On Fractal Spacetime and the Gauge-Gravity Duality

Ervin Goldfain

Global Institute for Research, Education and Scholarship (GIRES), USA

E-mail ervingoldfain@gmail.com

Abstract

We introduce here a speculative proposal for merging the gauge-gravity duality with the onset of fractal spacetime well above the electroweak scale. The proposal generates a positive cosmological constant in 1D spacetime, consistent with the dimensional reduction conjecture expected to take place near the Planck scale. Our proposal also unveils a) a possible path towards field unification in fractal dimensions and b), an unforeseen bridge between fluid/gravity correspondence and the complex Ginzburg-Landau equation (CGLE).

Key words: Holographic duality, cosmological constant, field unification, fractal spacetime, fluid/gravity correspondence, complex Ginzburg-Landau equation.

1. Introduction and Motivation

Echoing the points brought up in [1], large energy fluctuations on ultrashort distance scales lead to a breakdown of the smooth geometric structure typically assumed in classical spacetime models. Geodesics—paths representing the shortest trajectories in curved spacetime—are no longer smooth or continuous in the classical sense. The violent, scale-dependent oscillations in curvature render these curves *non-differentiable*, and in many cases, singular. The unavoidable conclusion is that, somewhere above the low energy scale of particle physics, the familiar four-dimensional spacetime turns into a *fractal manifold* endowed with evolving continuous dimensions, nonlocal behavior and memory effects. The concept of fractal spacetime reflects a scale-dependent geometry in which traditional notions of distance, time, and motion must be reformulated. By the same token, incessant fluctuations in energy and momentum *create* and *annihilate* quantum particles in and out of the quantum vacuum. It follows that fractal spacetime must be the primary source of both gravitational and particle phenomena in conditions extending into the realm of *complexity* and *emergence,* well beyond General Relativity (GR) and Quantum Field Theory (QFT). This perspective offers a compelling bridge between QFT, GR, and the novel mathematical framework of *complex dynamics* needed to describe the structure of spacetime at the most fundamental level. A key observation along these lines is that strongly fluctuating regimes tend to suppress quantum behavior on account of *environmental* and *gravitational decoherence.* In our interpretation, phenomena evolve in far from equilibrium settings which favor the onset of bifurcations and chaos, critical behavior and self-organization, several fundamental manifestations of complex dynamics [1-3].

Building off these premises, the goal of the paper is to integrate an important concept lying beyond the Standard Model of Particle Physics (SM), namely the *gauge-gravity duality*, with the idea of fractal spacetime well above the so-called electroweak scale defining SM.

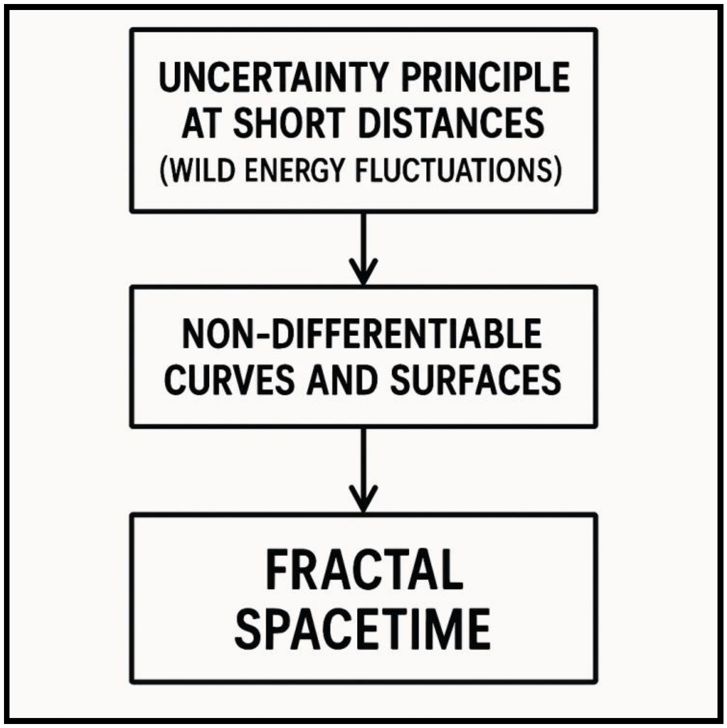

Fig. 1: From the uncertainty principle to fractal spacetime

The paper proceeds as follows: next section delves into the idea of holographic duality and the standard gauge-gravity conjecture. Section 3 elaborates on extending the conjecture to the realm of continuous dimensions. We caution that our contribution is purposedly designed as a "user friendly" introduction with emphasis on pedagogical clarity rather than formal mathematical exposition.

