The game show aka Monty Hall problem

Richard L. Hudson 5-12-2025

abstract

The game show problem aka Monty Hall problem [1], originated when Craig Whitaker posed a question of a winning strategy for a 3 door game show to Marilyn Savant who wrote articles for Parade magazine.
Her 1990 response was to switch doors when given the option. [2] [3]
The debate of probability of success as 2/3 vs 1/2 has continued until today.
This paper reveals errors in her response.

Whitaker's question

Suppose you're on a game show, and you're given the choice of three doors. Behind one door is a car, behind the others, goats. You pick a door, say #1, and the host, who knows what's behind the doors, opens another door, say #3, which has a goat. He says to you, "Do you want to pick door #2?" Is it to your advantage to switch your choice of doors?

Marilyn Savant's 1990 response [3]

Yes; you should switch. The first door has a 1/3 chance of winning, but the second door has a 2/3 chance. Here's a good way to visualize what happened. Suppose there are a million doors, and you pick door #1. Then the host, who knows what's behind the doors and will always avoid the one with the prize, opens them all except door #777,777. You'd switch to that door pretty fast, wouldn't you?
The benefits of switching are readily proven by playing through the six games that exhaust all the possibilities. For the first three games, you choose #1 and "switch" each time, for the second three games, you choose #1 and "stay" each time, and the host always opens a loser. Here are the results.. For the first three games, you choose #1 and "switch" each time, for the second three games, you choose #1 and "stay" each time, and the host always opens a loser. Here are the results.

	DOOR 1	DOOR 2	DOOR 3	RESULT
GAME 1	AUTO	GOAT	GOAT	Switch and you lose.
GAME 2	GOAT	AUTO	GOAT	Switch and you win.
GAME 3	GOAT	GOAT	AUTO	Switch and you win.
GAME 4	AUTO	GOAT	GOAT	Stay and you win.
GAME 5	GOAT	AUTO	GOAT	Stay and you lose.
GAME 6	GOAT	GOAT	AUTO	Stay and you lose.

the game

Initial conditions: the player does not know the location of the car, thus they can only make a random guess.

The host knows the location of the car, thus their choice is not random, acknowledged by Savant:

"So let's look at it again, remembering that the original answer defines certain conditions, the most significant of which is that the host always opens a losing door on purpose. (There's no way he can always open a losing door by chance!)"

The game rules are:

rule 1. the host cannot open the door from the players 1st choice.

rule 2. the host cannot open a door containing a car.

If c is the car, then the list of all possible events (games) is fig.1.

The player always chooses door 1. The host chooses door 2 or door3 (red) per the game rules.

Instead of playing a list twice, to compare stay vs switch results, each game can be played once, followed by a comparison of door 1 with door r, the remaining closed door.

e	1	2	3	r
1	a	b	c	c
2	a	c	b	c
3	b	a	c	c
4	b	c	a	c
5	c	a	b	b
6	c	a	b	a
7	c	b	a	a
8	c	b	a	b

fig.1

e	1	2	3	r
1	g	g	c	c
2	g	c	g	c
3	g	g	c	c
4	g	c	g	c
5	c	g	g	g
6	c	g	g	g
7	c	g	g	g
8	c	g	g	g

fig.2

e	1	2	3	r
1	g	g	c	c
2	g	c	g	c
3	c	g	g	g

fig.3

questions

1. Why are there 6 patterns (distributions)?

There are 3 possible prizes for door 1, 2 possible prizes for door 2, and 1 possible prize for door 3. Calculate 3!=6.

For the general game with 3 distinct prizes a, b, c, there are 6 patterns of prizes.
abc, acb, bca, bac, cab, cba.

2. Why are there 8 games for 6 patterns?
When c is behind door 1, there are 2 host choices.
The host opening 2 doors in 1 game would reveal the car location, end the game, and deny the player's 2nd guess. Thus opening 2 doors requires 2 additional games, 6 and 8.

3. Why is Savant's list (fig.3) small compared to fig.2?
A generic 'g' is substituted for 'a' and 'b' as shown in fig.2. Savant does not consider the goats as different, and only sees 1car and 2 identical goats. This only requires 3 patterns, equivalent to removal of duplications in fig.2.
She also does not understand the logical difference of (door 2 or door 3) vs (door 2 and door 3).

general game vs Savant game
Since each door has the same distribution of prizes, the player's 1st choice is irrelevant to the results.
The results depend on the host choices which depend on the game rules.

In fig.1, the wins are mutually exclusive for door 1 and door r.
There is no advantage to switch.

In fig.3, for door 1 the win ratio is 1/3 vs 2/3 for door r.
There is an apparent advantage to switch, but only because of excluding games, by not considering the goats as different. They exist simultaneously and therefore require separate identities, such as goat1 and goat2.

probability
Probability is a substitute for lack of knowledge, in an abstract mathematical form, and is not intended to be taken literally. The prizes are in fixed locations for the duration of a game. The doors never contain partial prizes. The measure of success is (guess car door)/(all possible guesses).

She was a victim of 'gambler's fallacy'.[4]

"The fallacy leads to the incorrect notion that previous failures will create an increased probability of success on subsequent attempts."

Her example of a million doors is total nonsense.
The following are true statements.
1. The set of n doors contains 1 car. Probability of car=1.
2. There is 1 door that contains a car. Probability of car=1.
3. The set of (n-1) doors does not contain a car. Probability of car=0
Probability of (1)= Probability of (2) + Probability of (3).
A probability of 1/n means, if the player guessed a different door for n games, they would win 1 game and lose n-1 games. With n possibilities, the player could win on the 1st or last guess or anywhere in between.

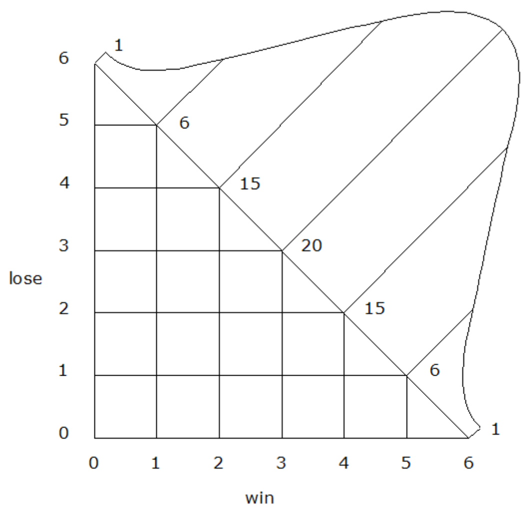

fig.4

Fig.4 shows all possible results for playing 6 games. Winning or losing a sequence of 6 games is rare, 1/64. The ideal average will be 20/64=5/16.
If 'win' and 'lose' are replaced with H and T, the curve is Pascal's triangle for a coin toss.

conclusion

The errors are due to Whitaker's choice of 2 goats as secondary prizes and Savant's lack of understanding simple probabilities.
Using the measure of success from 'probability',

the player doesn't win with their 1st guess with a 1 in 3 chance, since the host does not open that door to verify win or lose.
The player may win with their 2nd guess with a 1 in 2 chance, when the host does open that door.

The answer to Whitaker's question is no.

reference

[1} The American Statistician, August 1975, Vol. 29, No. 3
[2] game show problem, Wikipedia Sep 2024
[3] Marilyn vos Savant, https://web.archive.org/web/20130121183432/http://marilynvossavant.com
[4] Wikipedia, Gambler's fallacy, Apr 2025