	3,141… =1π	1.732… √3	4	6.882…	24	12,566… =4π	6,928… =√48	13,856… =√192	34,64… =√1200	82,583..
3	7,069… =2,25π	3,897…	9	15,485…	54	28,274… =9π	15,588…	31,177… =√972	77,94… =√6075	185,812…
4	12,566… =4π	6,928… √48	16	27,527…	96	50,265… =16π	27,713… =√768	55,426… =√3072	138,56… √19200	330,331…
5	19,635… =6,25π	10,825…	25	43,012…	150	78,54… =25π	43,301…	86,603… =√7500	216,5… √46875	516,143..

Based on the results of this study, it can be noted that regular geometric two-dimensional and three-dimensional shapes demonstrate some mathematical properties that differ from other shapes (not regular ones). In particular:

– the possibility of summing their areas by means of a right triangle.;

– the similarity of the size of the circle inscribed in them (with a diameter of 2 for regular two–dimensional shapes) and the inscribed sphere (with a diameter of 3 for regular three–dimensional shapes), provided that the area value is equal to the volume value (that is, the equality of the area and perimeter values for two–dimensional shapes, volume and area for three-dimensional [1, 2]).

It can be assumed that there is probably a similar "geometric way" to obtain the volume value of the third three-dimensional shape (correct) based on the volume values (or other parameter values) of similar two other regular shapes (n-shapes) by means of a certain geometric shape (probably already three-dimensional). In this regard, there is a certain freedom of action and a theoretical foundation for researchers of mathematical patterns.

References

[1] Ворон, А.В. Тождество значений площади и периметра ряда двумерных фигур, объема и площади – трехмерных // «Академия Тринитаризма», М., Эл № 77-6567, публ.25873, 14.11.2019.

[2] Ворон, А.В. Тождество значений площади и периметра ряда двумерных фигур (квадрат, круг, прямоугольный, тупоугольный, равнобедренный и равносторонний треугольник), объема и площади – трехмерных (платоновы тела, конус, цилиндр, 3-4-6-гранная пирамида и сфера) // «Академия Тринитаризма», М., Эл № 77-6567, публ.27338, 23.09.2021.