2. Holographic Duality and the Standard Gauge-Gravity Conjecture

The *holographic principle* asserts that all the information contained within a volume of space can be described by information on the boundary of that space—in an analogous way to a hologram, where a 3D image is encoded on a 2D surface. The principle was inspired by the study of Black Hole thermodynamics, where *entropy* (which is a measure of information content) scales not to its volume, but to the area of its event horizon.

The gauge-gravity duality (also referred to as the *AdS/CFT correspondence*) is the most representative example of the holographic principle: a gravitational theory in a D-dimensional Anti-de Sitter (AdS) space is dual to a conformal field theory (CFT) on its $D-1$-dimensional boundary. In the standard setup, D and $D-1$ are integers (typical case being AdS_5/CFT_4).

Key features of this duality include:

1) Matching symmetries (isometries of AdS become conformal symmetries of the boundary CFT).

2) Gauge fields in the bulk map to conserved currents (global symmetries) on the boundary, and gravity in the bulk encodes the stress-energy of the boundary theory. As a result, *gravity and gauge dynamics are unified* via this duality: a quantum gravity theory in AdS is equivalent to a non-gravitational (gauge) theory on the boundary.

3) Since the maximal entropy in a volume is proportional to its surface area, holography bounds the number of degrees of freedom available. The reasonable expectation is that this bound would necessarily change in non-integer dimensions. For instance, a fractal boundary of Hausdorff dimension D_H would lead to entropy scaling having the form R^{D_H-1} instead of R^2. Thus, extending AdS/CFT to a *fractional-dimensional* hologram means reinterpreting how bulk area and boundary entropy scale.

Below is a suggestive visualization of the AdS/CFT correspondence, mapping a hyperbolic-like manifold (AdS) to conformal field theory (CFT) living on its boundary:

