Title:
Last Base Mathematics: A Constructible Geometry of Number, Time, and Harmony

Author: Benjamin Xavier Last
Affiliation: Independent Researcher, Visual Systems Thinker, Artist

Preface

This work represents the articulation of a system I believe to be ancient in origin, yet largely unspoken in modern mathematical literature. I do not claim to have created Last Base Mathematics, but rather to have uncovered its presence—buried in clocks, calendars, harmonics, and geometry.

As an artist and systems thinker, I have long intuited patterns in number and proportion that seemed to transcend traditional notation. Through exploration, iteration, and assistance from computational tools, I found that these patterns could be formalized into a coherent structure. The result is what I now call The Last Eye of God—a radial, recursive framework for expressing number through geometry and resonance.

This paper is intended as a beginning, not a conclusion. I offer it to mathematicians, physicists, musicians, and artists alike—with the hope that you find something within these pages that sparks curiosity, and perhaps leads to deeper discovery.

— Benjamin Xavier Last

Abstract

Last Base Mathematics (LxB) is a proposed numerical and geometrical framework rooted in the base-12 number system and expanded through alternating base pairings. This system minimizes numerical footprint by leveraging circular and polygonal divisions as a foundational structure for counting, measurement, and transformation. At its core, LxB employs a dual-base architecture—typically base-12 and an alternating secondary base such as 5—to create layered, fractal-like representations of number through concentric or rotational subdivisions.

We introduce the central visual form of the system, referred to as The Last Eye of God, a radial structure that embodies both inward fractional divisions and outward whole-number rotations. This structure, when rotated iteratively through three axes in multiples of twelve, produces complex symmetrical forms that reveal harmonic and geometric coherence.

By uniting cyclic symmetry, polygonal geometry, and alternating base logic, LxB offers a compact and visually intuitive framework for understanding number, proportion, and physical transformation across disciplines. While this paper remains grounded in physical and mathematical expression, the implications of the system suggest broader avenues of inquiry into the inherent patterns of reality.

1. Introduction & Background

Throughout history, base systems have shaped the structure of mathematics, timekeeping, and physical measurement. From the Babylonians' base-60 to the globally adopted base-10, the way we count influences how we perceive and interact with the world. Yet, embedded quietly in our daily lives—through clocks, calendars, and musical scales—is the presence of base-12: a system whose divisibility, symmetry, and geometric compatibility suggest an overlooked elegance.

Last Base Mathematics (LxB) emerges from the hypothesis that base-12, when paired with a recursive or alternating secondary base (commonly base-5), produces a compact, harmonious, and fractal-like numerical system. The purpose of this system is not to supplant traditional base systems, but to offer a new way of interpreting number as motion, shape, and harmony—rooted in geometry rather than linear arithmetic.

