The Absolutely Precise Frequency of the Hydrogen 21-cm Line

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Abstract

The hydrogen hyperfine transition or the hydrogen 21-cm line is critically important in astronomic and cosmic observations and in quantum mechanics. In our previous papers, we calculated out and determined the atomic unit of time (t_{au}) to be $2.418884326586532 78\times10^{-17}$ s. With this value of t_{au}, we calculated out and determined the precise frequencies of some atomic transitions of some atoms such as ^{1}H, $^{27}Al^{+}$, ^{40}Ca, ^{87}Sr, $^{115}ln^{+}$, ^{171}Yb, $^{171}Yb^{+}$, ^{199}Hg and $^{199}Hg^{+}$ and the nuclear transition of $^{229}Th^{*}$. In this paper, we calculate out and determine the absolutely precise frequency of the hydrogen 21-cm line to be 1420405751.76690576 Hz in comparison to the most precise measured value 1420405751.768(2) Hz or 1420405751.767(4) Hz.

Keywords: the hydrogen 21-cntimeter line, the atomic unit of time, frequency.

1. The Hydrogen 21-centimeter Line

The hydrogen hyperfine transition or the hydrogen 21-cm line is the electromagnetic radiation spectral line that is created by spin-flip of neutral hydrogen atoms. Each hydrogen atom is made of one proton and one electron. Both these particles have a property called spin. The spin of the electron and proton in a hydrogen atom can be either aligned (higher energy state) or anti-aligned (lower energy state). When the spins flip from aligned to anti-aligned, the atom will shed the excess energy, emitting electromagnetic radiation with wavelength of about 21 cm as follows (**Fig. 1**).

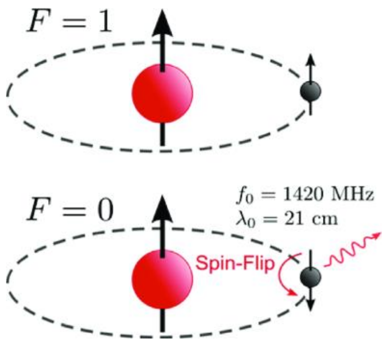

Fig. 1. Production of the hydrogen 21-cm line

This wavelength falls within the microwave region of the electromagnetic spectrum, its corresponding radio waves in the universe can penetrate the large clouds of interstellar cosmic dust that are opaque to visible light. So observation and study of this radio emission have immense significance in astronomy and cosmology. It helps astronomers map our galaxy and even peek into the early universe.

The frequency of this radiation is about 1420 MHz. The most precise measured values of this frequency are 1420405751.768(2) Hz [1] or 1420405751.767(4) Hz [2]. In this paper, we calculate out and determine it to be 1420405751.76690576 Hz.

2. The Absolutely Precise Frequency of the Hydrogen 21 cm Line

In our previous papers [3-6], we calculated out and determined the atomic unit of time (t_{au}) to be $2.41888432658653278\times10^{-17}$ s. With this value of t_{au}, we can calculate out the absolutely precise frequency of the hydrogen 21-cm (f_0) as follows.

$$\frac{1}{f_{0-meas/au}}=\frac{1}{f_{0-meas}t_{au}}$$

$$=\frac{1}{1420405751.767(4)\times 2.41888432658653278\times10^{-17}}$$

$$=29105326.617932(82)$$

$$\frac{1}{f_{0-calc/au}}=2(8\cdot3\cdot29+1)(16\cdot9\cdot5\cdot29-1)+\frac{3}{5}+\frac{1}{56}+\frac{3}{56(8\cdot3\cdot29+1)}$$

$$=2(8\cdot3\cdot29+1)(16\cdot9\cdot5\cdot29-1)+\frac{173}{4\cdot70}+\frac{3}{8\cdot7(8\cdot3\cdot29+1)}$$