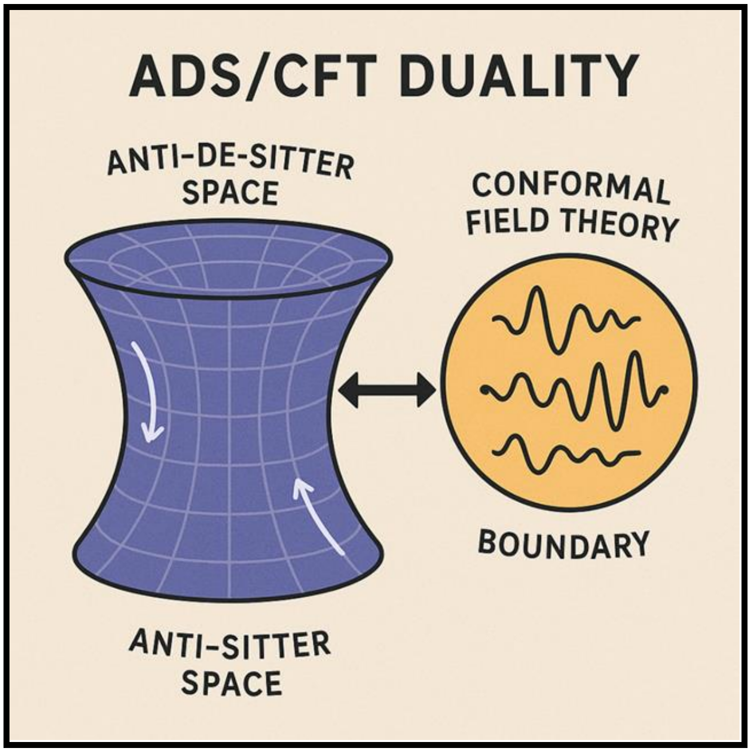

Fig. 2: Visualization of the AdS/CFT correspondence

3. Extending Standard AdS/CFT to Continuous Dimensions

There are several compelling arguments calling for the analytic continuation of AdS/CFT correspondence to *non-integer dimensions* [see e.g. 4 – 5]. The core idea of this proposal is that spacetime dimension is a *dynamic and continuous* parameter in the holographic duality. The usual lexicon of field theoretic concepts (such as correlation functions or anomalies) are analytically continued or “deformed” by $\varepsilon = 4 - D$. Likewise, according to this proposal, *dimensional flow* implies that the theory organizes itself so that at different scales it effectively has different dimensions [7]. At extremely large energies matching the ultraviolet (UV) sector of field theory and primordial cosmology, one may enter a regime of “high fractality” ($\varepsilon \propto O(1)$) where geometry is very irregular. As energy lowers, spacetime “smooths out” to $\varepsilon << 1$, approaching the standard four-dimensionality of spacetime in classical and quantum physics.

In this context it’s worth noting that, *at least in principle*, the drop in fractality in the late stages of Universe expansion can solve the S8 anomaly of cosmology and explain the Dark Energy weakening reported by DESI [10 – 11, 16].

Fig. 3 illustrates the basic idea behind the AdS/CFT correspondence in continuous dimensions.

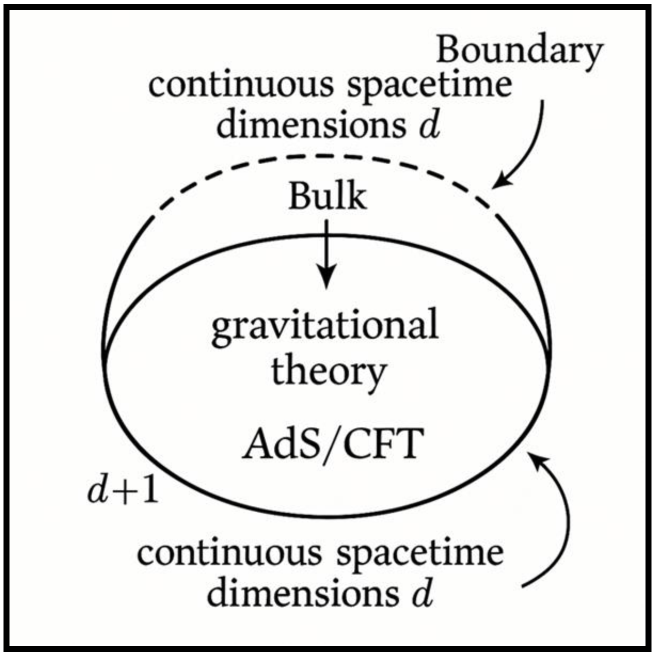

Fig. 3: Diagram of AdS/CFT in continuous dimensions

3. 1 Gauge-gravity Duality with a Positive Cosmological Constant

An obvious shortcoming of the standard AdS/CFT correspondence is that it operates with a *negative cosmological constant*, in manifest conflict with cosmological observations. The goal of this section is to indicate that extending the correspondence to continuous dimensions leads to a positive cosmological constant, whose small magnitude is consistent with observations.

To this end, we first posit that GR equations and metric differentiability remain *approximately valid* on energy scales approaching the Planck regime but staying sufficiently far below the Planck scale. According to a string theoretical formulation of the AdS/CFT duality, Einstein's equations in D spacetime dimensions contain a *negative cosmological constant* Λ and are written as [6, 8 - 9]:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 0 \tag{1}$$

in which,

$$\Lambda = -\frac{D(D-1)}{2R_{AdS}^2}; \ D = 0,1,2,... \tag{2}$$

with R_{AdS} denoting the AdS curvature radius, a parameter that can be conveniently set to unity. On a *minimal fractal spacetime* defined in one-dimensional spacetime, dimensional flow with respect to the Renormalization Group (RG) scale assumes the form,

$$D(\mu_{RG}) = 1 - \varepsilon(\mu_{RG}) \propto 1 - O[{m^2(\mu_{RG})}/{\Lambda_{UV}^2}]\ ; \ \varepsilon << 1 \tag{3}$$

where μ_{RG} stands for the RG scale and Λ_{UV} is the ultraviolet cutoff. In contrast with the conventional gauge-gravity duality, it follows from (3) that (2) turns into

$$\boxed{\overline{\Lambda} = \Lambda R_{AdS}^2 = O(\varepsilon) > 0} \tag{4}$$

(4) simply implies that the standard AdS/CFT correspondence with a negative cosmological constant gets replaced by a dS/CFT correspondence in a continuous one-dimensional spacetime endowed with a positive cosmological constant.