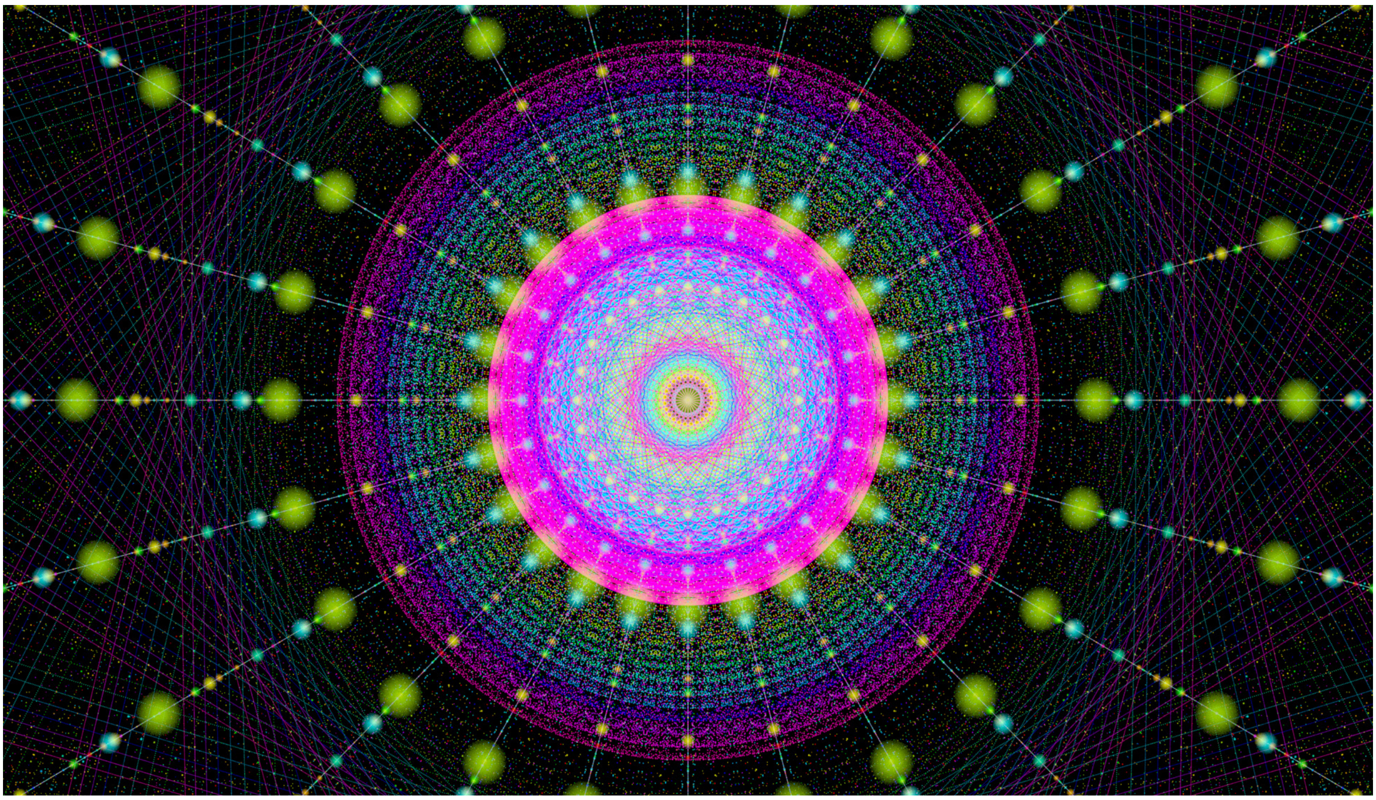
The central structure of LxB, referred to as The Last Eye of God, is a radial construct that allows both whole number counting (outward from center) and fractional division (inward toward center). It is formed by subdividing circles and rotating these structures across three dimensions in twelvefold symmetry.
This generates complex yet ordered forms that naturally align with existing structures in music, time, physics, and potentially quantum phenomena.

The motivation behind this work is twofold:

To formalize the mathematics underlying these geometric and numerical relationships, providing an academic framework for exploration.

To develop intuitive visual tools that make the system accessible not only to researchers, but to artists, educators, and curious minds across disciplines.

While similar concepts appear in sacred geometry, the present work takes a grounded, mathematical approach, stripping away mysticism in favor of precision, clarity, and testable structure. The deeper implications—philosophical, metaphysical, or otherwise—are acknowledged but remain beyond the scope of this paper.



2. Core Concepts of Last Base Mathematics

2.1 The Role of Base-12 as primary base

Base-12 (duodecimal) is chosen for its exceptional divisibility.
Twelve can be evenly divided by 2, 3, 4, and 6—more than any other number under 13—making it uniquely suited to circular division and rotational symmetry.

2.2 Alternating Base Pairs and Emergent Higher Bases

Each of the 12 segments of a circle is further divided into a secondary base. In the example below we do so with base 5, yielding 60 unique positions (fig 1). LxB5 mimics the resolution of a base-60 system while retaining a symbolic footprint of only 12 characters. The pattern continues recursively:

$12 \times 5 = 60$

$60 \times 12 = 720$

$720 \times 5 = 3{,}600$

$3{,}600 \times 12 = 43{,}200$
…and so on.

