Study of the Bug-Rivet Paradox through the Definition of Length

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Abstract

This research investigates the bug-rivet paradox through the definition of length. This approach differs from the conventional method of length contraction. Length contraction was initially proposed to explain adjustments in the path length of light, but it has resulted in numerous misconceptions in various contexts. By examining the light path from the beginning, this study confirms that there is no contradiction in the process of joining the hole and the rivet.

Introduction

There is a bug in a hole and a rivet is attached to it. The bug-rivet paradox arises when we try to answer the question, *'Can the bug in the hole survive?'* [1] If the hole moves towards the rivet at relativistic speed, the bug does not survive. Conversely, if the rivet moves towards the hole, the bug survives.

Figure 1: Comparison between a rivet frame and a hole frame.
(a) Rivet frame, (b) Hole frame

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This creates a paradoxical situation. A complete interpretation of this paradox without contradiction is not yet known. Unlike other paradoxes in special relativity, there have been few attempts to interpret or solve this problem. Although this paradox seems fatal, it is not impossible to resolve. I believe that in our world, there are no inherent contradictions among events that occur relativistically. And nature operates without contradiction, even if humans are unaware of the truth behind this paradox. The way to understand nature's behavior without contradiction is to naturally follow the path of light and follow the logic that the path of light tells us. Consequently, we can comprehend this paradox without contradiction and explain it logically and thoroughly. However, it is crucial to set aside our preconceptions temporarily and examine what the definition of length and the path of light reveal.

1 Definition of Length and Light-Clock

One second is defined as the time it takes for light emitted by a cesium-133 atom in a specific state to vibrate 9,192,631,770 times. Let's define the distance that light travels in one second as one *light-second* ($=$ *one ls*). Since measuring one second or one light-second of light traveling in one-way is challenging, let's assume it is reasonable to measure the distance in two-way using a mirror.

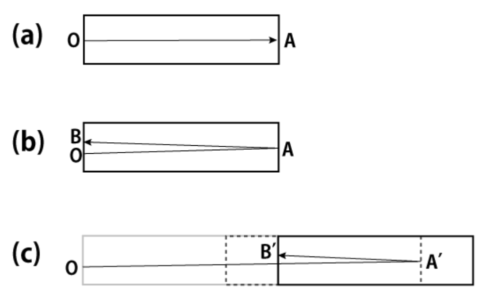

Figure 2: (a) One-way length of stationary system
(b) Two-way length of stationary system
(c) Two-way length of moving system

In an inertial frame, there is a light-clock that takes one second for light to travel from the left end to the right end (Figure 2-a). The length of this light-clock is L_o, and it takes two seconds for light to travel back and forth (Figure 2-b), assuming the light changes direction immediately without any time delay. Figure 2-(c) shows the light-clock in another inertial frame as observed by a relatively moving observer. In this situation, the unit length is L, not L_o. The lengths of L_o and L are different. When an observer at rest observes the length of their own light-clock and the length of another person's light-clock, their sizes are not the same. According to the principle of relativity, both lengths are valid as unit lengths. In Figure 2-(b) and (c), the ratio of the lengths of the light paths in $\overline{OAB}$ and $\overline{OA'B'}$ is equal to the ratio of the size of the unit time T_o of the light-clock at rest and the unit time T of the light-clock in motion. $\overline{OAB}$ and $\overline{OA'B'}$ represent the lengths of the light paths, and the fact that their lengths are different means that the size of the unit times for these two paths is different. It is widely known that time in the motion system is longer, commonly referred to as *time dilation*. If we accept the constancy of the speed of light along with time dilation, we can confirm the following facts. (To simplify the development of the logic, we assume that both the rivet and the hole are ideally rigid bodies. We further assume that there is no deformation, breakage, or electromagnetic interaction between these bodies at the moment of impact.)