An appealing consequence of minimal fractality of spacetime embodied in (3) is that Dark Matter represents a large-scale topological condensate of relic dimensions left over from the primordial Universe and referred to as "*Cantor Dust*" [13]. In this context and according to [12], (4) reproduces the cosmological constant interpretation as overall contribution of dimensional fluctuations encoded in Cantor Dust, that is, $\overline{\Lambda} = O(\sum_1^\infty \varepsilon_i)$. Specifically, on scales far larger than the electroweak scale, the energy content of the cosmological constant comes from the cumulative contribution of energies stored in the continuous dimensionality of Cantor Dust.

3.2 Potential Field Unification in Continuous Dimensions

An intriguing upshot of AdS/CFT correspondence in continuous dimensions is that gravity and gauge interactions may be unified in some unexpected ways. For example, proceeding by analogy with [14], we may demand that dimensional fluctuations corresponding to $D-\varepsilon$ and $D+\varepsilon$ are *indistinguishable* from each other in a range of energies far above the electroweak scale. The holographic duality then implies,

$$D-\varepsilon = D+\varepsilon+1 \Rightarrow \varepsilon = -1/2 \tag{5}$$

and,

$$D+\varepsilon = D-\varepsilon+1 \Rightarrow \varepsilon = 1/2 \tag{6}$$

By [14], if condition (5) is fulfilled, bosons and fermions overlap in a *single phase*. This, in turn, implies that gravity, along with gauge and fermion fields, appear to *emerge as a sole entity* when $\varepsilon = -1/2$.

3.3 Fluid/Gravity Correspondence from CGLE

CGLE is a paradigm of non-equilibrium statistical physics and dynamic critical phenomena. It encodes many key properties of collective phenomena with space-time dependence, and it models the generic onset of chaos, turbulence, and spatiotemporal patterns in extended systems. In particular, CGLE describes the formation of dissipative spacetime structures in Reaction-Diffusion (RD) processes. Ref. [15] sets up the link between CGLE and the RD model of *evolving dimensional fluctuations* conjectured to arise far above the electroweak scale.

Proceeding along these lines, one may bridge the gap between CGLE and the high temperature / long wavelength limit of GR. Below is a condensed account of this scenario.

Start by recalling that an arbitrary solution of RD model of dimensional fluctuations lying near the bifurcation point can be expressed through a *complex-valued function* $W(r,\tau)=U(r,\tau)+iV(r,\tau)$ obeying the CGLE [15],

$$\boxed{\frac{\partial W}{\partial \tau}=W+(1+ic_1)\frac{\partial^2 W}{\partial r^2}-(1+ic_2)W\left|W\right|^2} \tag{7}$$

in which τ, r are the temporal and radial components of spacetime and c_1, c_2 are real-valued coefficients. Then, following the derivation detailed in [8], CGLE (7) can be shown to map to a Navier-Stokes (NS) like equation,

$$\frac{du}{d\tau}=\frac{\partial u}{\partial \tau}+u\cdot\nabla u=-\frac{1}{\rho}\nabla p+\nu\nabla^2 u \tag{8}$$

where $u(r,\tau)$ stands for the flow velocity, ρ is the fluid density, p its pressure and ν the viscosity. Finally, using (8), one may uncover an intriguing path linking NS and GR via the *fluid/gravity correspondence*. Fig. 4 below visualizes the overall steps leading from CGLE to the fluid/gravity correspondence and GR.

