AN ACUTE-ANGLED TRIANGLE AS AN ADDER OF THE VOLUME OF REGULAR THREE-DIMENSIONAL GEOMETRIC SHAPES

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Abstract

The article shows the possibility (using an acute-angled triangle, the logic of the Pythagorean theorem) of finding the volume of regular three-dimensional geometric shapes based on the mathematical equation $c=\sqrt[3]{(a^3+b^3)}$. A number of theorems have been formulated that complement the Pythagorean theorem.

1 Introduction

The Pythagorean theorem is supposed to have some incompleteness: it "works" in the classical formulation for the area of squares or three regular similar shapes. There is probably a pattern for a triangle and the volume of three similar figures based on the idea that the laws of dialectics apply to mathematical objects-ideas: "as we relate dialectical laws to material objects, so, accordingly, we relate these laws to selected objects-ideas" [1, 2]. In this regard, such values of geometric shapes as "Area" and "Volume" can be attributed to such idea objects. For example, when this pair of opposites is equal, there is a certain similarity in regular two-dimensional and three–dimensional geometric shapes: when the values of area and volume are equal, the radius of the inscribed circle for two-dimensional shapes is 2, and the inscribed sphere for three-dimensional is 3 [1, 2]. In this regard, a mathematical object – a right-angled triangle (in which the areas of two similar regular two–dimensional figures are equal to the area of a third similar regular two-dimensional figure) can be considered as dual to another mathematical object-the idea of a triangle, in which the volumes of two regular similar three-dimensional figures are equal to the volume of a third regular similar three-dimensional figure.

2 The main part

It is assumed that the Pythagorean theorem is a special case of a more general mathematical pattern of summing in a certain geometric way the area of regular two–dimensional and three-dimensional geometric shapes, and the volume of regular three-dimensional geometric shapes [3]. In this regard, the classical Pythagorean theorem is formulated in a new way, as well as the generalized Pythagorean theorem for calculating the area of regular two-dimensional and three–dimensional geometric shapes.:

The Pythagorean theorem, formulated in more detail in comparison with the classical version of the formulation. In a right-angled triangle, the area of a square with a side length equal to the length of the hypotenuse is equal to the sum of the areas of squares with side lengths equal to the other two sides of the triangle.

Generalized Pythagorean theorem for calculating the area of regular two-dimensional and three-dimensional geometric shapes. In a right-angled triangle with a hypotenuse equal to the diameter of a circle inscribed in a two-dimensional figure or in a three-dimensional sphere with the corresponding area of the figure, the areas are equal to two other similar figures with circles or spheres inscribed in them with diameters equal to the other two sides of the triangle (Figure 1). In this theorem, the choice of diameter as a criterion for the dimension of the sides of a triangle is determined by the convenience of representation. In other cases, it is possible to choose other criteria for the geometry of a regular shape – radius, height, and the like.