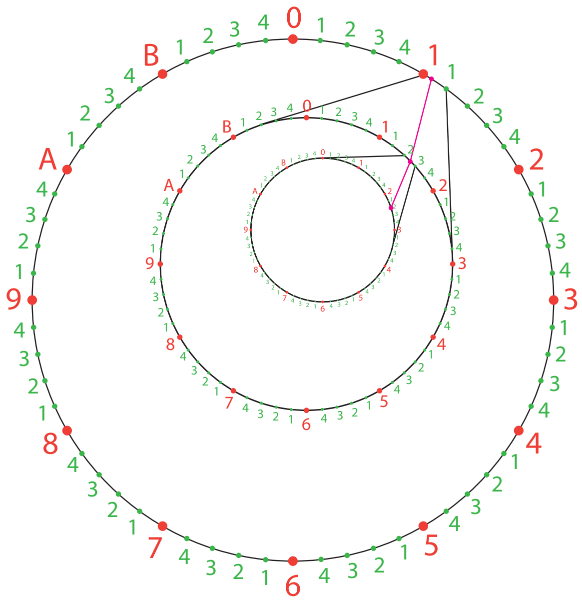


fig 1.

Here we can see how the repeating 12/5 constructable structure appears in LxB5 as a subdivion.

The magenta marked points would read as a value of 1.01222 as we read into the recursive subdivisions alternating between the base 12 ‘hour’ and the base 5 ‘minute’

E.g.

alternating base:	12	5	12	5	12	5
LxB5 value:	1.	0	1	2	2	2

2.3 Numerical Footprint Minimization

Rather than expanding notation, LxB uses layer depth to increase precision. For example, 1/3 is exactly 4 rather than an endless 0.333… in decimal (fig 2). It allows geometric snapping of values to appropriate subdivision layers.

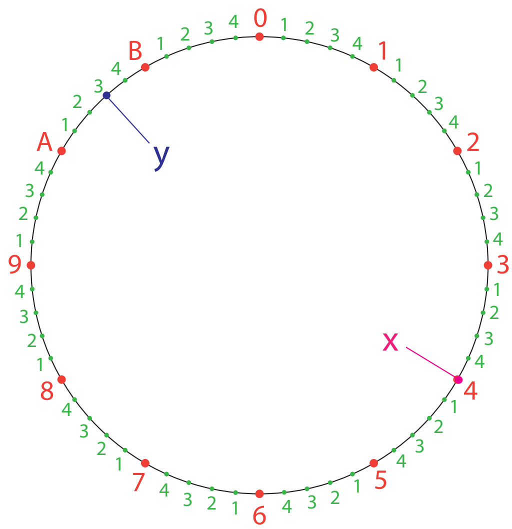


fig 2.

x = 4
y = A.3

3. Last Base Constructible Numbers and Radial Equivalence

3.1 Constructible Numbers as a Foundation

All divisions are made using straightedge and compass: halves, thirds, fifths, etc. Each creates a polygon, which maps to a base.

3.2 Every Base is Size One

All LxB base divisions operate within a unit circle. "1" in base-12 is maintained as an equal size regardless of which alternating number is used. This maintains geometric equibavalence across differing numbering bases.

3.3 Inferring Values Through Geometry

Arithmetic can be bypassed via spatial inference. A pointer in LxB3 at 2.2 also sits at LxB5 2.323.... recurring (fig 3, fig 4).

This enables base-translations and calculations without symbolic operations. It also means numbers can be stored at the lowest appropriate base value minimising footprint for use in such applications as large particle simulations, orbital dynamics, nano-scale measurements, or alternatively simply as a form of compression.