1. Time dilation causes time intervals to appear as $T_{(OA'B')} > T_{(OAB)}$
2. The speed of light remains invariant across all reference frames.
3. Therefore, the measurement of lengths is represented as $L_{(OA'B')} > L_{(OAB)}$

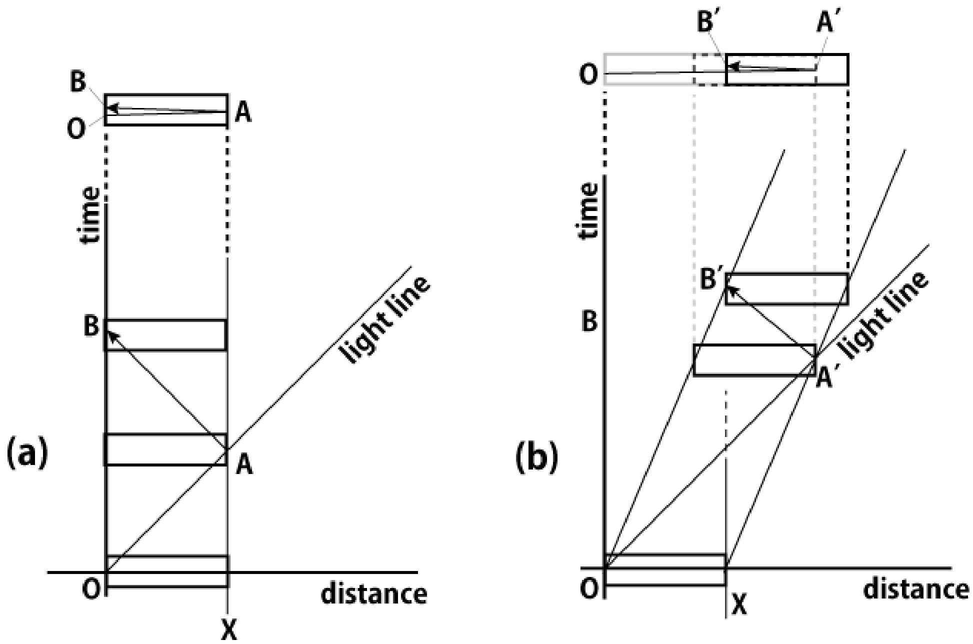

Figure 3: The world line of light in space-time.
(a) Path of light in a stationary system
(b) Path of light in a moving system.

2 The length of the light path rather than the length contraction

We have a fixed perception that objects moving at relativistic speeds contract in length. We will not proceed with this logic in this case. Length contraction was proposed to explain the results of the Michelson-Morley experiment but lacks experimental proof.

Also, it has become a major area of science because Lorentz and Einstein acknowledged it. Instead of accepting length contraction outright, we'll investigate this issue using the fundamental path length of light. If we do so, we will discover that there is no contradiction in this thought experiment. Figures 3-(a) and (b) show the path of light in a four-dimensional spacetime diagram. Figure 3-(a) shows the path of light for a stationary light-clock, and (b) shows the path of light for a moving light-clock. The length of the light-clock L_o, when at rest, is denoted as its stationary length, while the length of the light-clock in motion is determined to be L. These two are related by equation (1). Here, L is the length of the moving light-clock. ($\gamma = 1/\sqrt{1-\beta^2}$, $\beta = v/c$, c is the speed of light, v is the speed of the light-clock) There are multiple methods available to demonstrate the validity of this equation (1).

$$L = \gamma L_o \tag{1}$$

- K Calculus [2]
- Relativistic Doppler effect [3]
- Constancy of the speed of light and time dilation

The purpose here is not to prove equation (1), so only the simplest third method will be introduced. Since time dilation has been proven through many experiments so far, let's accept it as correct. And since the constancy of the speed of light has also been repeatedly proven, let's accept it as correct. If these two facts are correct, equation (1) will naturally be proven correct. An observer in an inertial frame observes their own time and length as T_o and L_o, and observes the other party's time and length as T and L. In various inertial frames, the speed of light follows a relationship represented by equation (2).

$$c = c' = c'' = c_o = c = ... \tag{2}$$

The speed of light is constant for all observers, regardless of their motion. Thus, the following relationship holds between length, time, and the speed of light for a stationary observer.

$$\frac{L_o}{T_o} = c_o \tag{3}$$

And if the speed of light is constant for all observers in inertial frames, the following equation also holds.

$$\frac{L}{T} = c \tag{4}$$

This is so obvious. And since $c_o = c$, the following equation also holds.