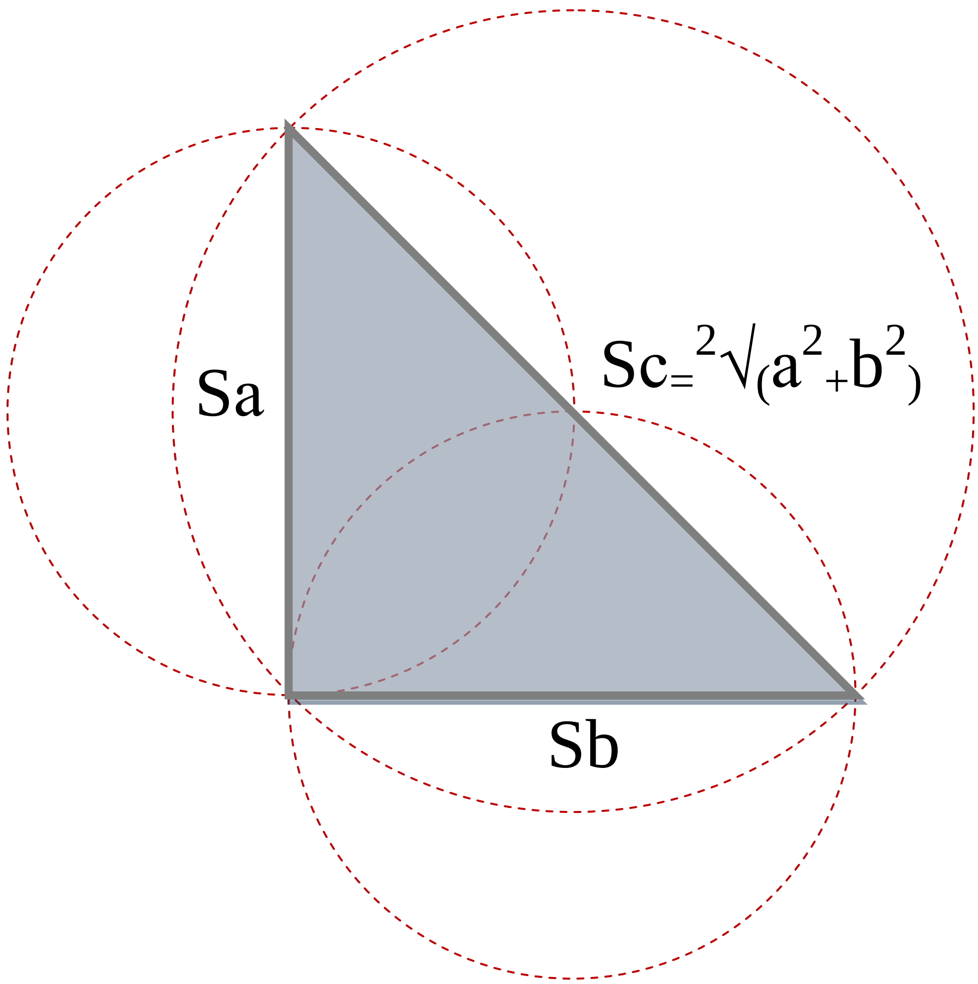

Figure 1 is a visualization of a geometric model of a right–angled triangle, where one side is equal to the other two based on the mathematical equation $c=\sqrt[2]{(a^2+b^2)}$, and the lengths of the sides of the triangle are equal to the diameters of the spheres.
The figure shows a triangle where: a=b

The logic of the Pythagorean theorem can also be applied to find the sum of the volumes of two regular three-dimensional geometric shapes if linear values of the lengths of one of the parameters of these shapes (edge, radius, or diameter of a sphere inscribed in the shape) are applied as the dimension of the sides of a triangle. The figure shows a geometric model of using the logic of the Pythagorean theorem to find the volume of regular three–dimensional geometric shapes based on the mathematical equation – $c=\sqrt[3]{(a^3+b^3)}$ (Figure 2). In this regard, the Pythagorean theorems for calculating the volume of regular three-dimensional geometric shapes and the Pythagorean theorem for cubes are formulated.:

Generalized Pythagorean theorem for calculating the volume of regular three-dimensional geometric shapes. In a triangle with one side equal to the value of the cubic root of the sum of the cubes of the lengths of its other two sides, the volume values of two regular similar three-dimensional figures of the third are equal (based on the diameters of the spheres inscribed in them equal to the sides of the triangle).

Pythagorean theorem for cubes. In an acute-angled triangle in which the equality $c=\sqrt[3]{(a^3+b^3)}$ holds, a cube with an edge equal to the larger side of the triangle is equal to the sum of cubes with edges equal to its smaller sides. In the theorems for calculating the volume of regular three-dimensional geometric shapes, the choice of diameter as a criterion for the dimension of the sides of a triangle is determined by the convenience of representation (Figure 2).

