Geometric rationale for the Riemann hypothesis

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Abstract

This paper constructs a "three-dimensional complex coordinate system" and proposes that the non-trivial zeros of the Riemann Zeta function lie on demarcation lines within the critical region. By infinitely projecting the singular point s=1 between complex planes in the 3D complex space and the standard complex plane, we derive regions where non-trivial zeros cannot exist. The boundaries of these regions (including Re(s) =1/2) are identified as potential loci for zeros.

Key words: Complex three-dimensional coordinate system; Infinite Projection
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1. Introduction

The Riemann hypothesis posits that all non-trivial zeros of the Riemann Zeta function $\zeta(s)$ lie on the critical line Re(s)=1/2. While computational verification supports this claim for over 10^{13}zeros, a rigorous mathematical proof remains elusive.

This work introduces a geometric framework using a **3D complex coordinate system** Abbreviated as (3D-CCS). By projecting the singularity s=1 between 3D-CCS and the standard complex plane, we demonstrate that non-trivial zeros must reside on boundaries separating "zero-free regions" (ZFRs).

2. Definitions

2.1 Three-Dimensional Complex Coordinate System

Let the 3D-CCS consist of: *Complex number three-dimensional coordinates system ,* (hereinafter referred to as 3D)

In fig1*, Re(x) and Re(y) form a real number plane"*$Re(x) - Re(y)$*plane" the imaginary number axis Im perpendicular to O point is establish， this is complex number* 3D Fig1

The K(X， Y， bi) coordinate is a three-dimensional complex space，

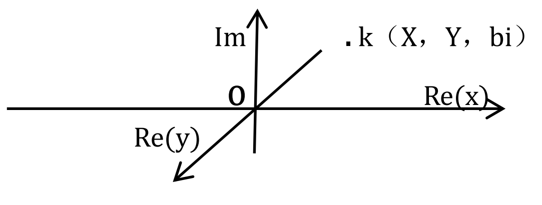

Fig1

The essence of three-dimensional complex space is that there are infinitely many complex planes distributed in three-dimensional space with the Im axis as the center, perpendicular to the real number plane.

This is just replacing the Z-axis in the 3D real space coordinate system with the Im axis, All projection rules are the same as the 3D real space coordinate system.

This article only involves the geometric projection of singularities and is not related to any parameters of the complex plane. The projection rules are consistent with 3D real coordinates. Not related to quaternions or Clifford algebras.

2.2 Basic complex plane

In an infinite number of two-dimensional complex planes, if we set a basic complex plane such as "$Im - Re(x)$plane" in the figure1, all coordinate points in 3D can be obtained by projecting onto the basic complex plane.

In future discussions, all complex planes refer to fundamental complex planes.

LEMMA 1.

All coordinate points on the complex plane (Except for points on the base plane) are projections of points in 3D, and these points on the complex plane are virtual images

Therefore, when we set the complex plane as the fundamental complex plane observed in the two-dimensional world (for example "Im- Re(x)plane"),After providing additional conditions ， we can provide coordinate points for any point in three-dimensional space on the fundamental complex plane, but these coordinate points are virtual images. Its real coordinate point is K in Fig1.

For example, if the point on the Basic complex plane is (x, b_i), we can label it as $(x, b_i)\mu$. when μ=0, it is a point on the Basic complex plane, and when μ ≠0, it represents a point in 3D space. μ is the additional condition.

Explanation: As this article does not involve the application of additional conditions, no in-depth research has been conducted. Because it conforms to the rules of 3D real coordinates. Therefore, Lemma 1 does not have a written proof step, and in future proofs, Lemma 1 will only serve as a basis for projecting points in 3D onto a two-dimensional plane.

2.3 Singularity of $\zeta(s)$ in 3D-CCS

The singularity s=1is represented as an arc s =1 on the real plane (Fig. 2). Points k_i(x，y）on this arc satisfy

Fig2 shows the pole [1] in 3D，On the real plane, pole are distributed on arcs with a radius of S=1. Point k_i(X，Y) is a point on the arc.

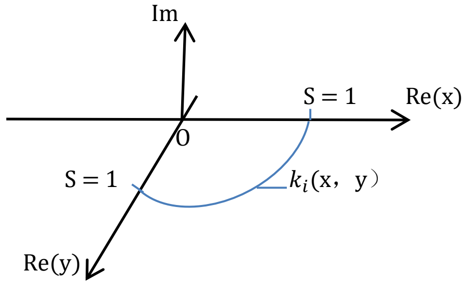

Fig2

In 3D, only this quadrant satisfies the definition domain of the critical region.