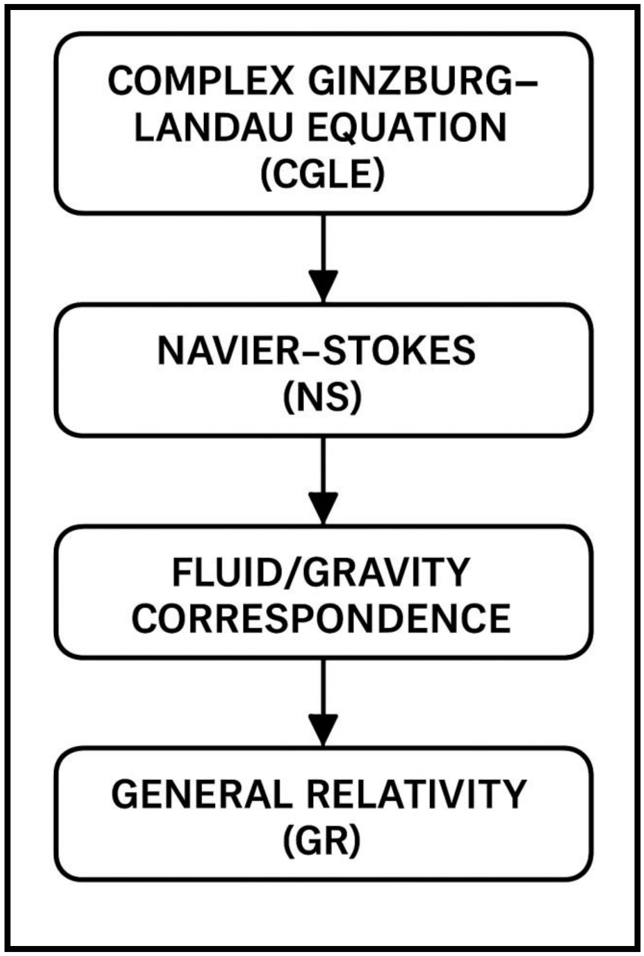

Fig. 4: The path From CGLE to GR

References

1. Goldfain, E., Fractal Spacetime from Uncertainty Relations at Ultrashort Distances, (2025), preprint http://dx.doi.org/10.13140/RG.2.2.14435.46883

2. Goldfain, E., Three Routes to the Dynamics of Continuous Spacetime Dimensions, (2025), http://dx.doi.org/10.13140/RG.2.2.16611.87844/1

3. Goldfain, E., Introduction to Fractional Field Theory (consolidated version), (2015), http://dx.doi.org/10.13140/RG.2.1.1694.2249

4. Skvortsov E. and Tran T., AdS/CFT in Fractional Dimension and Higher Spin Gravity at One Loop, (2017), preprint https://arxiv.org/pdf/1707.00758

5. Goldfain, E., Fractal Spacetime and the Dynamic Generation of Mass Scales in Field Theory, Prespacetime Journal| September 2014 | Volume 5 | Issue 9 | pp. 843-851…

6. Goldfain, E., On the Gauge-Gravity Duality in Fractal Dimensions, (2022), http://dx.doi.org/10.13140/RG.2.2.16806.19528

7. Eckstein, M. and Trześniewski, T., Spectral dimensions and dimension spectra of quantum spacetimes, (2020), https://arxiv.org/pdf/2005.14210

8. Hubeny, V. E. *et al.*, The Fluid/Gravity Correspondence, (2011), http://dx.doi.org/10.1142/9789814350525_0014

9. Goldfain, E., Nonequilibrium Dynamics and General Relativity, (2023), http://dx.doi.org/10.13140/RG.2.2.18233.65125/1

10. Adil, S. A. *et al*, S_8 increases with effective redshift in ΛCDM cosmology, (2024), https://doi.org/10.1093/mnrasl/slad165

11. Goldfain, E., Potential Evidence for Evolving Spacetime Fractality from DESI, (2025), http://dx.doi.org/10.13140/RG.2.2.30476.42884/1

12. Goldfain, E., Cosmological and Fermi Scales from Hamiltonian Chaos, (2022), http://dx.doi.org/10.13140/RG.2.2.30556.40328

13. Goldfain, E. From Cantor Dust to Fuzzy Dark Matter, (2022), http://dx.doi.org/10.13140/RG.2.2.27469.61920

14. Goldfain, E., Spin-statistics Theorem from the Stuart Landau Equation, (2023), http://dx.doi.org/10.13140/RG.2.2.23193.08802

15. Goldfain, E., Fractional Dynamics and Physics Beyond Effective Field Theory (Part 1), (2024), http://dx.doi.org/10.32388/MHWR52

16. Goldfain, E. Potential Evidence for Dark Matter as Dimensional Condensate, (2023), http://dx.doi.org/10.13140/RG.2.2.28553.01129/1