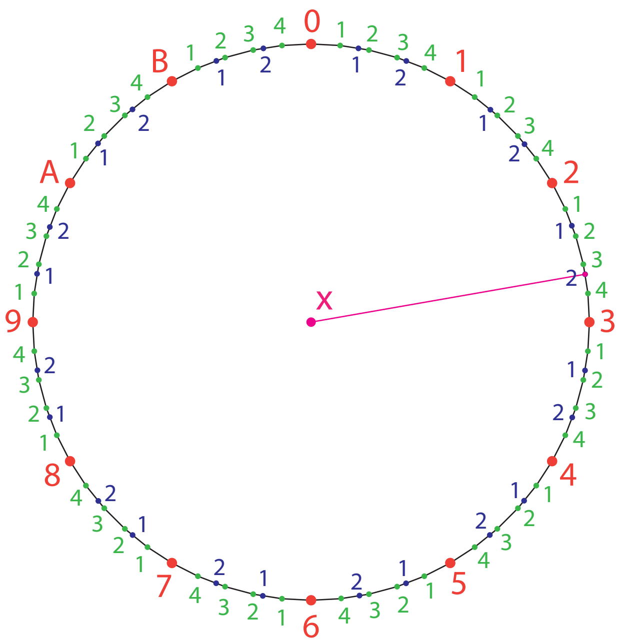


fig 3.
LxB3 (12/3) overlayed with LxB5 (12/5)

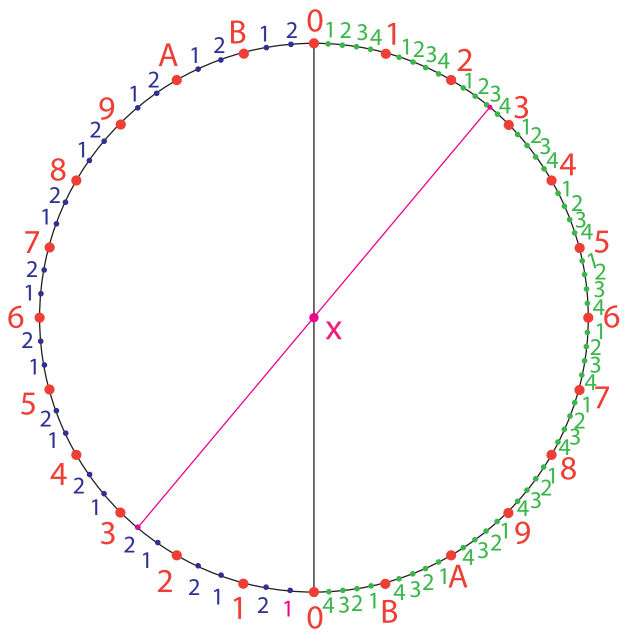


fig 4.
LxB3 (12/3) mirrored against LxB5 (12/5)

In the diagrams above we see the same representation oof LxB3 against LxB5 represented as an overlay on the left and as a mirrored circle on the right. We see that the value of one is equivalent in both LxB3 and LxB5 as both start with the same base 12 subdivion. That means that we can store the lowest whole value of whichever number system. Further alternating bases can also be used so that stored values can be represented with the lowest whole number footprint. In the example above X is 2.2 in LxB3, and 2.323.... recurring in LxB5, so in the case of storing vectors in a simulation, we could do the calculation in LxB 5, but store the smaller LxB3 number.

4. Mathematical Framework and Base Interactions

4.1 The Alternating Base Engine
LxB alternates between base-12 and base-5 at each recursive layer. Each level increases resolution while retaining simple notation.

4.2 Positional Logic Without Positional Burden
Numbers are stored as stacked coordinates:
Example: [7][3][B] = (7×5×12) + (3×12) + 11 = 467

4.3 Reversible Resolution
467 decomposes exactly back to [7][3][B], showing the system's reversibility and modular logic.

4.4 Base Interoperability via Modular Mapping
Values can be matched across base layers using modular congruences. This allows base fusion, phase tracking, and system synchronization.

4.5 Arithmetic on the Circle

Addition = rotation

Multiplication = repetition

Division = subdivision
Values are determined and validated through angular movement.

