Quantization of Gravity

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1 Introduction

In the previous article, we explored how gravity can exist in a quantum superposition and how, within a single orbital or energy eigen state, this corresponds—conceptually—to a superposition of stress-energy tensors. In this article, we aim to take this further by examining how gravity could exhibit quantum behavior and by calculating the curvature during the evolution of a particle in energy eigen states or superpositions thereof.

2 Proof of Quantization Properties of Gravity

Let us consider a quantum particle of mass m existing in a specific energy level. For simplicity, consider a superposition of two quantum states:

$$|\psi\rangle = \alpha|A\rangle + \beta|B\rangle \tag{1}$$

Each of these states represents a particle of mass m, albeit in a different configuration. Importantly, we do not obtain a mass of $\frac{1}{2}m$ for any individual instance; the mass remains m in all realizations.

Now, consider the same superposition in terms of energy. We have:

$$\hat{H}|E_n\rangle = E_n|E_n\rangle \tag{2}$$

This implies that the energy operator yields a definite value for each eigen state. The total energy is not divided among the superposed states in terms of potential energy and charge. However, the kinetic energy does vary spatially at a given instant:

$$T(\vec{r}) = -\frac{\hbar^2}{2m}\psi^*(\vec{r})\nabla^2\psi(\vec{r}) \tag{3}$$

This means that kinetic energy is position-dependent even at the same moment in time. However, the variations are so minute compared to the particle's mass, charge, and total kinetic energy that they can safely be neglected.

Therefore, we can assert that mass and energy remain effectively constant throughout the spatial extent of the energy eigen state, and each measurement yields the same result. Nonetheless, the particle can exist at multiple locations simultaneously. Since the stress-energy tensor depends directly on mass and energy, we conclude that it too can exist at multiple spatial locations at the same time and approximately retains the same value at each point.

The key idea is that the stress-energy tensor is distributed over the orbital shell with nearly uniform values from point to point. Thus, it exists in a quantum superposition—and so does the curvature it induces. In flat Euclidean spacetime, we can write the invariance of the stress-energy tensor under coordinate translation as:

$$T^{\mu\nu}(x+a) = T^{\mu\nu}(x') = T^{\mu\nu}(x) \tag{4}$$

This invariance suggests that the metric should exhibit similar behavior. However, in our quantum context, this distribution exists *simultaneously*. Therefore, curvature too may exist in a superposition.

3 Calculating Curvature During Energy-State Evolution

Assume a particle evolves between energy levels and is in a superposition of, for instance, the 1s and 2s orbitals, which have different energy eigenvalues. The stress-energy tensors for all superpositions within the 1s orbital will differ from those in the 2s orbital. When a measurement is made, a single definite value is observed.

Mathematically, this means:

$$T_1^{\mu\nu} \neq T_2^{\mu\nu} \quad \text{where} \quad T_i^{\mu\nu} = \langle E_i|\hat{T}^{\mu\nu}(x)|E_i\rangle, \quad i = 1, 2 \tag{5}$$

This supports the idea that distinct combinations of mass and energy correspond to distinct spacetime curvatures. While this may seem counterintuitive classically, in quantum mechanics it is consistent: if two mass-energy configurations coexist in superposition, so can their corresponding curvatures.

To compute the expected curvature, we begin by evaluating the stress-energy tensors for each state:

$$T_1^{\mu\nu}(x) = \langle E_1|\hat{T}^{\mu\nu}(x)|E_1\rangle, \quad T_2^{\mu\nu}(x) = \langle E_2|\hat{T}^{\mu\nu}(x)|E_2\rangle \tag{6}$$

The total expectation value becomes:

$$T^{\mu\nu}(x) = \langle\psi|\hat{T}^{\mu\nu}(x)|\psi\rangle = |c_1|^2\langle E_1|\hat{T}^{\mu\nu}(x)|E_1\rangle + |c_2|^2\langle E_2|\hat{T}^{\mu\nu}(x)|E_2\rangle + 2\,\mathrm{Re}\left(c_1^* c_2 \langle E_1|\hat{T}^{\mu\nu}(x)|E_2\rangle\right) \tag{7}$$

We can then calculate the Einstein tensor using the weak-field or linearized gravity approximation:

$$G_{\mu\nu}^{(1)}(x) = 8\pi G\, T_{\mu\nu}^{(1)}(x), \quad G_{\mu\nu}^{(2)}(x) = 8\pi G\, T_{\mu\nu}^{(2)}(x) \tag{8}$$

The expectation value of the Einstein tensor is thus:

$$\langle G_{\mu\nu}(x)\rangle = G^{(1)}_{\mu\nu}(x) + G^{(2)}_{\mu\nu}(x) + \delta G^{\text{int}}_{\mu\nu}(x) \tag{9}$$

where $\delta G^{\text{int}}_{\mu\nu}(x)$ represents the perturbative contribution from the interference term in the stress-energy tensor.

Although linearized gravity is employed here, other frameworks may also be used. This method enables us to describe the evolution of energy states in a gravitational context.

4 Conclusion

We have shown how gravity may exhibit quantum properties and simultaneously maintain classical behavior. By incorporating superpositions and interference into the stress-energy tensor, and using linearized gravity, we find a consistent picture in which curvature can reflect the quantum structure of matter.

In conclusion, While total energy remains fixed within an energy eigen state, the kinetic energy density varies slightly from point to point. By neglecting these small fluctuations, we can more clearly interpret the electron's presence as a superposition of localized probability amplitudes. Consequently, the associated curvature is not uniformly spread over the entire orbital but arises locally—corresponding to the electron's possible positions—thereby allowing spacetime curvature itself to exist in a quantum superposition tied to the particle's probabilistic nature.

[1] [2] [3] [4] [5]

References

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