3. Projection of S=1 and Zero-Free Regions

In Fig2, set the Re(θ) axis through the origin O,
Re(θ) on real number "Re(x)- Re(y)plane", Re(θ) average segmentation angle $\pi/2$, Re(θ) intersects with the arc at point k_i Fig3

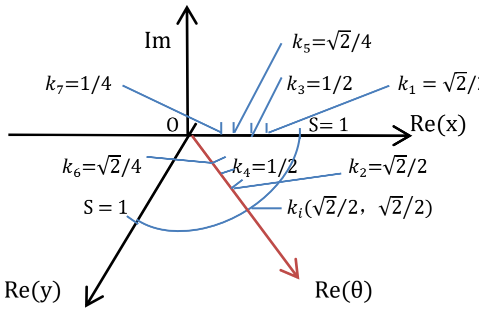

Fig3

$\because Ok_i = 1$, $\therefore k_i(x, y) = k_i(\sqrt{2}/2, \sqrt{2}/2)$.

3.1 Infinite projection of singularity

According to the lemma: Project the points on the arc S = 1~ k_i onto Re (x), (S = 1 ~ $k_i$represents between point S = 1 and point k_i, "~" The following meanings are the same)

∵ in real number "Re(x)-Re(y)plane", All complex planes within quadrant "Re(x)-Re(y)plane" satisfy the domain of " pole and Non trivial Zeros of Riemann Zeta Functions",

∴These complex planes are interrelated.

∵ on Re(x) , the segments k_1=$\sqrt{2}/2$ ~ S = 1 are all projections of S =1 on Re(x), that is in the segments k_1=$\sqrt{2}/2$ ~ S =1, all points are equivalent to pole, and S=1is Points where the Zeta function is Undefined or not parsed, [1] if these points are meaningful on Re(x), then they are also meaningful on the arc, when S=1, This is obviously contradictory to the non trivial zero critical zone Re(x)< 1.

∴ There cannot be any 'non trivial zeros' within segment:

k_1=$\sqrt{2}/2$ ~ S = 1 of *"Im − Re(x)plane"*.

This defines nested **zero-free regions** (ZFRs)

3.2 Boundaries of (ZFRs)

∵ These complex planes are interrelated, ∃ (ZFRs) on Re(x) ,

∴ Corresponding to, of k_1=$\sqrt{2}/2$~S=1 on Re (x), there should also be a (ZFRs) on Re(θ).

it is k_2=$\sqrt{2}/2$~S=1 in Fig3.

Project k_2 from Re(θ)onto Re(x)and mark it as k_3, $k_3 = 1/2$.

1/2 ~ $\sqrt{2}/2$ is a newly added (ZFRs)on Re (x) and the newly added the (ZFRs)on Re(x) will also affect the (ZFRs)on Re(θ).

This forms a connection between two (ZFRs)'s, where $k_1 = \sqrt{2}/2$ is their Boundaries.

∵ the angle between Re(x)and Re(θ) is $\pi/4$

∴when projecting from Re(x) onto Re(θ). each time the projection will reduce the projection value by $\sqrt{2}$ times.

The mathematical expression is $k_n = \sqrt{2}^{-n}$, when $n \to \infty$, $\sqrt{2}^{-n} = \sqrt{2}^{-\infty} = 0$.

That is to say, after infinite projection $\sqrt{2}/2$~S=1 (ZFRs) between Re (x) and Re (θ), in 0 <Re (x) < 1is composed of countless different(ZFRs). so non trivial zeros can only appear on the boundary where two(ZFRs) intersect. For example, the first boundary line is$\sqrt{2}/ 2$.

In 0 <Re (x) < 1, we can get boundary line : $\sqrt{2}/2$, 1/ 2, $\sqrt{2}/ 4$, 1/ 4, $\sqrt{2}/ 8 \ldots \sqrt{2}^{-n}$, n=1,2,3,4…

∵ the non trivial zeros of the Zeta function in 0 <Re (x) < 1, [2]and the range between 0 and 1 is(ZFRs),

∴ the non trivial zero point of the Zeta function is located on the boundary between two (ZFRs) in 0 <Re (x) < 1.

∵ in$\zeta(s) = 2^s \pi^{s-1} \sin\frac{\pi s}{2} \boldsymbol{\tau}(1-s)\zeta(1-s)$, $\zeta(s) = \zeta(1-s)$ Symmetry constrains the position of the zero point, For the critical line 1/2, there is only a $\sqrt{2}/2$ boundary line on the right side, but $\frac{\sqrt{2}}{2} - \frac{1}{2} = \frac{\sqrt{2}-1}{2}$, $\left(\frac{1}{2} - \frac{\sqrt{2}-1}{2}\right) = \frac{2-\sqrt{2}}{2}$ is not the boundary of (ZFRs).

∴ in critical line 1/2, Unable to find symmetrical boundary lines.

4. Conclusion

The infinite projection of S=1 in 3D-CCS generates ZFRs whose boundaries coincide with the critical line Re(s)=1/2R. This provides a geometric rationale for the Riemann hypothesis.**#**

References

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