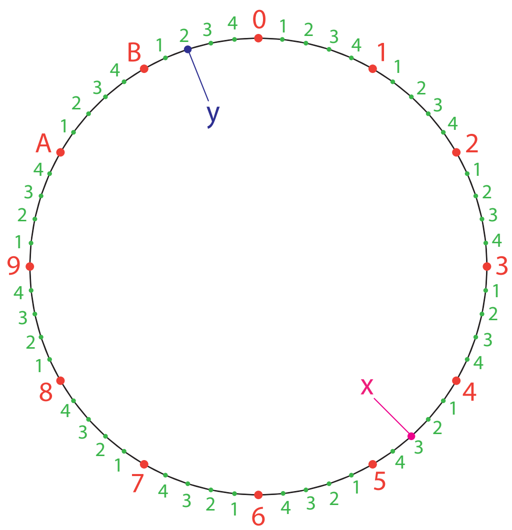


fig 5.

x = 4.3

y = B.2

Addition
4.3 + B.2 = 14

Subtraction
B.2 - 4.3 = 6.4

Multiplication
4.3 * B.2 = 38.6

Division
B.2 / 4.3 = 2.2435...(repeating)

4.6 Arithmetic in Last Base Mathematics (LxB)

In Last Base Mathematics, all arithmetic operations are performed directly within the alternating base structure, without conversion into decimal, base-10, or any intermediary system.

Each positional digit obeys its local base:

The first (leftmost) digit operates in base-12 (values 0–B).

The second digit operates in base-5 (values 0–4).

The third digit operates in base-12 again.

The pattern alternates indefinitely: 12, 5, 12, 5, 12, 5...

4.6.1 General Rules

Addition: Perform digit-wise addition. If the sum equals or exceeds the local base, carry 1 to the next higher digit. The carried value obeys the base of the next digit.

Subtraction: Perform digit-wise subtraction. If the minuend digit is smaller than the subtrahend, borrow from the next higher digit, adjusting according to its base.

Multiplication: Perform digit-wise multiplication, carrying overflows according to each digit's base. Alternatively, multiply expanded values then re-collapse.

Division: Perform long-form division while respecting the alternating base system for remainders and positional shifts.

Decimal expansion is neither necessary nor permitted within internal LxB calculations.
The system is closed, coherent, and self-sustaining.

4.6.2 Worked Examples in LxB5 (12/5 alternating base)

Example 1: Addition

Add [7][4][A] + [2][3][5]:

Place	Base	Operation	Result	Carry
3rd (rightmost)	12	A (10) + 5 = 15 → 15 ÷ 12 = 1 remainder 3	3	1
2nd	5	4 + 3 + 1 (carry) = 8 → 8 ÷ 5 = 1 remainder 3	3	1
1st (leftmost)	12	7 + 2 + 1 (carry) = 10 (no carry needed)	A	0

Thus:

[7][4][A] + [2][3][5] = [A][3][3]

Example 2: Subtraction

Subtract [9][1][6] – [4][3][9]:

Place	Base	Operation	Result	Borrow
3rd	12	6 – 9 → Borrow 1 from second place. (6+12)−9 = 9	9	1
2nd	5	(1-1) – 3 = -3 → Borrow from first place. (1+5)−3 =3	3	0
1st	12	(9-1) – 4 = 4	4	0

Thus:

[9][1][6] – [4][3][9] = [4][3][9]

Example 3: Multiplication

Multiplying [4][7] × 3 inside LxB manually:

Place	Base	Operation	Result	Carry
2nd (rightmost)	12	7 × 3 = 21 → 21 ÷ 12 = 1 remainder 9	9	1
1st (next left)	5	(4 × 3) + 1 carry = 13 → 13 ÷ 5 = 2 remainder 3	3	2
Extra place (left)	12	2 (carry)	2	0

Thus:

Final Result:

[4][7] x 3 = [2][3][9]

Example 4: Division

Divide [8][2][0] ÷ 2:

Expand thinking (without decimal!):

Rightmost place (base-12):
0 ÷ 2 = 0 (no carry).

Middle place (base-5):
2 ÷ 2 = 1 (no carry).

Leftmost place (base-12):
8 ÷ 2 = 4.

Thus:

[8][2][0] ÷ 2 = [4][1][0]

4.6.3 Carry and Borrow Charts

For clarity:

Base	Carry When	Borrow When
12	≥ 12	When minuend < subtrahend
5	≥ 5	When minuend < subtrahend

The base at each position must always be consulted individually during carry or borrow operations.

5. Applications in Timekeeping, Music, and Measurement

5.1 Timekeeping

12 × 5 = 60 minutes

60 × 60 = 3,600 seconds

12 × 5 × 12 × 5 × 12 = 86,400 seconds in a day
Time is already operating in an LxB-aligned system.