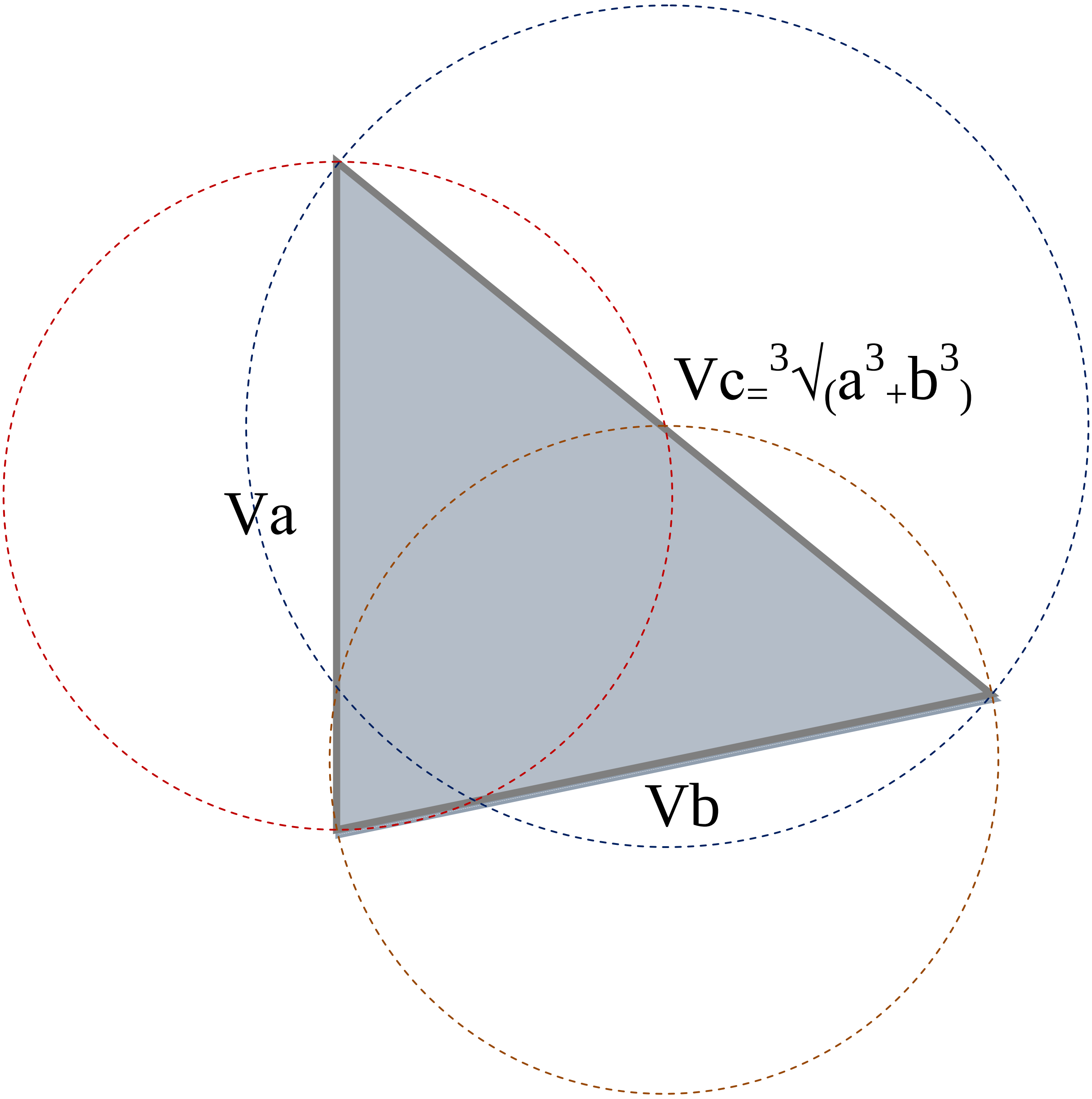

Figure 2 is a visualization of a geometric model of an acute–angled triangle, where the volume of one sphere is equal to the other two based on the mathematical equation – $c=\sqrt[3]{(a^3+b^3)}$, and the lengths of the sides of the triangle are equal to the diameters of the spheres.
The figure shows a triangle where: a=b

References

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