$$=29105326.6179340029$$

$$\frac{1}{f_{0-calc/au}}=2(8\cdot3\cdot29+1)(16\cdot9\cdot5\cdot29-1)+\frac{5}{8}-\frac{1}{3\cdot47}+\frac{103}{8\cdot3\cdot5\cdot47(8\cdot3\cdot29+1)}$$

$$=2(8\cdot3\cdot29+1)(16\cdot9\cdot5\cdot29-1)+\frac{8\cdot3\cdot29+1}{8\cdot3\cdot47}+\frac{103}{8\cdot3\cdot5\cdot47(8\cdot3\cdot29+1)}$$

$$=29105326.6179340029$$

Note: $141=3\cdot47,\ 188=4\cdot47,\ 87=3\cdot29$

Relationships with nuclides:

$^{56,58}_{26}Fe_{30,32}$ $^{63,65}_{29}Cu_{34,36}$ $^{82,83,84}_{36}Kr_{46,47,48}$ $^{100}_{44}Ru_{56}$ $^{103}_{45}Rh_{58}$ $^{107,109}_{47}Ag_{60,62}$ $^{112}_{48}Cd_{64}$

$^{136,137,138}_{56}Ba_{80,81,82}$ $^{140,142}_{58}Ce_{82,84}$ $^{5\cdot29}_{61}Pm^{*}_{84}$ $^{173}_{70}Yb_{103}$ $^{188}_{76}Os_{112}$ $^{208}_{82}Pb_{126}$ $^{209}_{83}Bi^{*}_{126}$

$^{209}_{84}Po^{*}_{125}$ $^{223,224}_{87}Fr^{*}_{136,137}$ $^{231,232}_{91}Pa^{*}_{140,141}$ $^{264,265}_{103}Lr^{*}_{161,162}$ $^{285}_{112}Cn^{*}_{173}$ $^{314}_{126}Ch^{ie}_{188}$

$^{344,2\cdot173,12\cdot29}_{136,137,138}Fy^{ie}_{208,209,210}$ $^{357}_{141}Ch^{ie}_{216}$ $^{437,438}_{173}Ch^{ie}_{264,265}$

$$f_{0-calc} = \frac{f_{0-calc/au}}{t_{au}}$$

$$= \frac{1}{29105326.6179340029 \times 2.41888432658653278 \times 10^{-17}}$$

$$= 1420405751.76690576 \text{ Hz}$$

$$\lambda_{0-calc} = \frac{c}{f_{0-calc}} = \frac{299792458}{1420405751.76690576}$$

$$= 21.1061140541760589 \text{ cm}$$

3. Discussion and Conclusion

In the above formulas of the reciprocal of the frequency of the hydrogen 21-cm line in atomic units (au), there are some characteristic factors such as 29/58/87/232, 45, 56/112, 70, 103, 47/141/188 and 173. It is supposed that these characteristic factors are related to nuclides according to our theories described in our previous papers [3-6]. In comparison with the formula of the reciprocal of ^{1}H 1S - 2S atomic transition frequency in atomic units [3, 5], some common typical factors such as 70, 103, 47 and 173 miraculously exist as follows.

^{1}H 1S - 2S atomic transition frequency ($f_{^1H}$):

$$\frac{1}{f_{^1H-calc/au}} = 17 - \frac{1}{4} + \frac{1}{70} - \frac{1}{37 \cdot 173} + \frac{1}{11 \cdot 47 \cdot 97 \cdot 103}$$

$$= 16.7641296822937984$$

$$f_{^1H-calc} = \frac{f_{^1H-calc/au}}{t_{au}}$$

$$= \frac{1}{16.7641296822937984 \times 2.41888432658653278 \times 10^{-17}}$$

$$= 2466061413187018.02 \text{ Hz}$$

These miraculous coincidences are strong proof that these formulas are correct. So we suppose that the above formulas and methodology to calculate and determine the frequency of the hydrogen 21-cm line should be reasonable and absolutely precise.

Reference

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