5.2 Music

12 semitones (chromatic scale)

5 tones (pentatonic)

Harmonic intervals appear as polygonal alignments
LxB provides tools to build harmonic visualizations and sonic interfaces.

5.3 Measurement

Many ancient and modern systems are based on LxB principles:

12 inches in a foot

360 degrees in a circle

43,200 seconds = ½ sidereal day
LxB offers scalable, modular measurement with visual logic.

Babylonians used an alternating 10/6 system for measurement as recently uncovered by UNSW

5.4 Simulations and compression

As whole numbers can be geometrically derived across differing Last Bases LxB there is an oportunity to save the lowest footprint whole numbers for vectors in aplications like physics, fluid and particle simulations.

This also has implications for the purposes of data compression and storage.

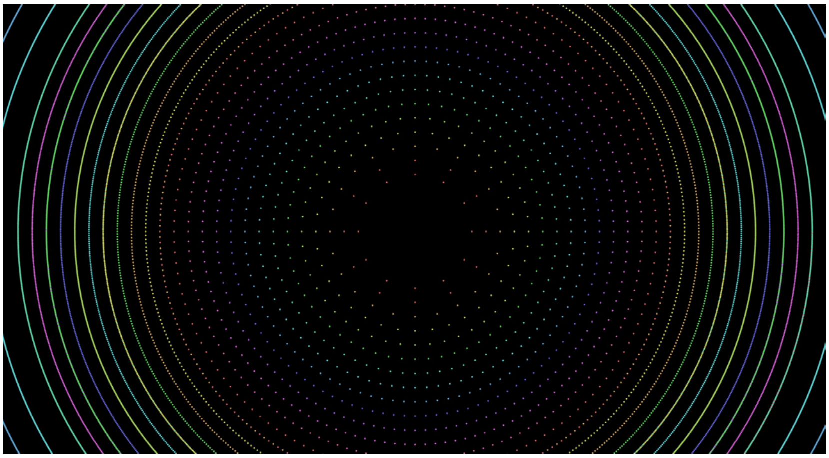
6. Visualizations and Computational Tools

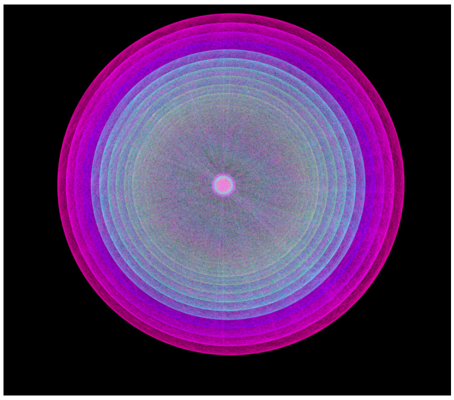
6.1 Radial Calculators
Base-12 and base-5 rings with input/output snapping. Displays stacked digit values and angular equivalences.

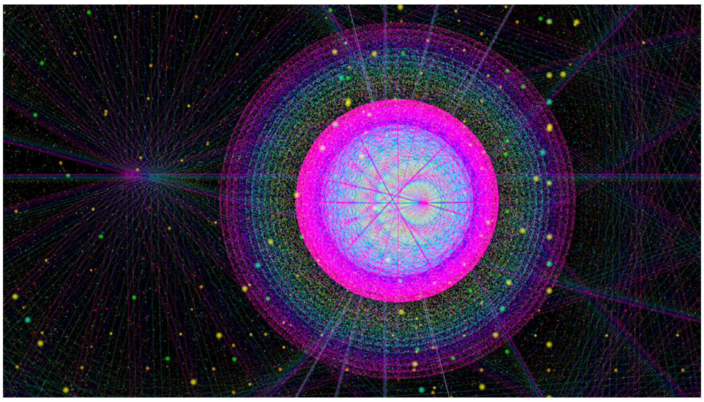
6.2 Polygonal Overlay Engine
Base values represented as constructible polygons. Useful for showing modular symmetry and harmonic resonance.