$$\frac{L}{T} = c = c_o = \frac{L_o}{T_o} \tag{5}$$

Here, since the time dilation that we are all familiar with is in the relationship $T = \gamma T_o$, the relationship $L = \gamma L_o$ must also hold. Consequently, the subsequent relationship is also established:

$$c = \frac{L}{T} = \frac{\gamma L_o}{\gamma T_o} = \frac{L_o}{T_o} = c_o \tag{6}$$

If $T = \gamma T_o$, which indicates time dilation, is correct, then $L = \gamma L_o$ must necessarily be true for the speed of light to be invariant. If this is not true, then the theory of relativity has serious problems, and its foundation collapses. Therefore, if time dilation and the constancy of the speed of light are accurate, equation (1) must necessarily be true. If we accept that equation (1) is correct, then the bug-rivet paradox can be easily understood.

2.1 Observation in the Rivet Reference System

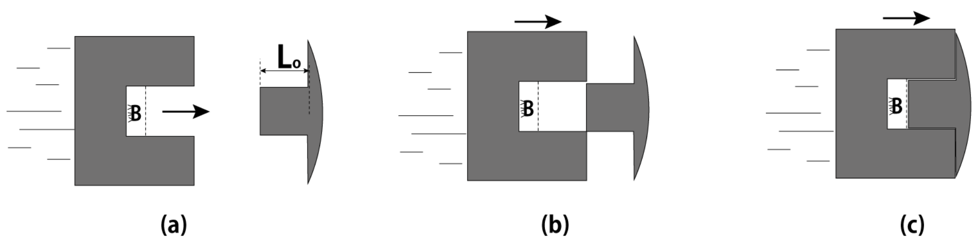

Figure 4: From the rivet's perspective, the bug survives.

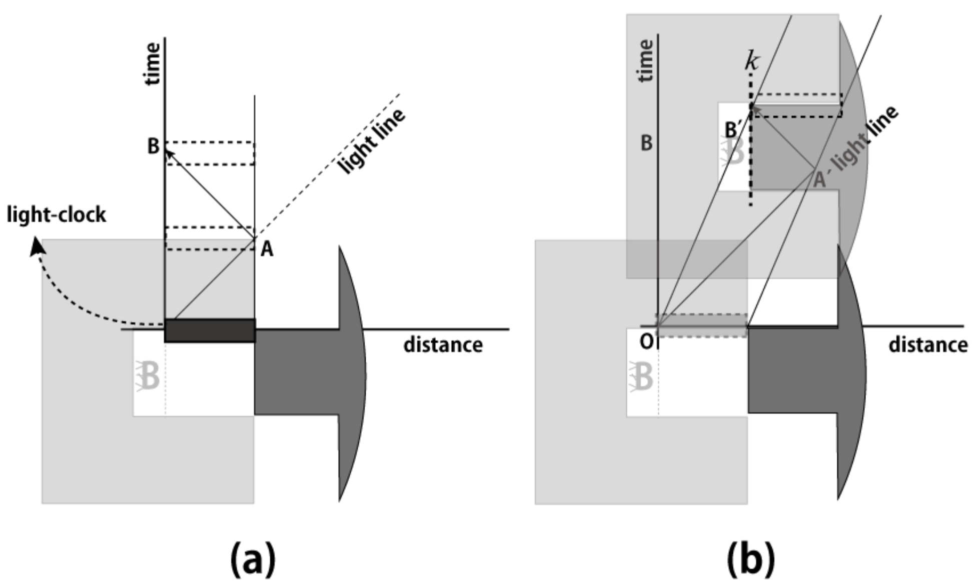

Figure 5: Surviving Bug and Relativistic Length of Hole (Rivet's View)

Figure 4 illustrates an observation from the rivet frame. It depicts a scenario in which the bug within the hole survives as the hole approaches and merges with the rivet. The sequence of events is represented in the order of (a), (b), and (c). To address this problem effectively, it is crucial to move away from the conventional notion that the length of the moving hole contracts. Instead of adhering to the fixed idea of length contraction, it is essential to focus on the path of light and how the length changes as the hole and the rivet combine. This emphasis is important because the length of the light path represents the true length. Figure 5 illustrates the scenario depicted in Figure 4 using a space-time diagram that incorporates a hole-rivet. Specifically, Figure 5-(a) represents the stationary system, while Figure 5-(b) depicts the situation in the moving system. Let us examine Figure 5-(b). When measuring the length of the hole, the light that originated from the starting point eventually returned to its original position, allowing for the full length to be measured. The other party recognizes this length as γL_o. In Figure 5-(b), at the final moment of the combination, the depth at which the rivet penetrated the hole is precisely the same as that in the stationary system. Consequently, the bug within the hole persists in this scenario.