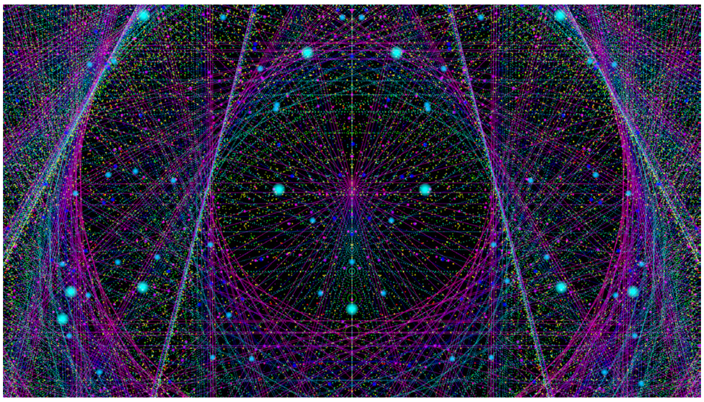
6.3 The Last Eye of God – Interactive Generator
Visualizer to rotate and explore layered constructs. Useful for simulations, clocks, or compositional tools.

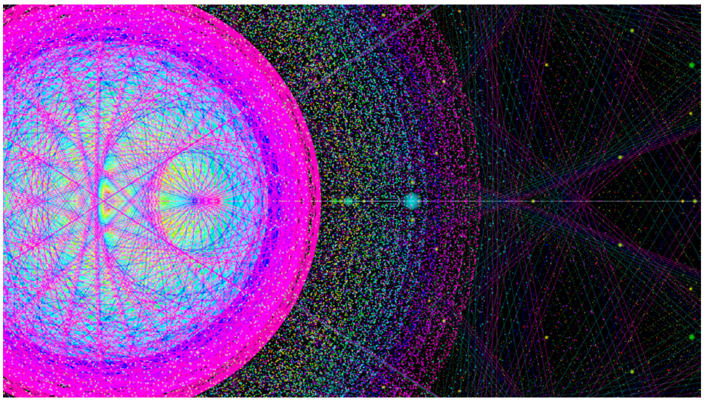
This is created by laying out the alternating bases and stepping out by the LxB base value. Eg. First ring is 12/1, second ring 12/2 3rd ring 12/3 and so on, we then count out the rings by the 12 or 5 alternating base value. This is then rotated 12 times in each axis. This creates a fractal like sherical grid object. The below images have been created out to a depth of 4 iterations. This could continue out infinitely with sufficient computing power. Images taken from the object below rendered in Houdini using Mantra.

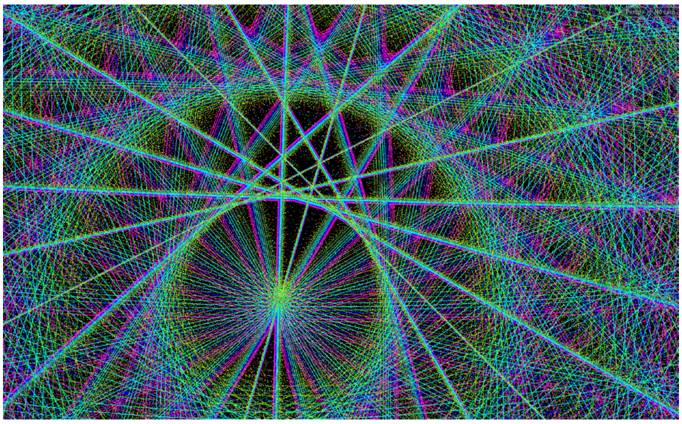












7. Further Implications and Future Work

7.1 Future Computational Experiments
LxB could inform rotary computing, analog modeling, and data compression through base-layer encoding.

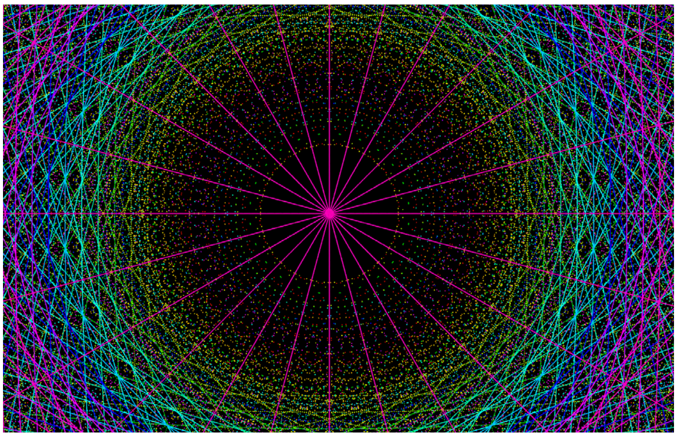
7.2 Harmonic Measurement Model
LxB provides a way to model preference for whole-number ratios in physical systems.

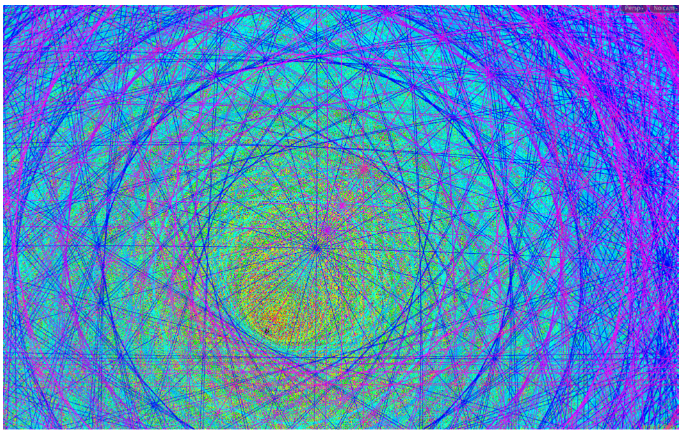
7.3 Reframing Computation
Proposes rotational computation systems, modular memory, and analog hybrid logic.

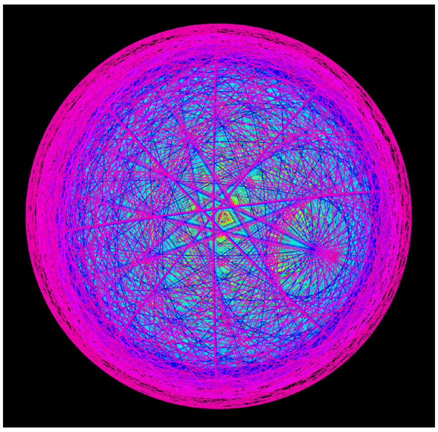
7.4 Educational and Artistic Integration
Ideal for visual math, music theory, and art installations.

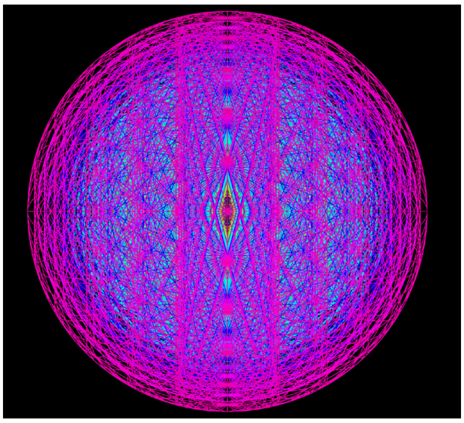
7.5 Call to Exploration
The door is open for future research into:

- Musical tunings
- Modular logic
- Quantum phase spaces
- Ancient numerical systems
- Visual pedagogy









Glossary

Footprint: Minimal number of digits needed

Snap-to-grid: A value falls cleanly on a base layer

Radial Depth: Layer of precision in the system

Constructible Number: Can be drawn with compass/straightedge

[X][Y]: Layered digit format, each in a different base

References / Notes

Duffin, Ross. How Equal Temperament Ruined Harmony (and Why You Should Care)

Eliade, Mircea. The Sacred and the Profane

Stillwell, John. Mathematics and Its History

Neugebauer, Otto. The Exact Sciences in Antiquity

Ghyka, Matila. The Geometry of Art and Life

Heath, Thomas. A Manual of Greek Mathematics