2.2 Observation in the Hole Reference System

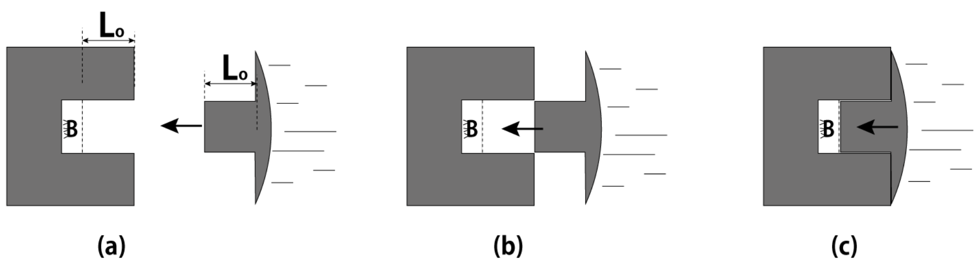

Figure 6: From the perspective of the hole, the bug survives.

The same conclusion holds true when viewed from the opposite perspective. This time, we look at the case from the perspective of the hole. Here, the time order is (a), (b), (c) in Figure 6. Figure 6 illustrates the situation in which the hole remains stationary while the rivet moves at relativistic speed. In Figure 7-(a), the path of light in the stationary system is illustrated. The time it takes for light to traverse the light-clock is denoted as T_o, and the length of the light-clock is represented as L_o. It takes $2T_o$ to travel back and forth through the light-clock, and the round-trip distance is $2L_o$. In Figure 7-(b), the light that started from point O of the rivet returns through a path $OA'B'$, representing unit time T, with the relationship $T = \gamma T_o$. And the length is also the length L of the light-clock of a relatively moving person. Since the time is in the relationship $T = \gamma T_o$ and the speed of light is constant, the length is also in the relationship $L = \gamma L_o$. If that does not hold, the constancy of the speed of light does not hold either. In Figure 7, does the rivet overlap with the bug's space when combined with the hole? No. The length L is the total length measured from the time the light is at point O until it reaches B' after a round trip. It does not instantaneously transform into γL_o when the hole and

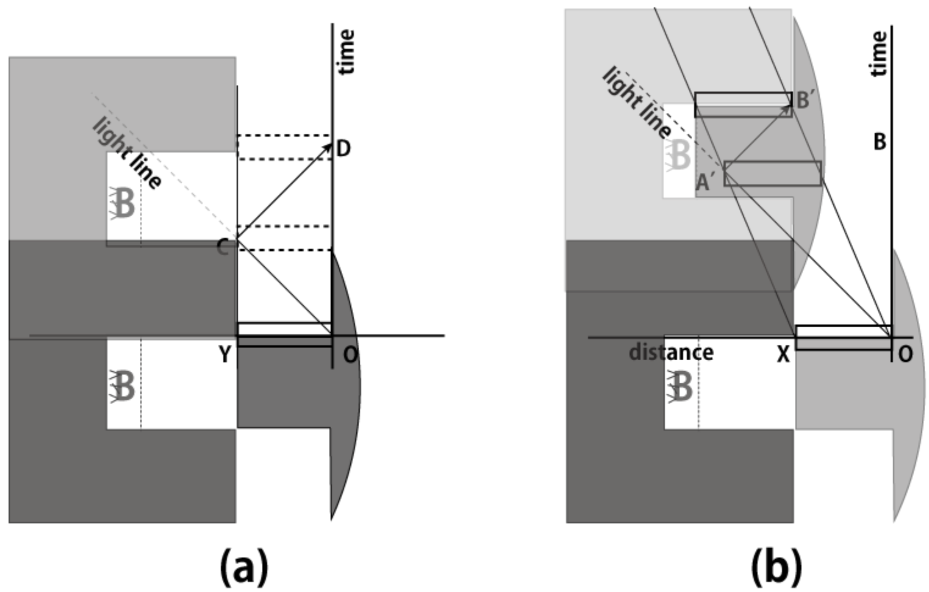

Figure 7: Surviving Bug and Relativistic Length of Rivet (Hole's View)

the rivet are combined. When the end of the last hole reaches the dotted line(k), the bug's space is still protected. Therefore, even when the hole and the rivet are completely combined, the bug can survive without being destroyed. This is the same whether you look at it from the hole's perspective, the rivet's perspective, or the stationary system. Therefore, there is no paradox in this problem. We now need to consider the relativistic length and the rivet length together. We examined four length models in Figure 8 to understand the correct relativistic length concept.

3 Correct Relativistic Length Model

Figure 8-(a) illustrates the length of the rivet as observed from a stationary inertial frame or by an observer in an inertial frame. Figure 8-(b) presents a model based on the hypothesis of length contraction. In this scenario, the bug survives in the reference frame of the hole but perishes in the reference frame of the rivet. This discrepancy leads to a contradiction, suggesting that the model is invalid. A relativistically moving object adheres to the relationship $L = \gamma L_o$, but this is not accurately depicted in Figure 8-(c). Instead, it is represented as shown in Figure 8-(d). Because the rivet is in motion while light travels to and from a reference point, the trajectory of the light differs from that in a stationary frame. In spacetime, this path is illustrated in Figure 7-(b), which pertains to relativistic length. At the precise moment when the hole and the rivet coincide, the length of the rivet remains unchanged. Therefore, if the bug survives in the stationary system, it will also survive in the moving system.

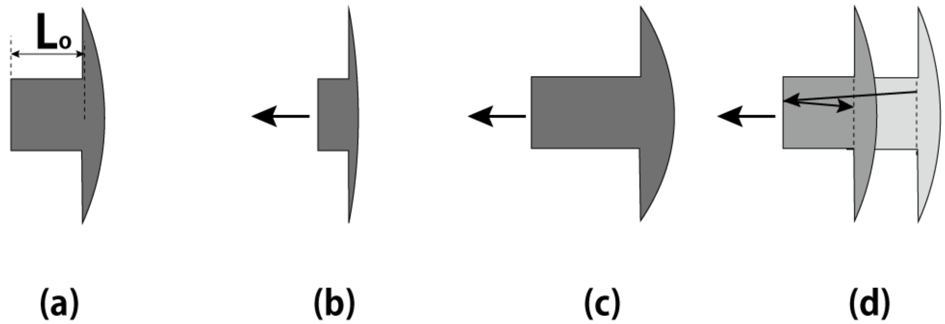

Figure 8: Relativistic Model of Length:
(a) Length of a rivet in a stationary inertial frame
(b) A vague imaginary model of the length contraction hypothesis
(c) A vague imaginary model of the length expansion
(d) Length expansion model based on the path of light
If the speed of light is infinite, Model (d) becomes Model (a).

Conclusion

The crux of the problem lies in the fact that while light measures the length of the light-clock, the hole or rivet continues to move. Consequently, the length observed in the stationary frame and the length in the moving frame are different. If the speed of light were infinite, this discrepancy in length would not occur, as is the case in classical mechanics. However, since the speed of light is finite, the ongoing movement of the hole or rivet during the measurement results in a difference from the length in the stationary frame. This difference adds to the original length, making it necessarily longer than the proper length. Therefore, in the context of the special theory of relativity, length expansion is a more accurate concept than length contraction. When analyzing the path of light in a space-time diagram and examining the relationship between length and the hole-rivet, length contraction proves invalid in all scenarios. The hole-rivet paradox arises from the initial assumption of length contraction. If we discard this assumption, the paradox ceases to exist. By accepting length expansion as the correct interpretation, the hole-rivet paradox becomes easily comprehensible, transforming it from a paradox into a straightforward concept.

References

[1] HyperPysics. The bug-rivet paradox. `http://hyperphysics.phy-astr.gsu.edu/hbase/Relativ/bugrivet.html`, 2025. May 5.

[2] Kwak KyeongDo. A study on the causes of the constancy of the speed of light. *Vixra, 2107.0063*, 1:8, 2017.

[3] Kwak KyeongDo. *Length Expansion.* Amazon.com, 2018.